Frequency Responses

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Relation s $\longleftrightarrow$ $j\omega$

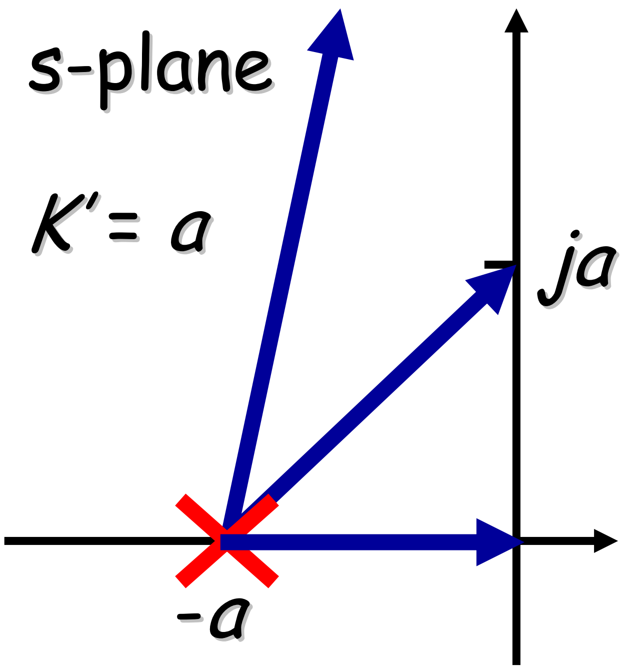
$$s = \alpha + j\omega$$

for $\alpha = 0$ $\quad s = j\omega$

Contents

- Relation $s \leftrightarrow j\omega$
- Frequency responses:
  - Nyquist (polar plot)
  - Bode
  - Nichols

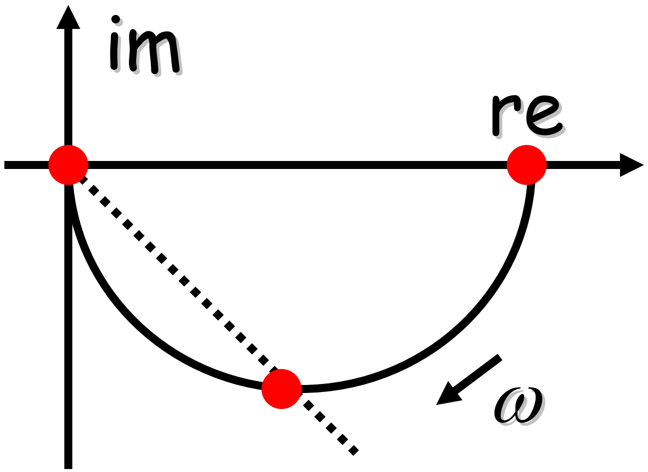
Relation s $\leftrightarrow j\omega$



$$\frac{K'}{s+a}$$

ω	$\lvert H(j\omega)\rvert$	$\arg\left(H(j\omega)\right)$
0	$\frac{a}{a}$	0
a	$\frac{a}{a\sqrt{2}}=\frac{1}{\sqrt{2}}$	$-\frac{\pi}{4}$
∞	0	$-\frac{\pi}{2}$

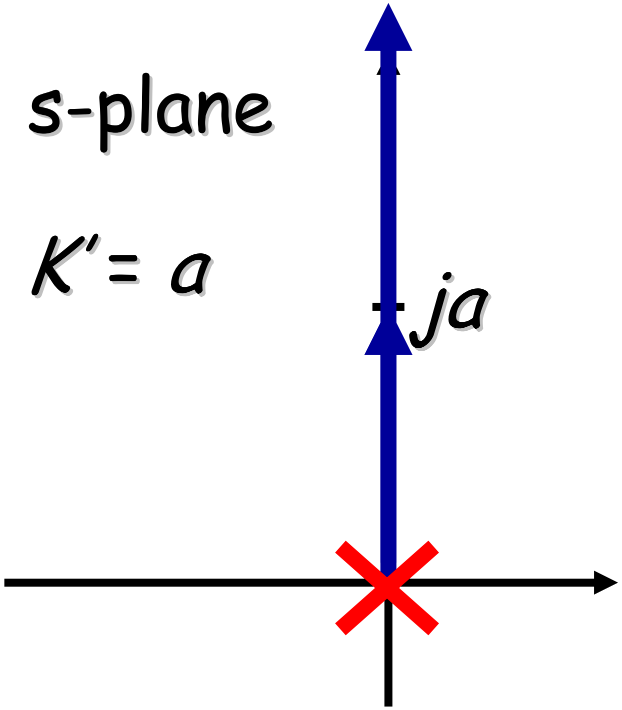
Nyquist plot (a/(jω+a))



Nyquist diagram
Polar plot

ω	$\lvert H(j\omega)\rvert$	$\arg(H(j\omega))$
0	$\frac{a}{a}=1$	0
a	$\frac{a}{a\sqrt{2}}=\frac{1}{\sqrt{2}}$	$-\frac{\pi}{4}$
∞	0	$-\frac{\pi}{2}$

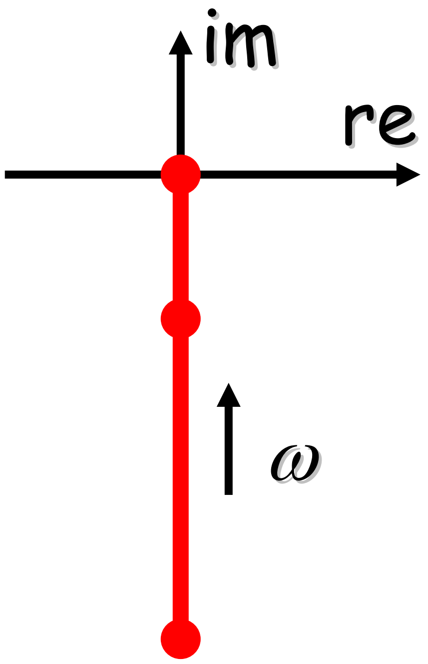
Relation s ↔ $j\omega$



$$\frac{K'}{s}$$

ω	$\lvert H(j\omega)\rvert$	$\arg\left(H(j\omega)\right)$
0	$\frac{a}{0} \to \infty$	$-\frac{\pi}{2}$
a	$\frac{a}{a} = 1$	$-\frac{\pi}{2}$
∞	0	$-\frac{\pi}{2}$

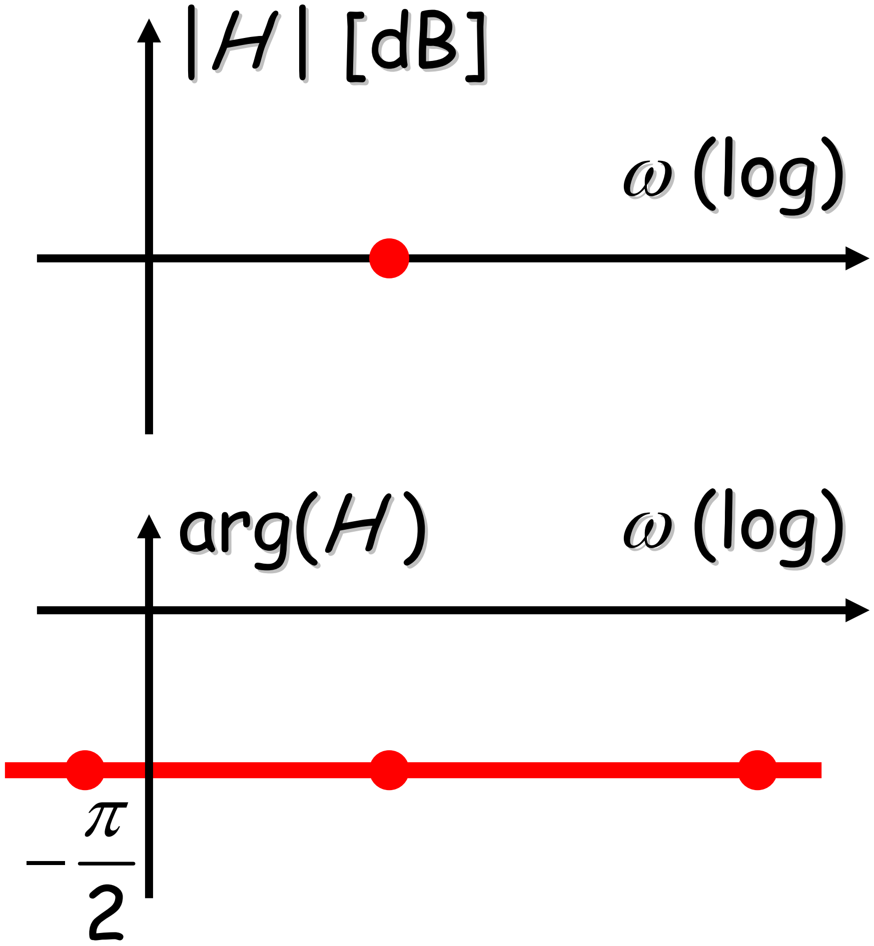
Nyquist plot



Nyquist diagram
Polar plot

ω	$\lvert H(j\omega)\rvert$	$\arg(H(j\omega))$
0	$\frac{a}{0} \to \infty$	$-\frac{\pi}{2}$
a	$\frac{a}{a} = 1$	$-\frac{\pi}{2}$
∞	0	$-\frac{\pi}{2}$

Bode plot (a/jω)



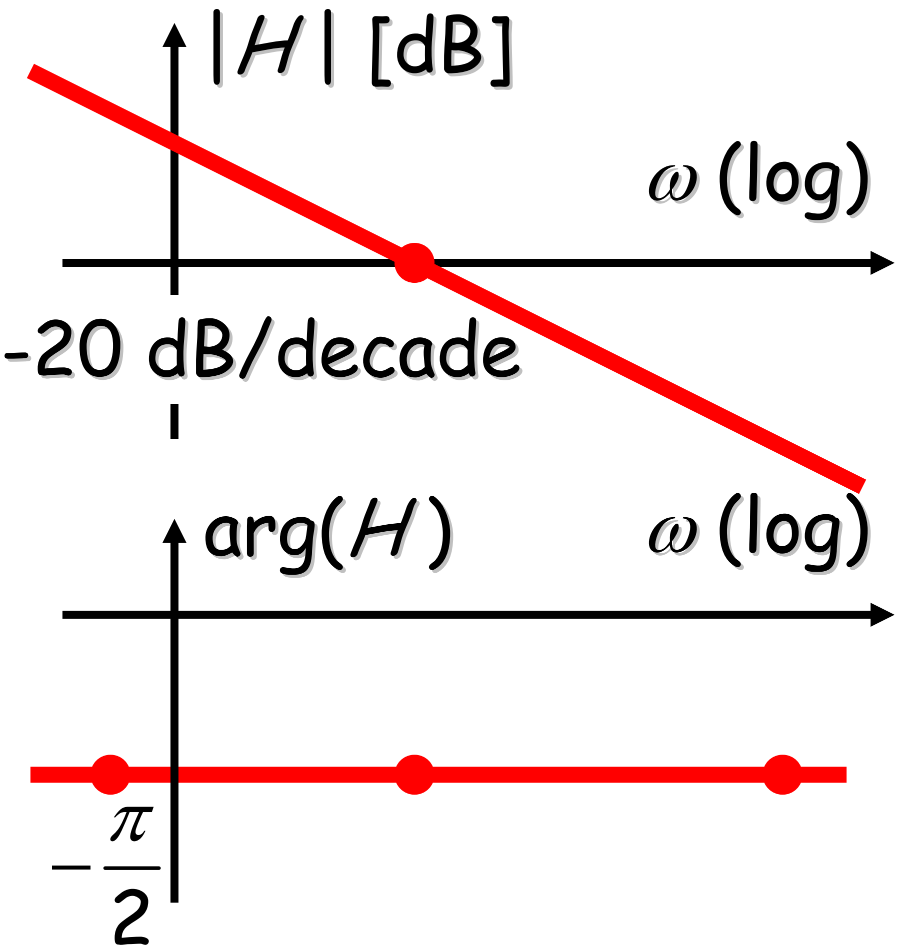
ω	$\lvert H(j\omega)\rvert$	$\arg(H(j\omega))$
$\omega \to 0$	$\frac{a}{0} \to \infty$	$-\frac{\pi}{2}$
a	$\frac{a}{a} = 1$	$-\frac{\pi}{2}$
∞	0	$-\frac{\pi}{2}$

Bode diagram:

$20\,{}^{10}\log|H|$ versus ω(log)

$\arg(H)$ versus ω(log)

Bode plot ($a/j\omega$)

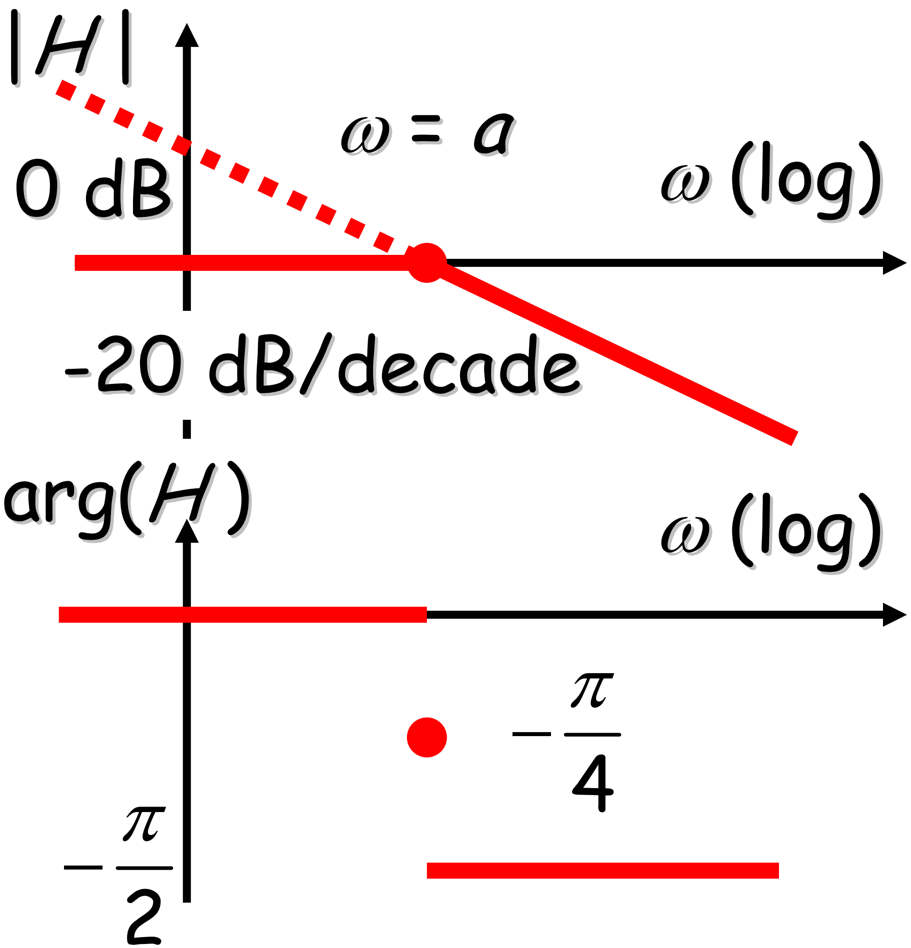


If ω ten times larger

$a/j\omega$ ten times smaller:

straight line with a slope
of $-20\,{}^{10}\log(10)$ =
-20 dB/decade
or -6dB/octave

Bode plot (a/(jω+a))



$$H(j\omega) = \frac{a}{j\omega + a}$$

asymptotic approximation:

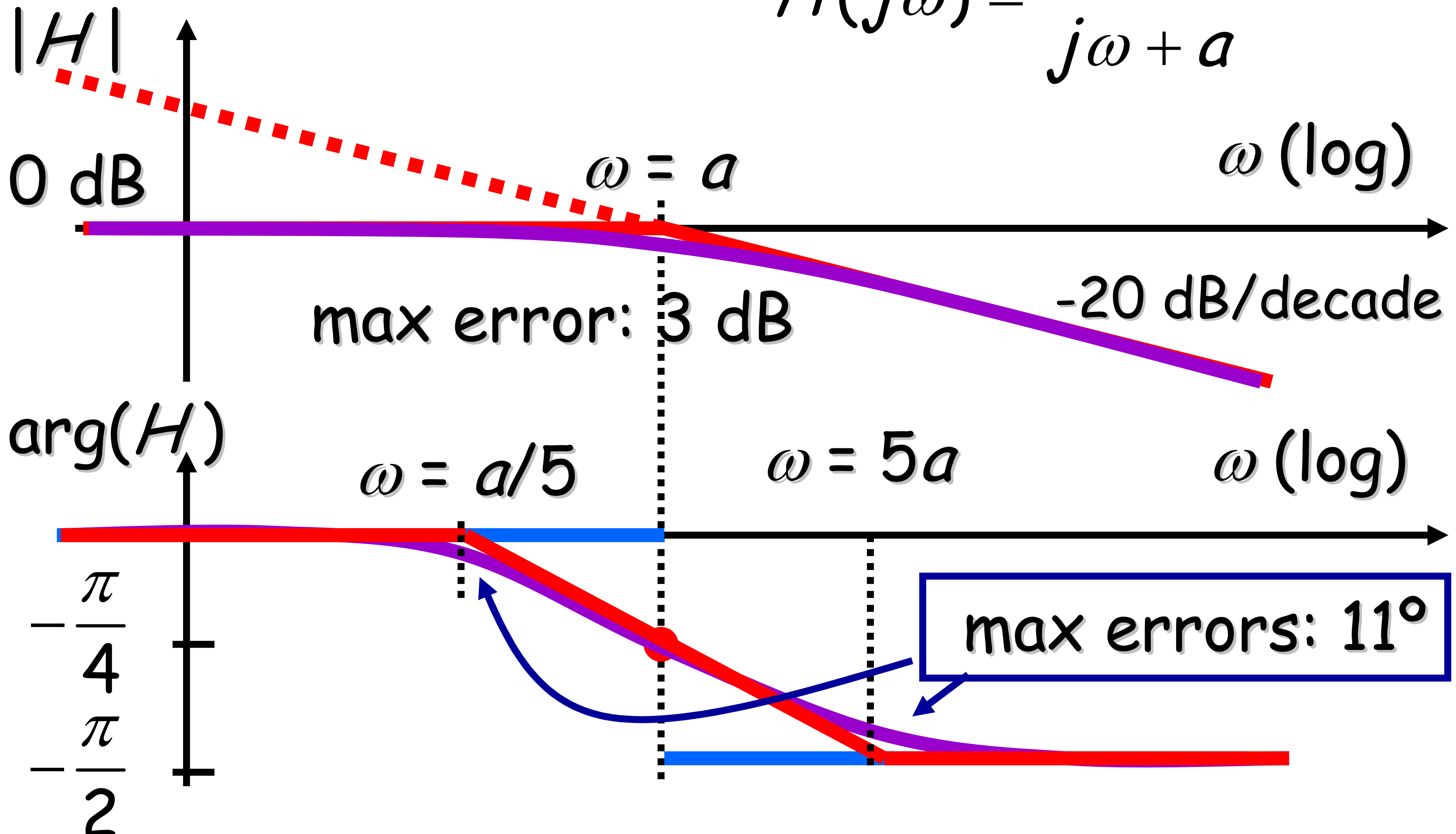
$$\omega < a: \quad H(j\omega) = \frac{a}{a}$$

$$\omega > a: \quad H(j\omega) = \frac{a}{j\omega}$$

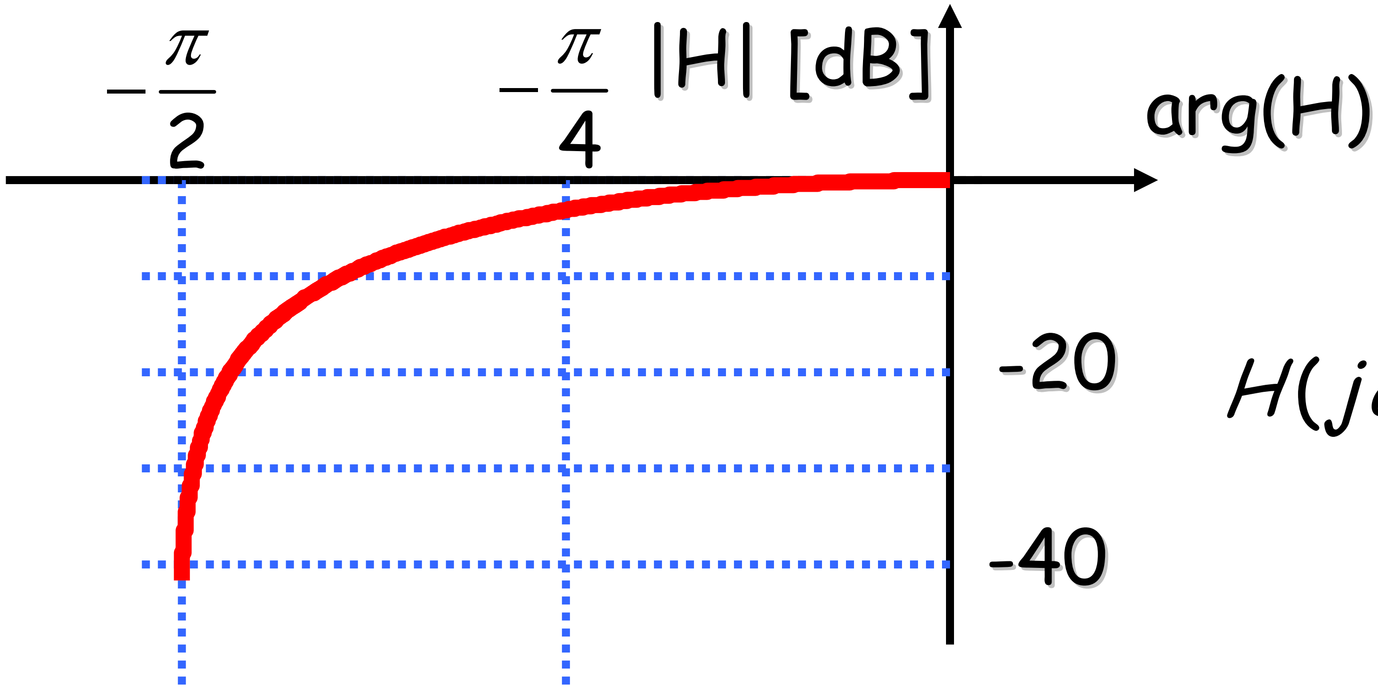
$$\omega = a: \quad H(j\omega) = \frac{a}{j\omega + a}$$

Bode plot (a/(jω +a))

$$H(j\omega) = \frac{a}{j\omega + a}$$



Nichols diagram



$$H(j\omega) = \frac{a}{j\omega + a}$$

Demo 20-sim

First-order system

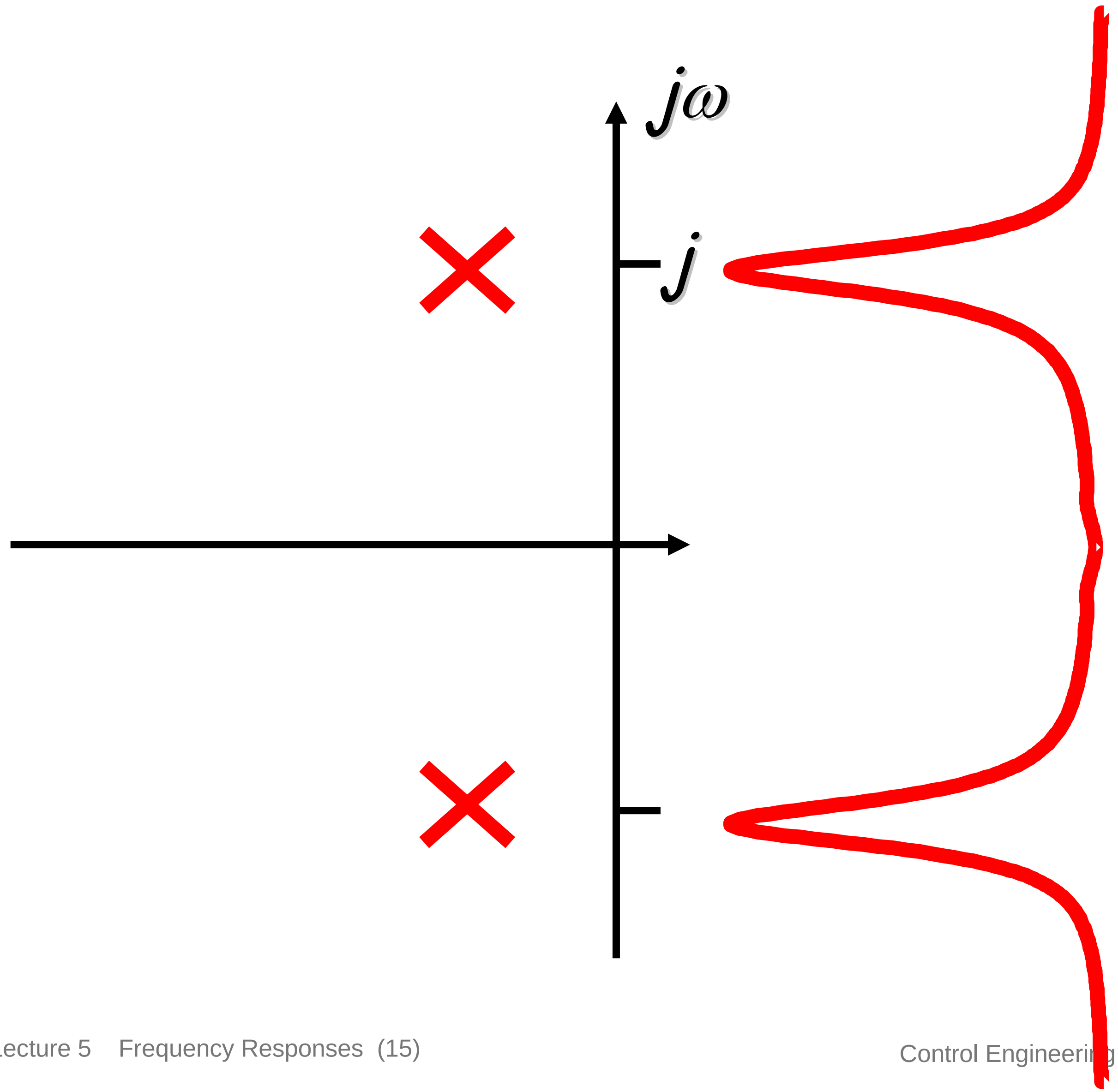
- Nyquist
- Bode
- Nichols

20-sim
demo

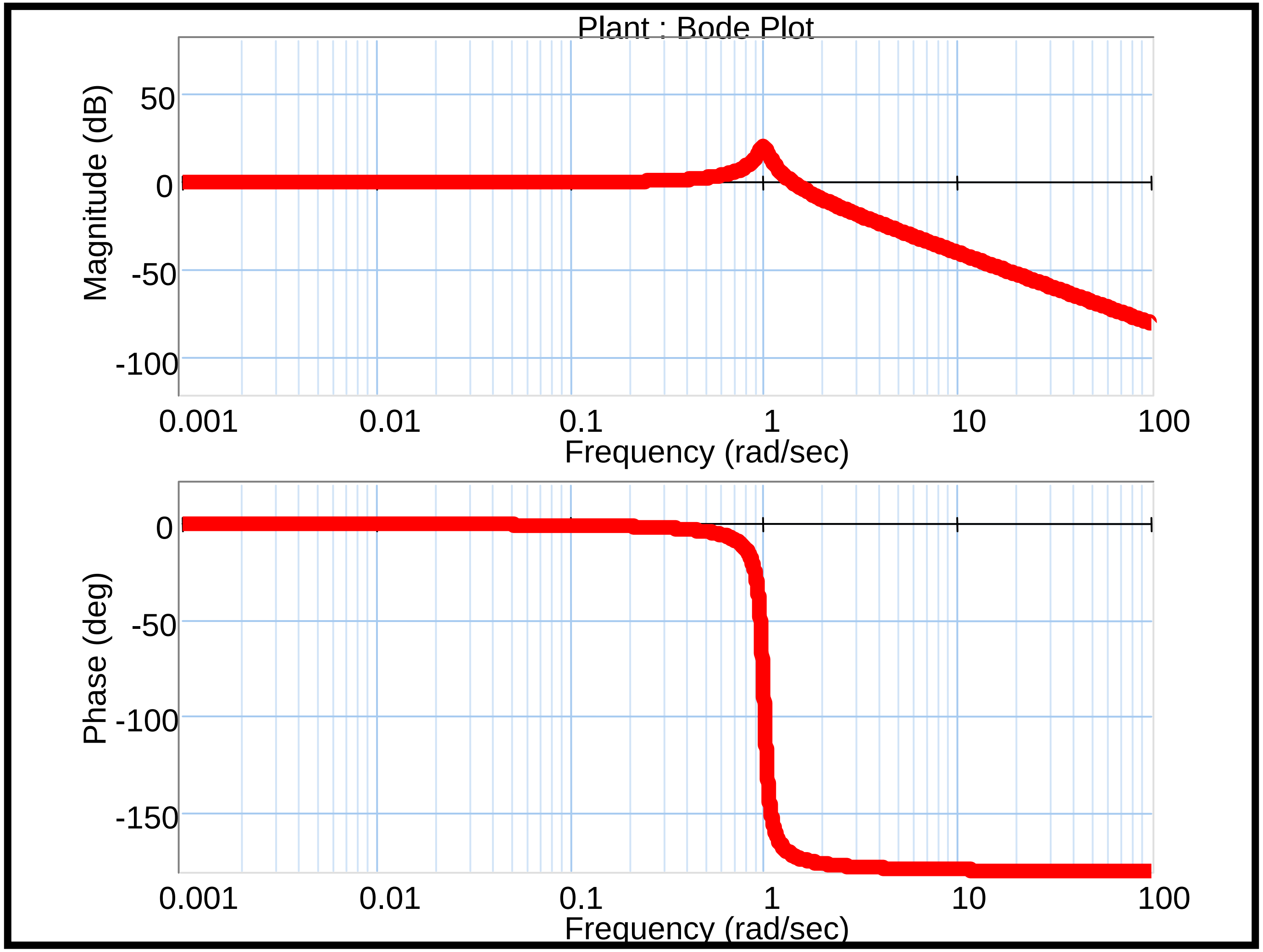
Rubber membrane model

- s-plane can be seen as a rubber membrane
- poles are needles under the membrane
- zeros are push pins in the membrane
- push pins in infinity
- $H(j\omega)$ is a cross cut through the membrane at the $j\omega$ axis

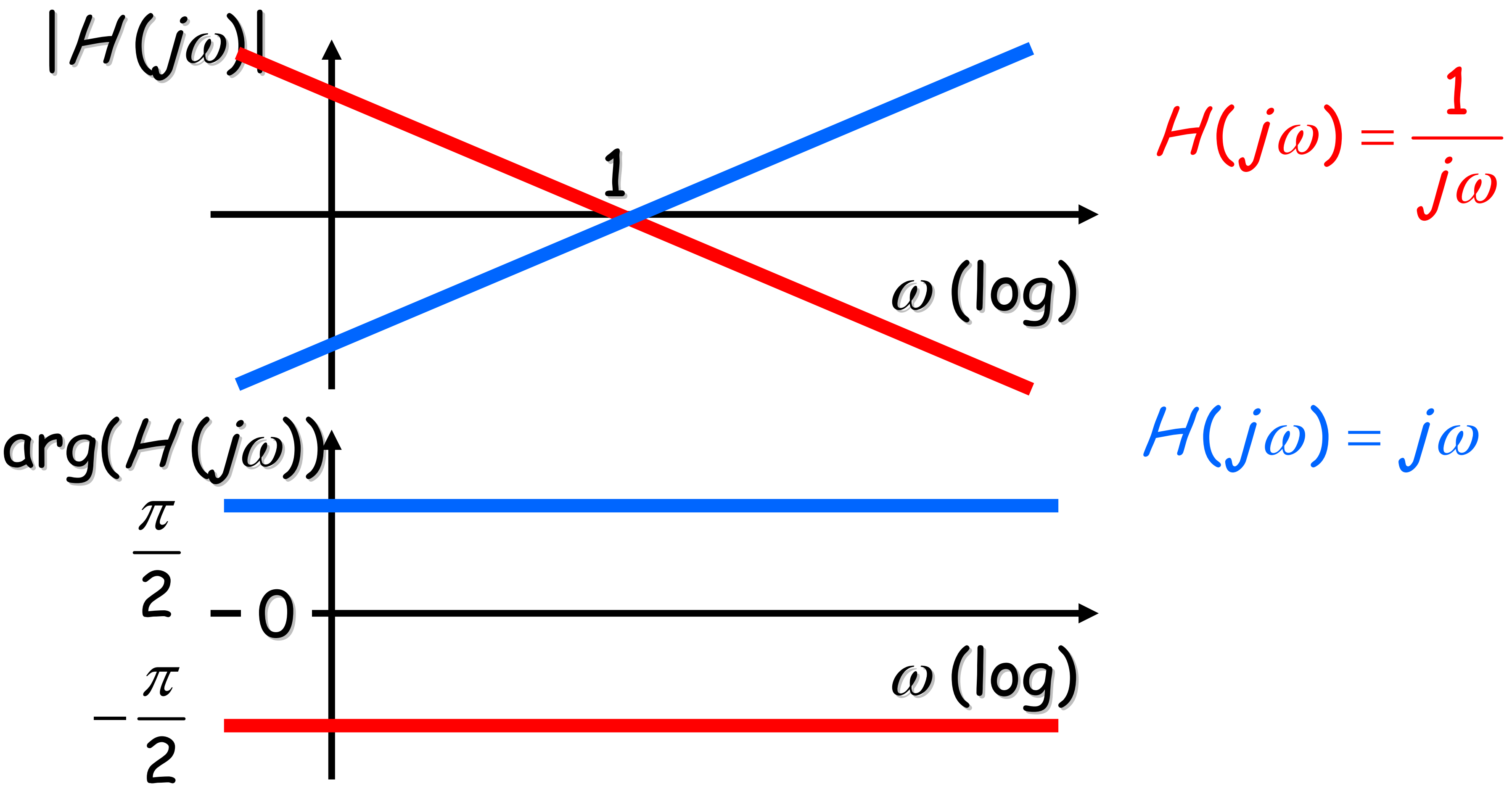
Complex poles



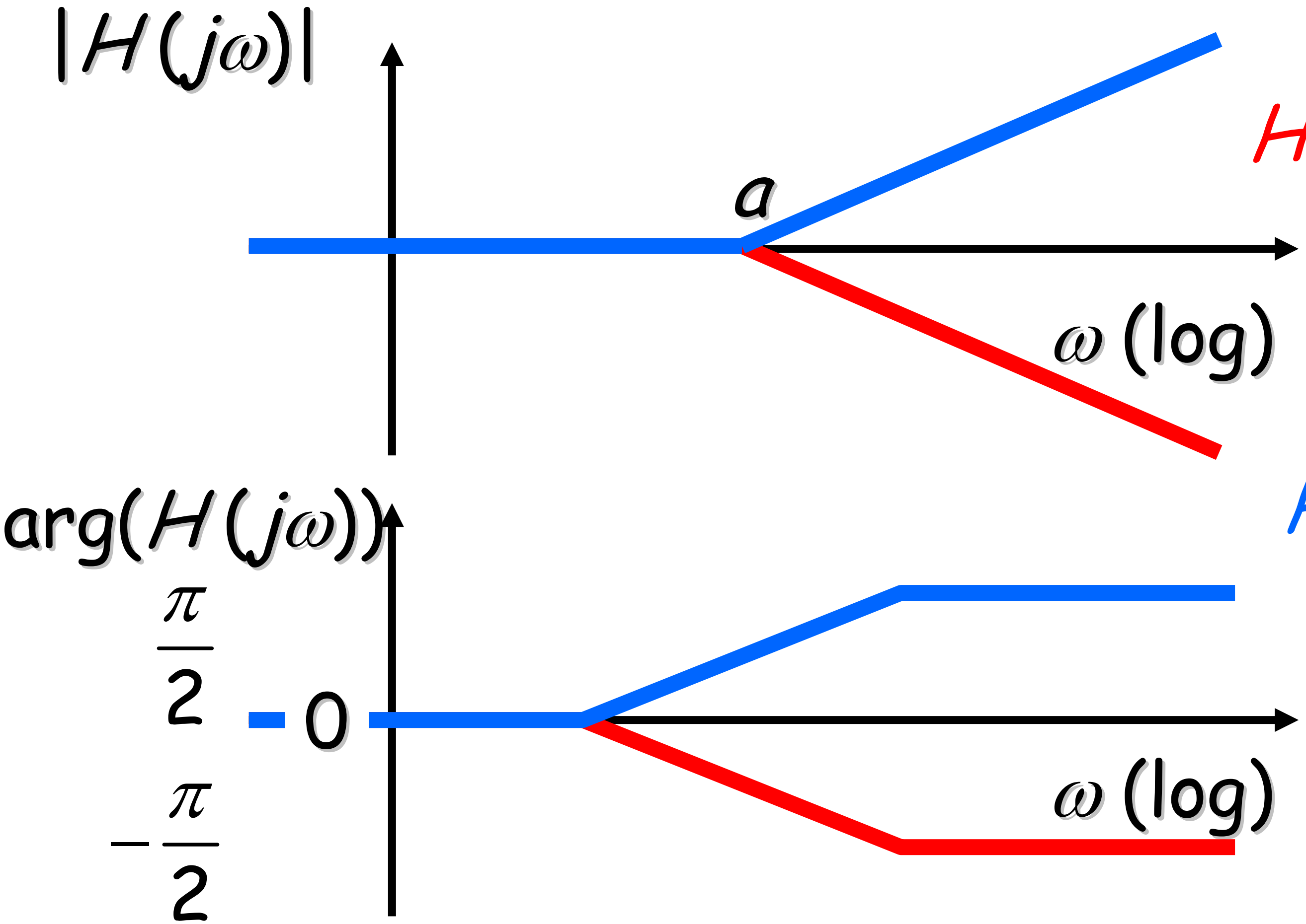
Bode plot (complex poles)



Basic elements ($j\omega$)



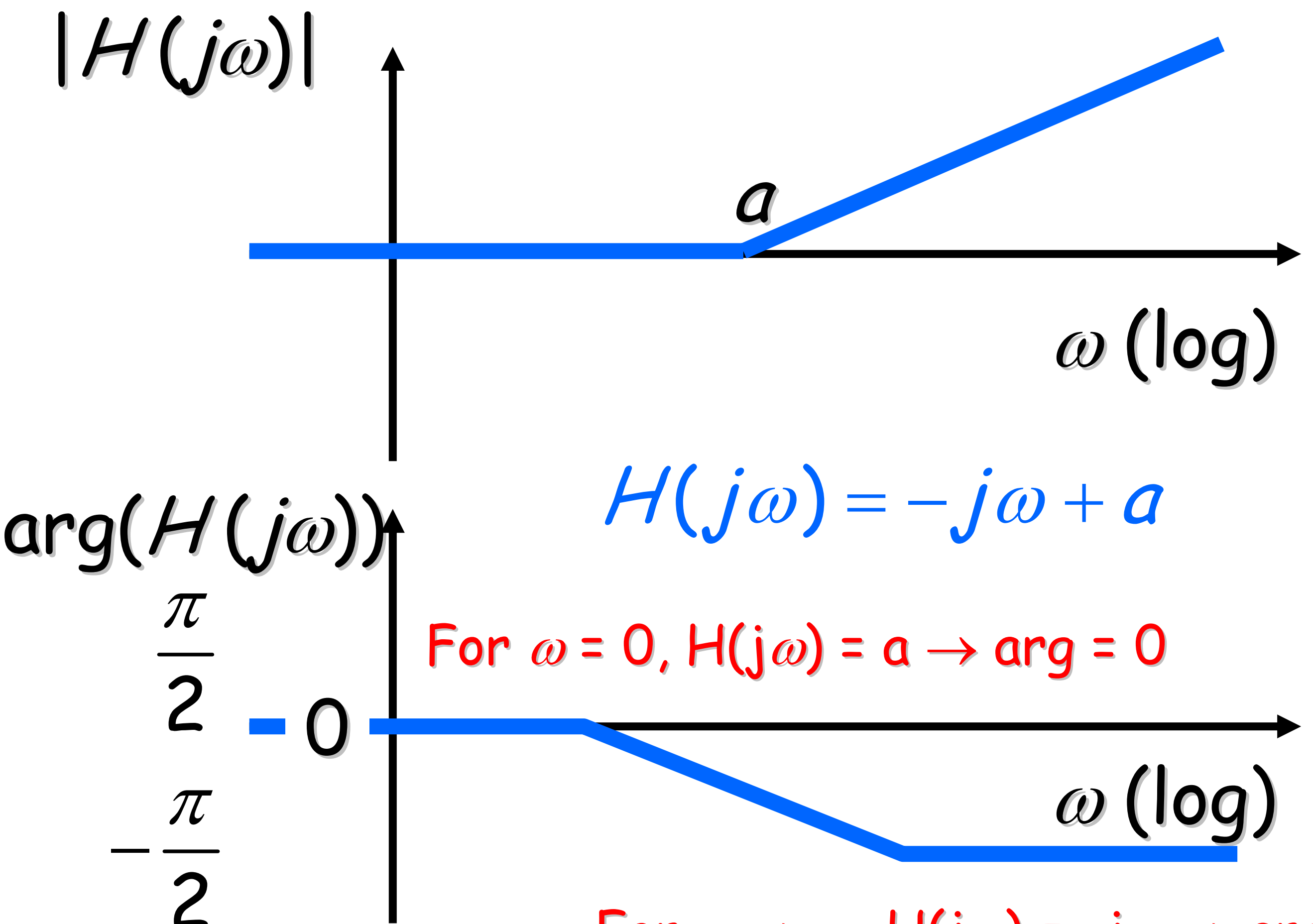
Basic elements ($j\omega + a$)

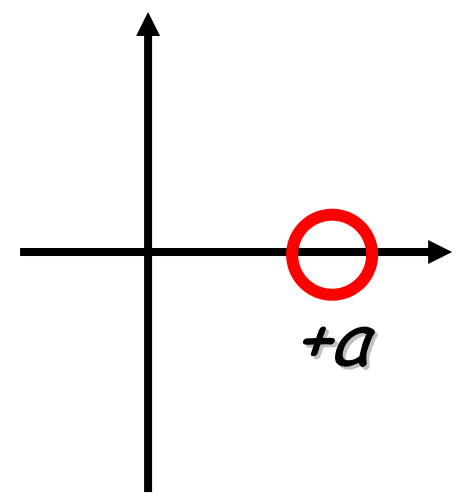


$$H(j\omega) = \frac{a}{j\omega + a}$$

$$H(j\omega) = \frac{j\omega + a}{a}$$

Basic elements (*-jω+a*)



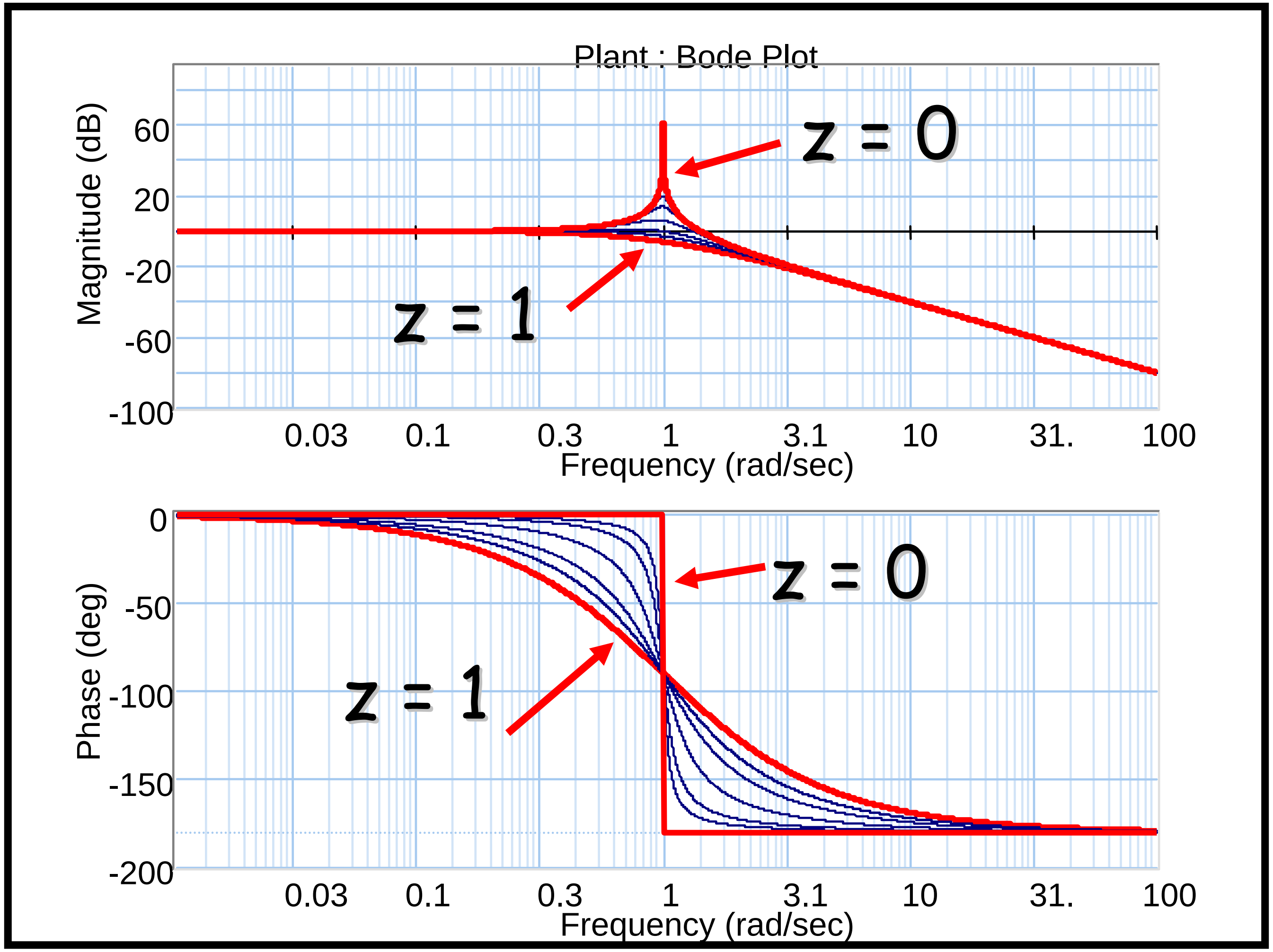


$$H(j\omega) = -j\omega + a$$

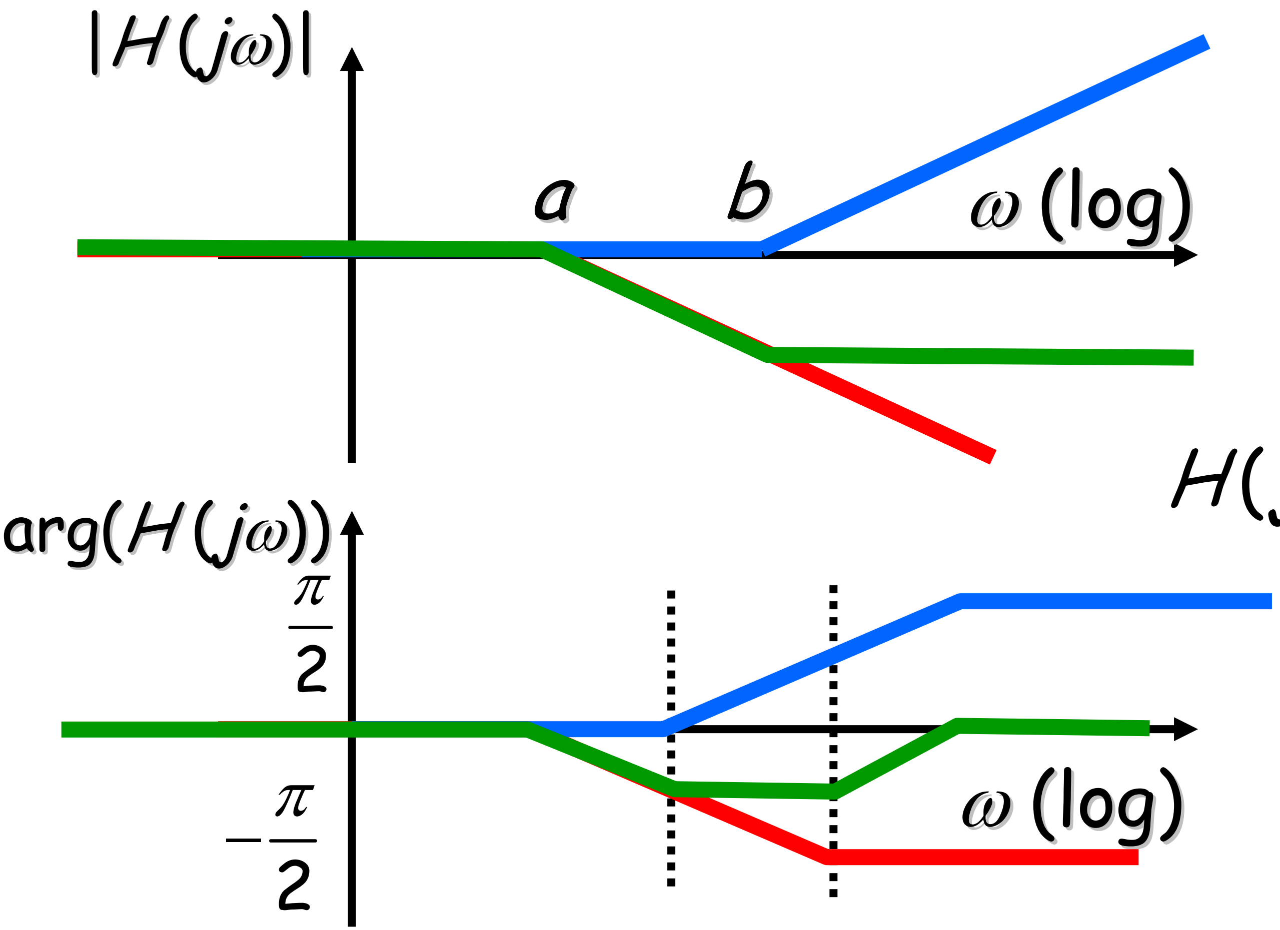
For $\omega = 0$, $H(j\omega) = a \rightarrow \arg = 0$

For $\omega \rightarrow \infty$, $H(j\omega) = -j\omega \rightarrow \arg = -\pi/2$

Basic elements (2nd order)



Combinations

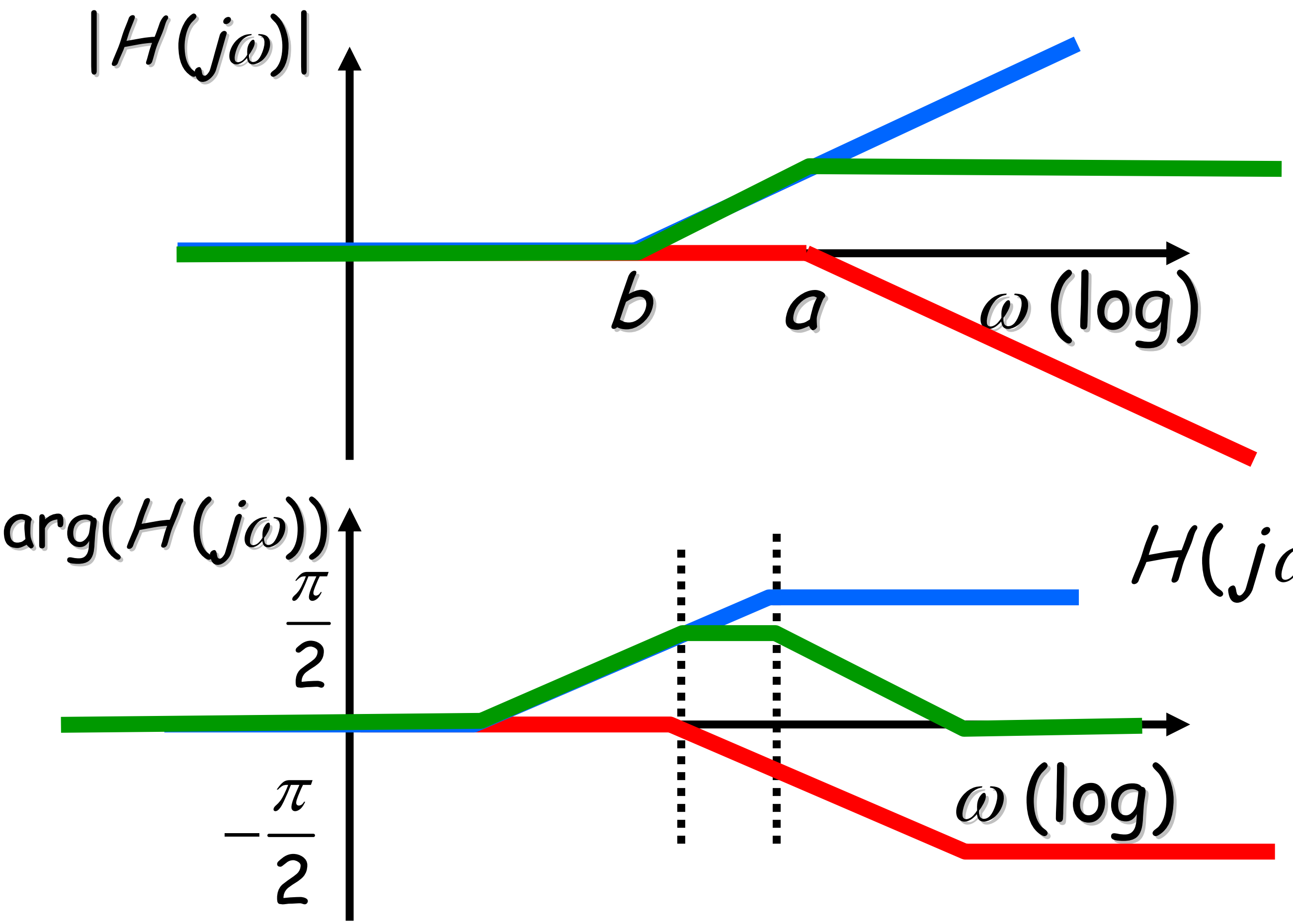


Lag network

$$H(j\omega) = \frac{a}{b}\frac{j\omega + b}{j\omega + a}$$

$$a < b$$

Combinations



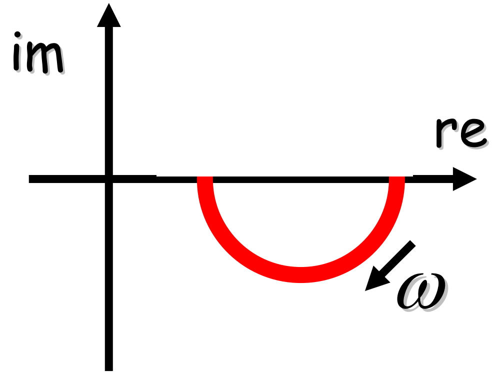
Lead network

$$H(j\omega) = \frac{a}{b}\frac{j\omega + b}{j\omega + a}$$

$$b < a$$

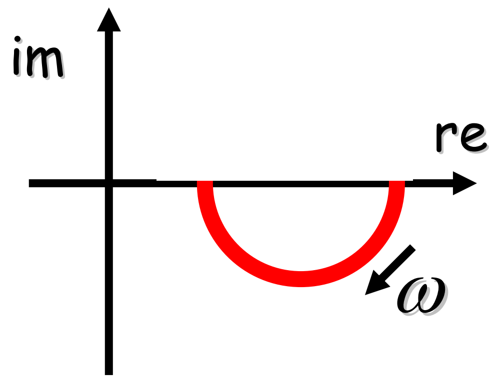
Relation s ↔ $j\omega$

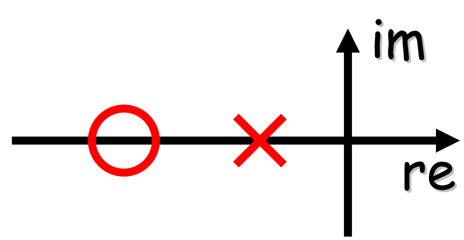
- Relation s ↔ $j\omega$ for lag and lead networks



Relation s ↔ $j\omega$

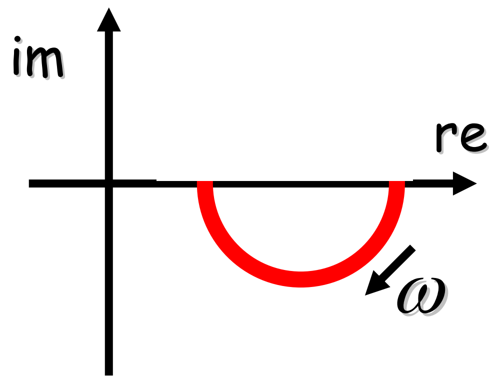
- Relation s ↔ $j\omega$ for lag and lead networks

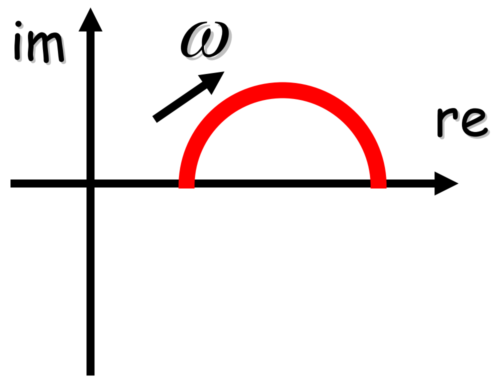


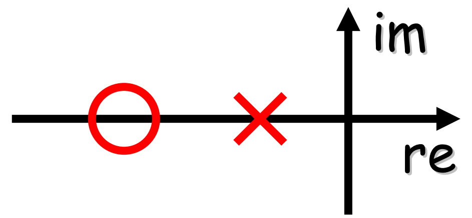


Relation s ↔ $j\omega$

- Relation s ↔ $j\omega$ for lag and lead networks

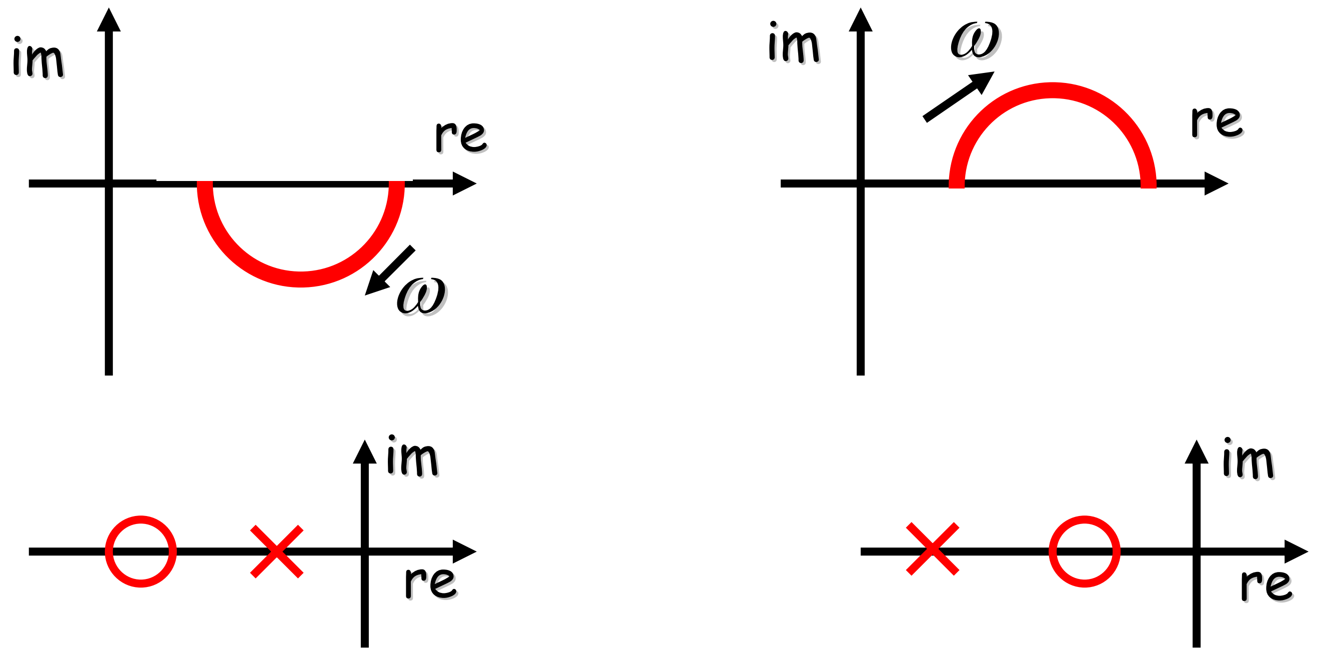




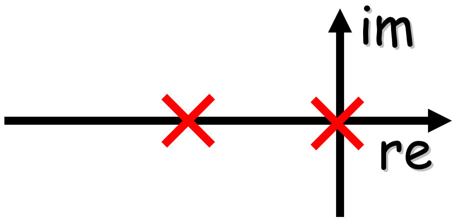


Relation s ↔ $j\omega$

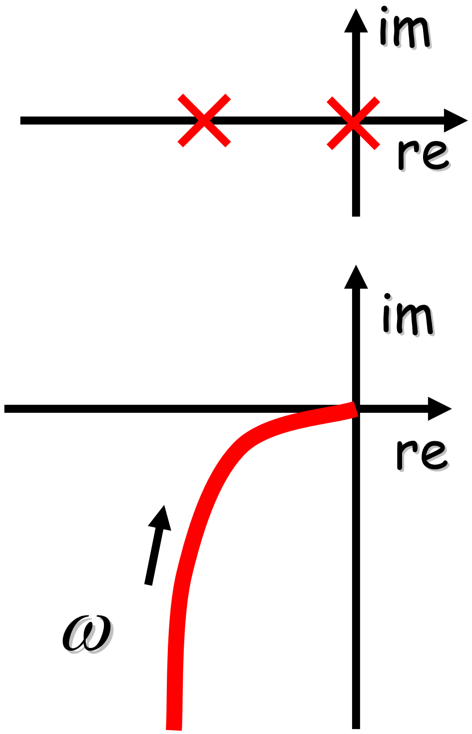
- Relation s ↔ $j\omega$ for lag and lead networks



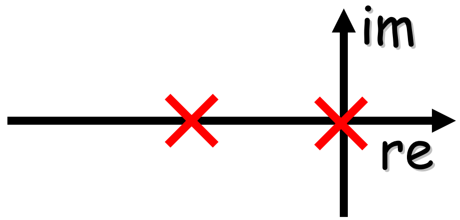
Relation s ↔ $j\omega$

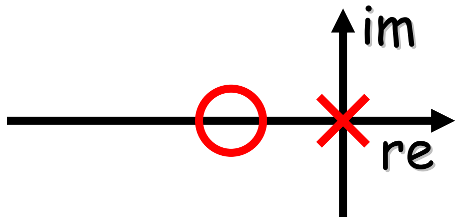


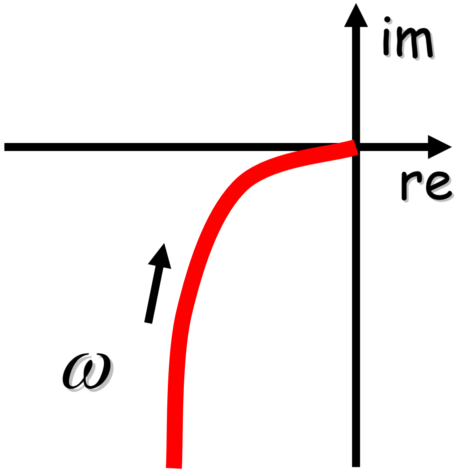
Relation s ↔ $j\omega$



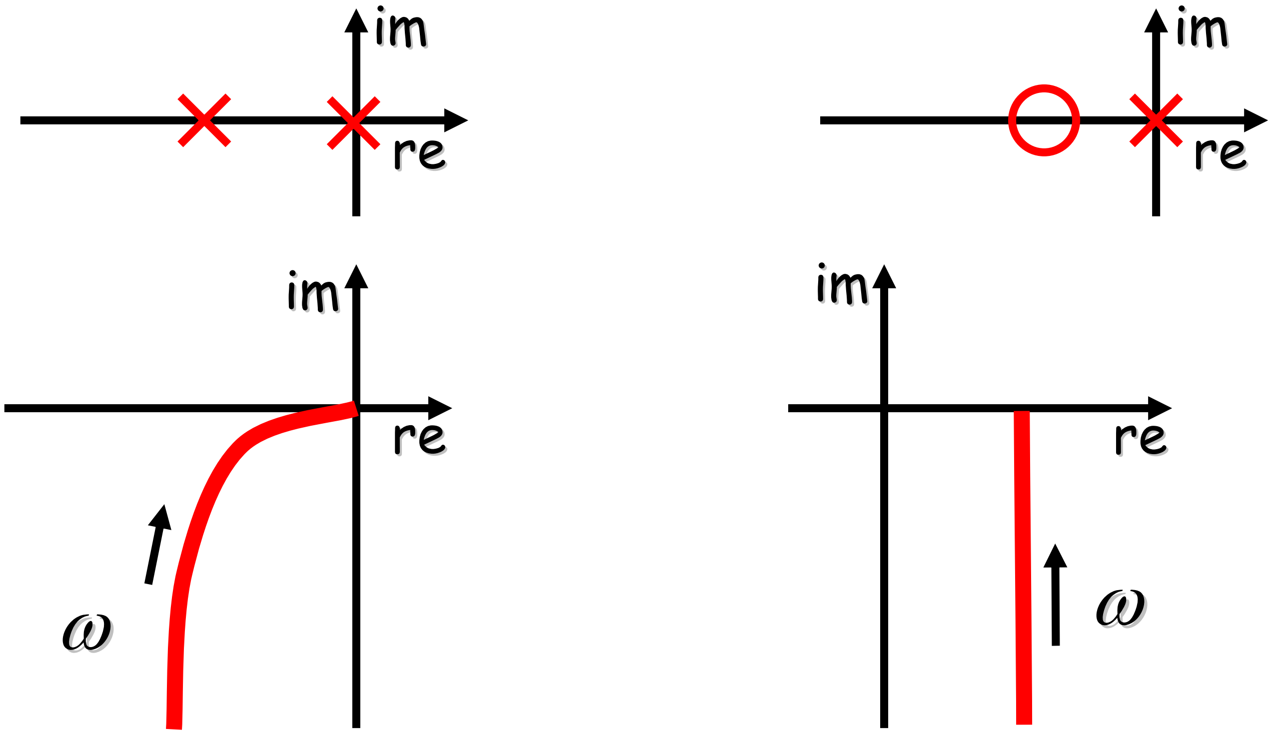
Relation s $\leftrightarrow j\omega$



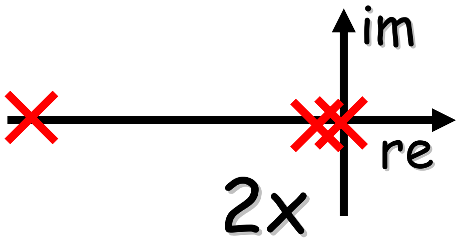




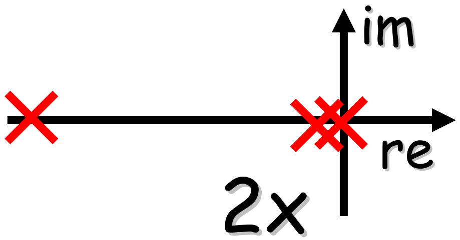
Relation s ↔ *jω*

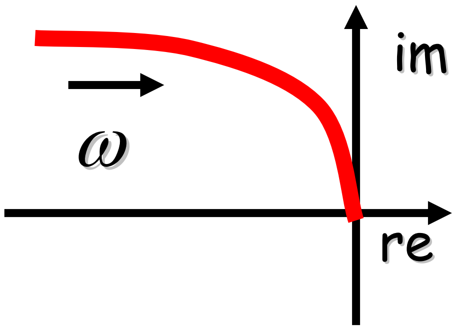


Relation s ↔ $j\omega$



Relation s ↔ *jω*





Relation s $\leftrightarrow j\omega$

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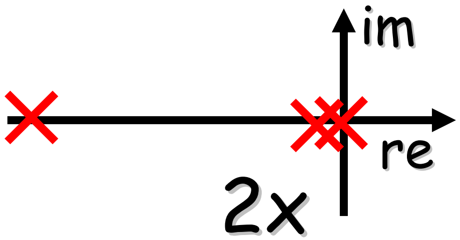
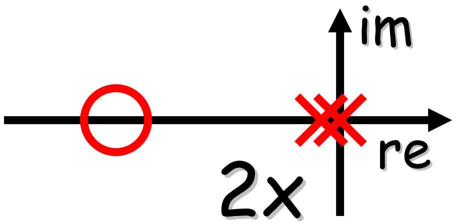
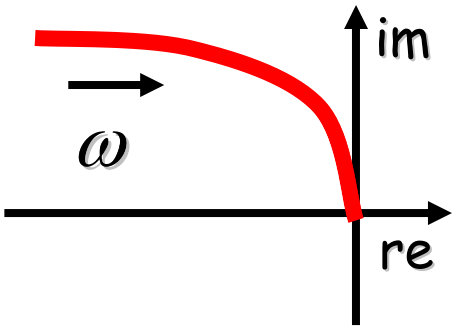
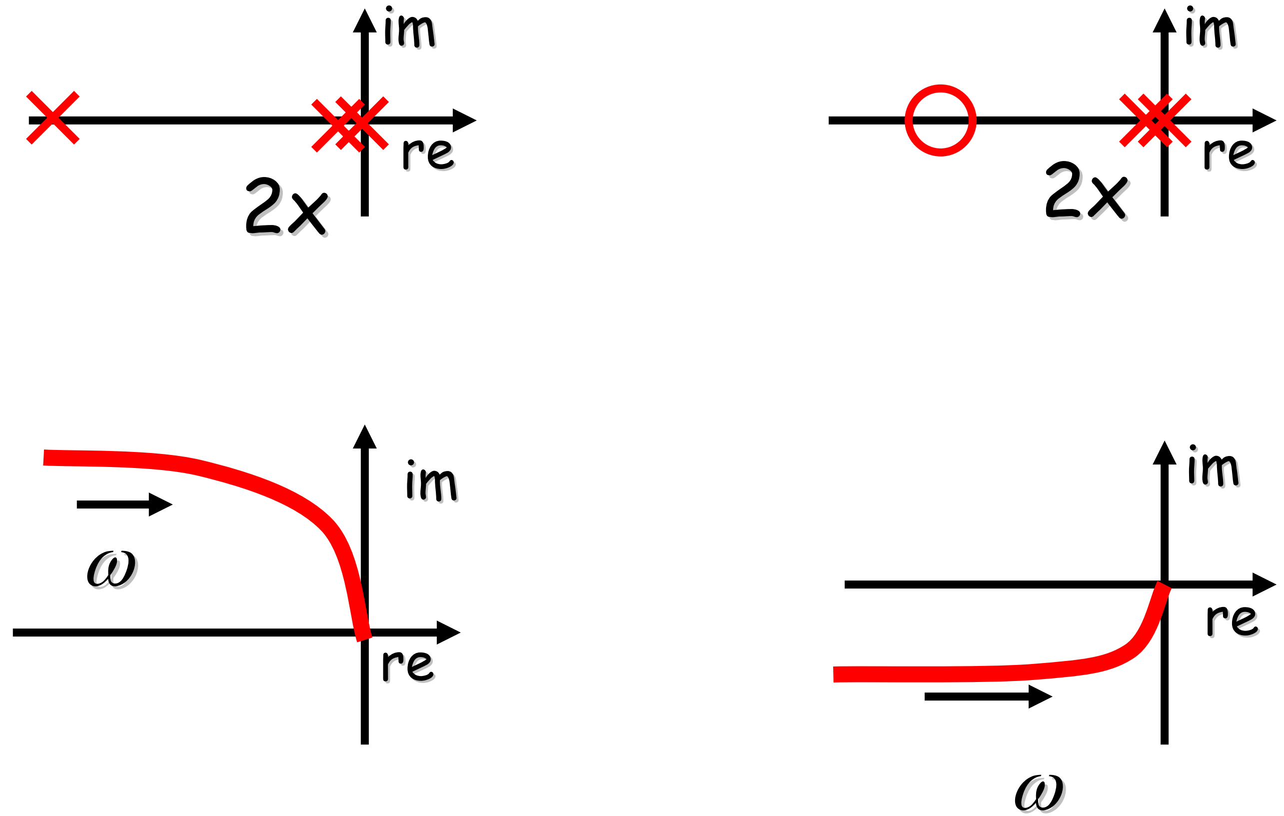






Relation s $\leftrightarrow j\omega$



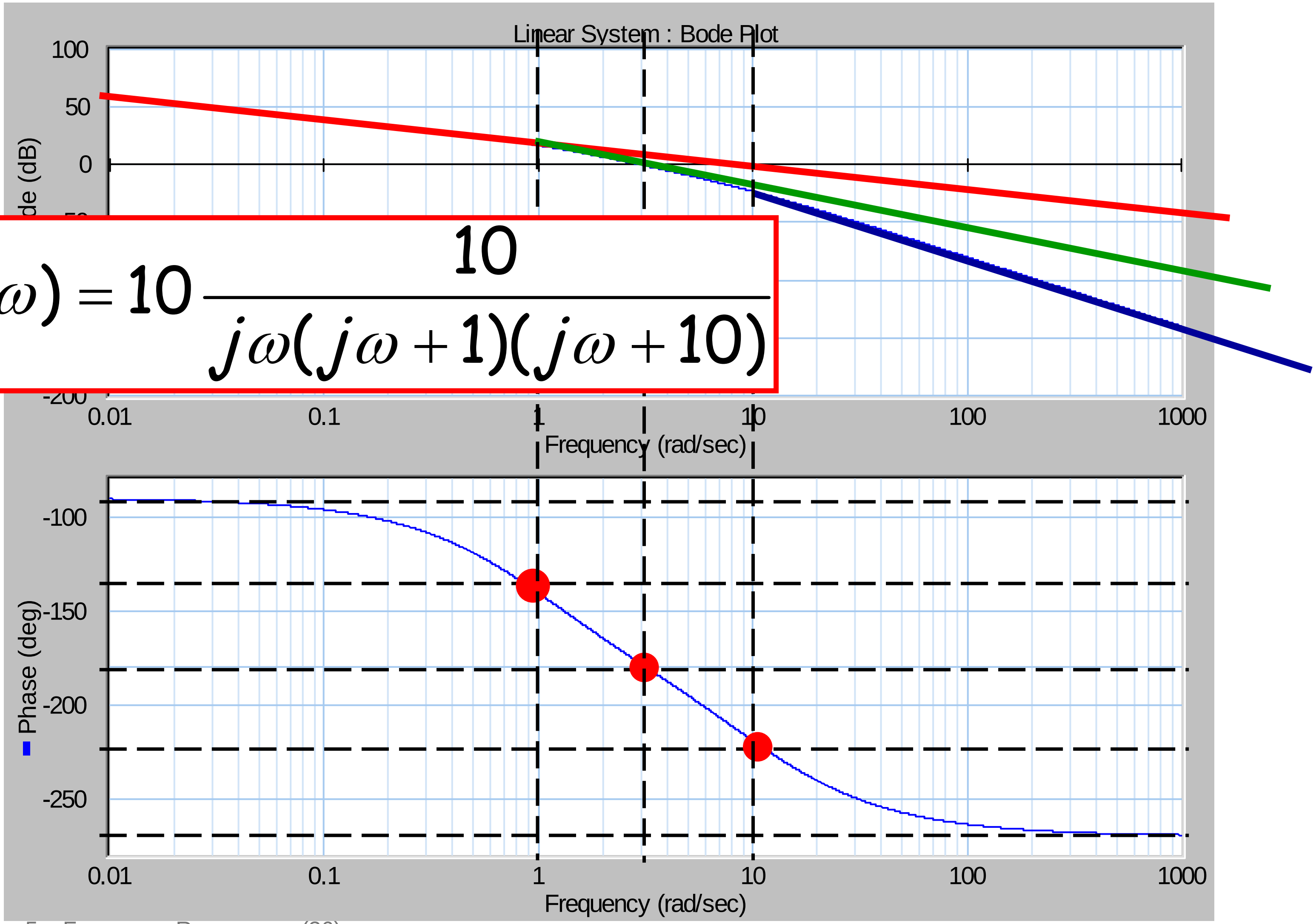
Demo's 20-sim

- lag network
  - bode
  - nyquist
  - nichols

20-sim
demo

- lead network
  - bode
  - nyquist
  - nichols

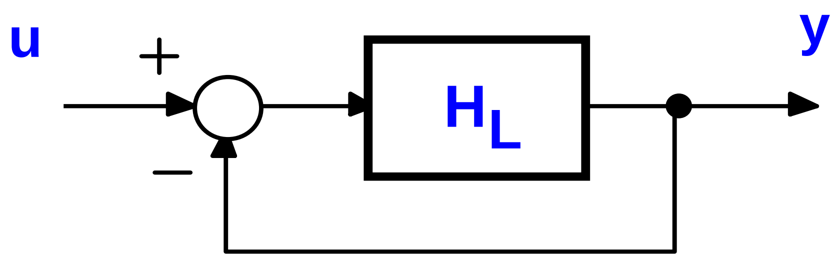
20-sim
demo



$$H(j\omega) = 10\frac{10}{j\omega(j\omega+1)(j\omega+10)}$$

Stability

We consider the following feedback system



The system is on the border of instability when:

$$|H_L(j\omega)| = 1 \text{ and } \arg\left(H_L(j\omega)\right) = -\pi$$

$$\text{or } H_L(j\omega) = 1e^{-j\pi}$$

Stability

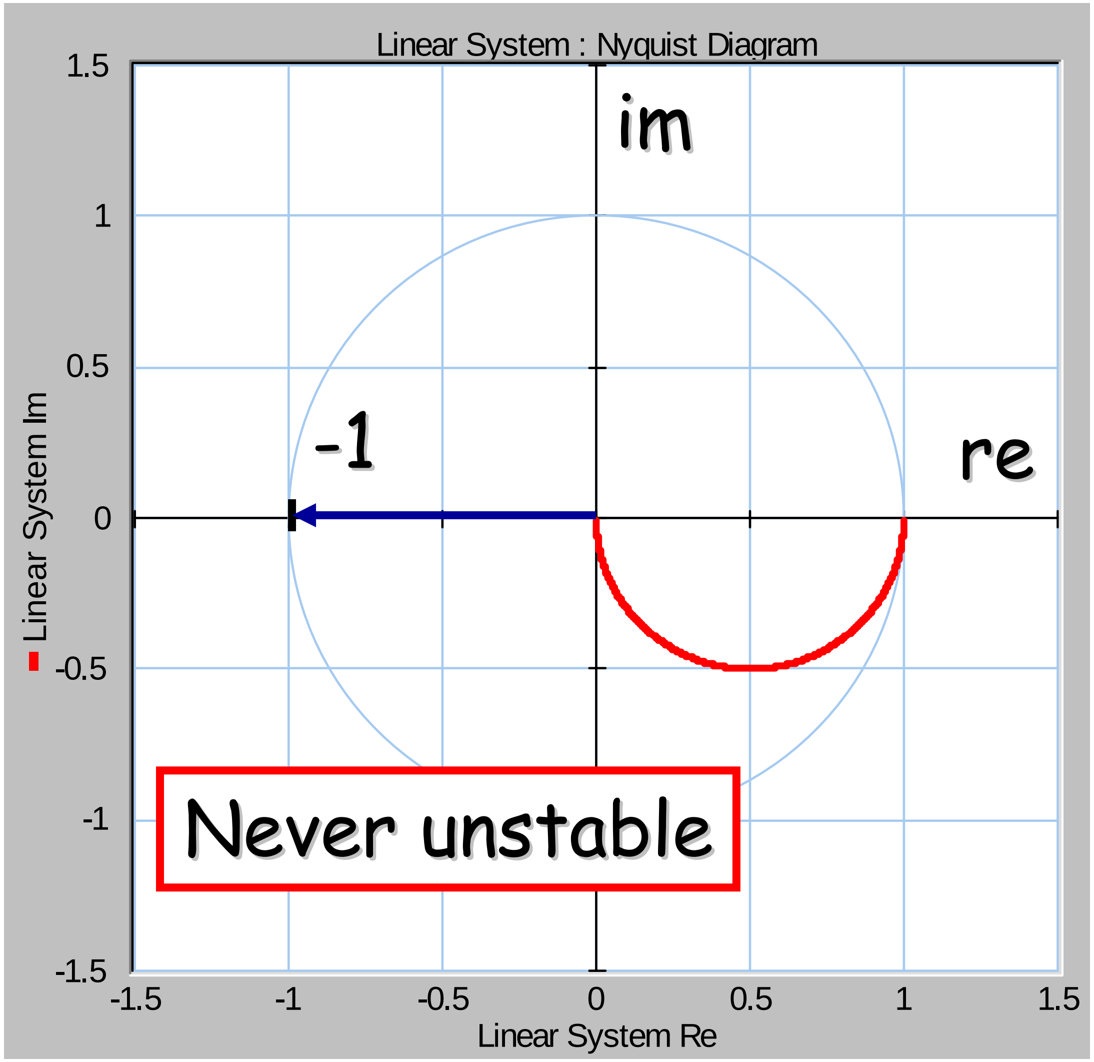
$$H(j\omega) = \frac{1}{(j\omega + 1)}$$

vector $= 1e^{-j\pi}$

if $H(j\omega) = 1e^{-j\pi}$

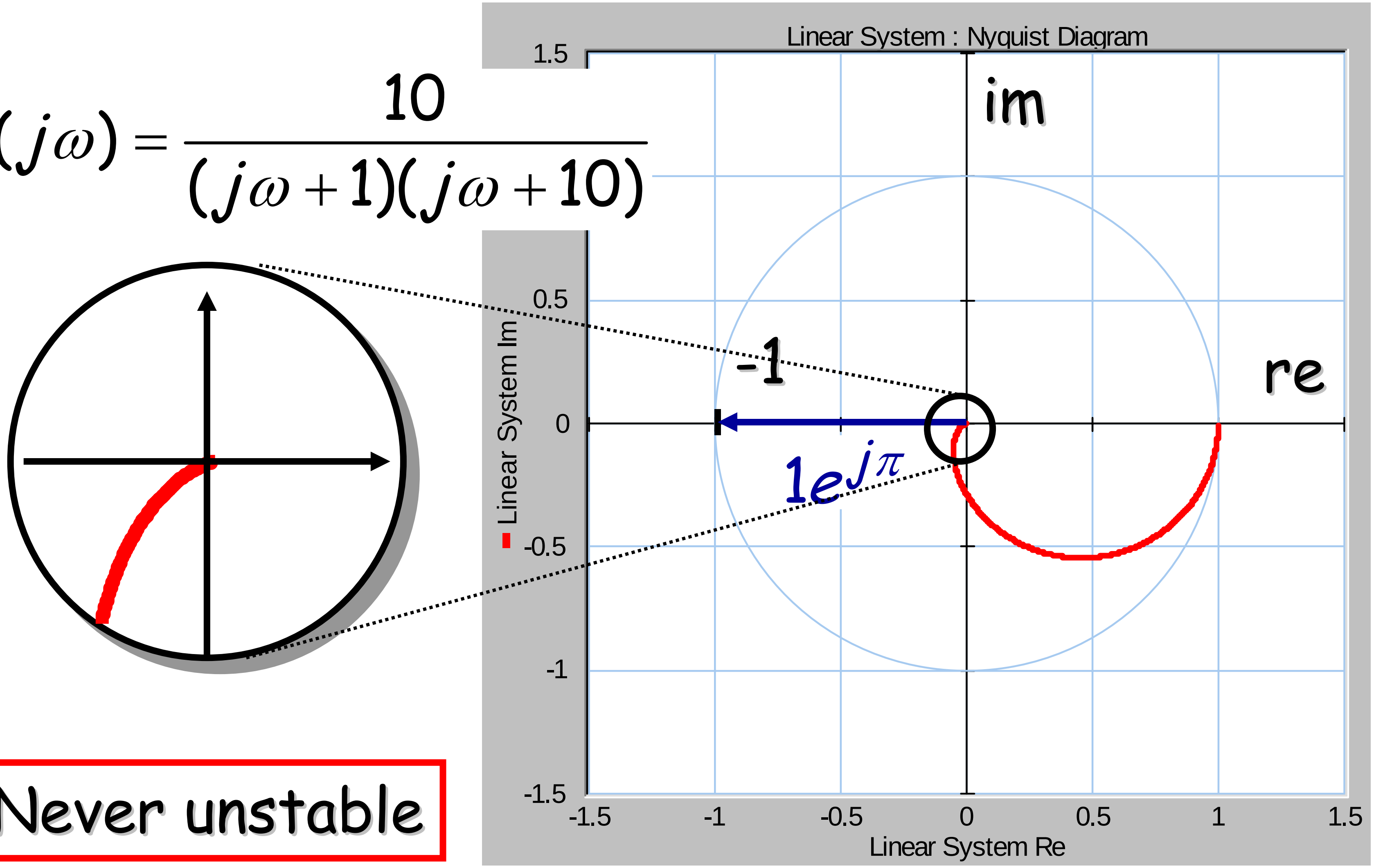
closed loop system

on border of stability



Stability

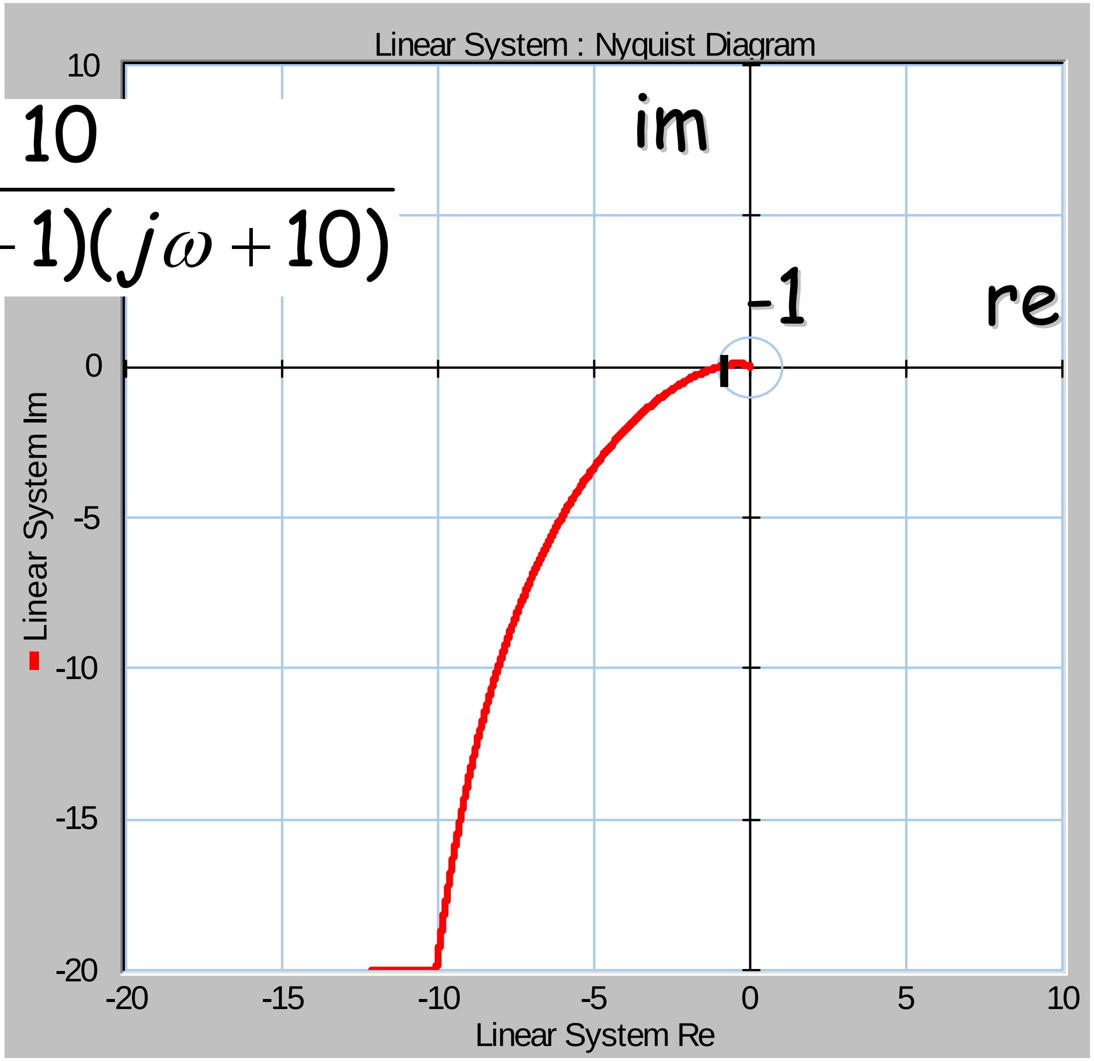
$$H(j\omega) = \frac{10}{(j\omega+1)(j\omega+10)}$$



Never unstable

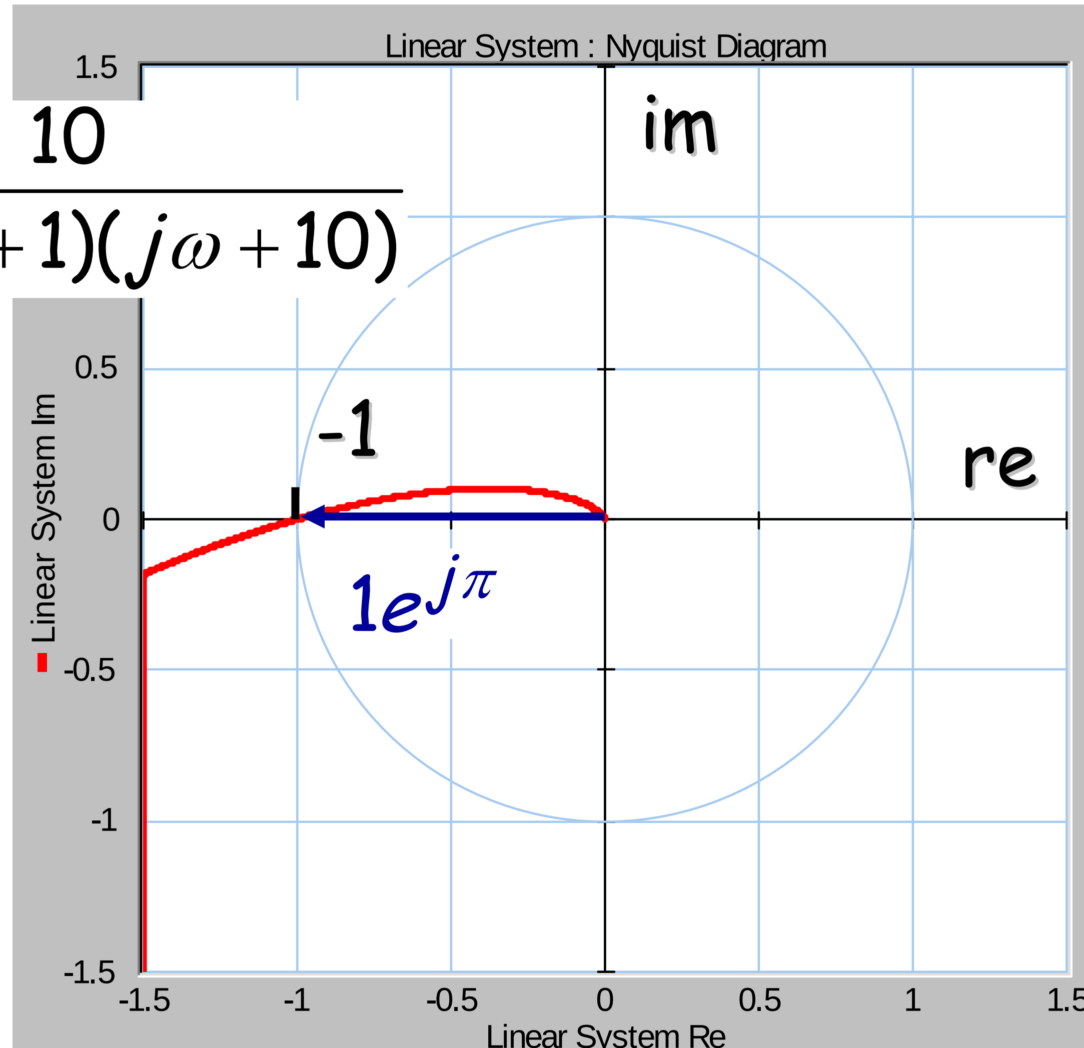
Stability

$$H(j\omega) = 11\frac{10}{j\omega(j\omega+1)(j\omega+10)}$$



Stability

$$H(j\omega) = 11\frac{10}{j\omega(j\omega+1)(j\omega+10)}$$



Border of instability

Compare

Root loci (34)

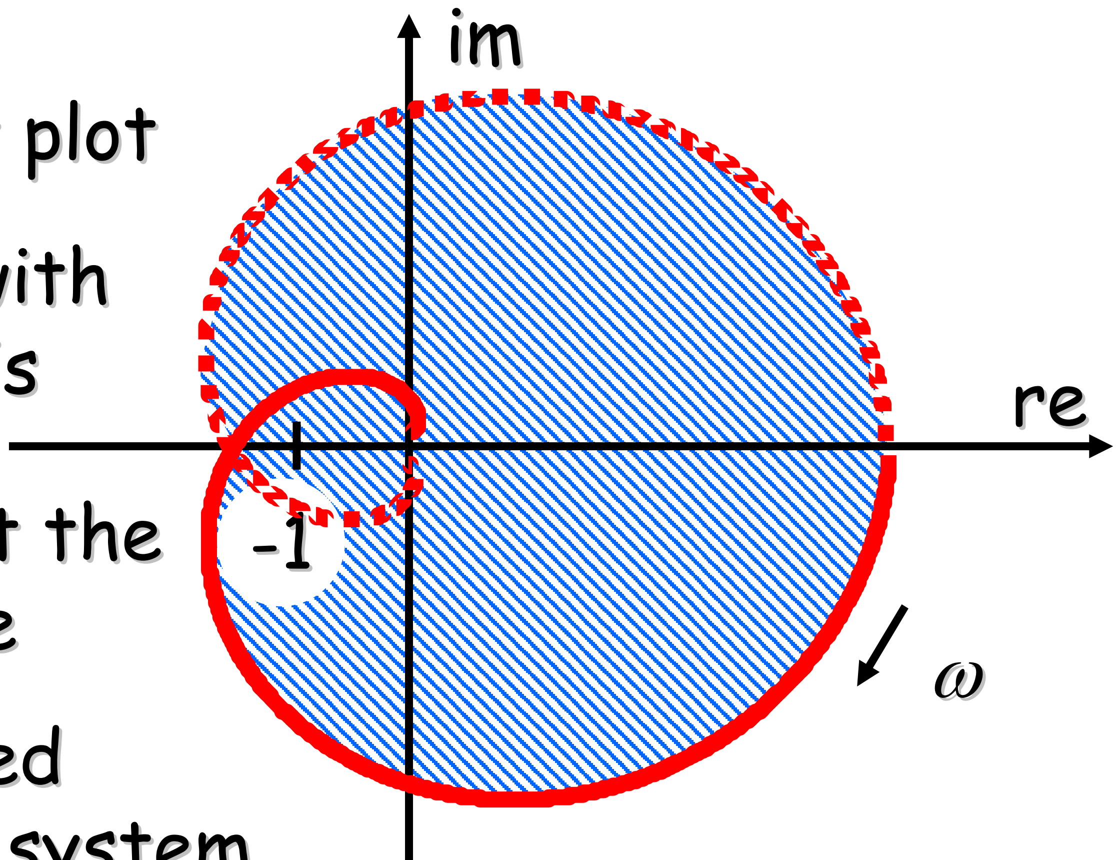
-1 Stability criterion

Draw the Nyquist plot

Mirror the plot with respect to re-axis

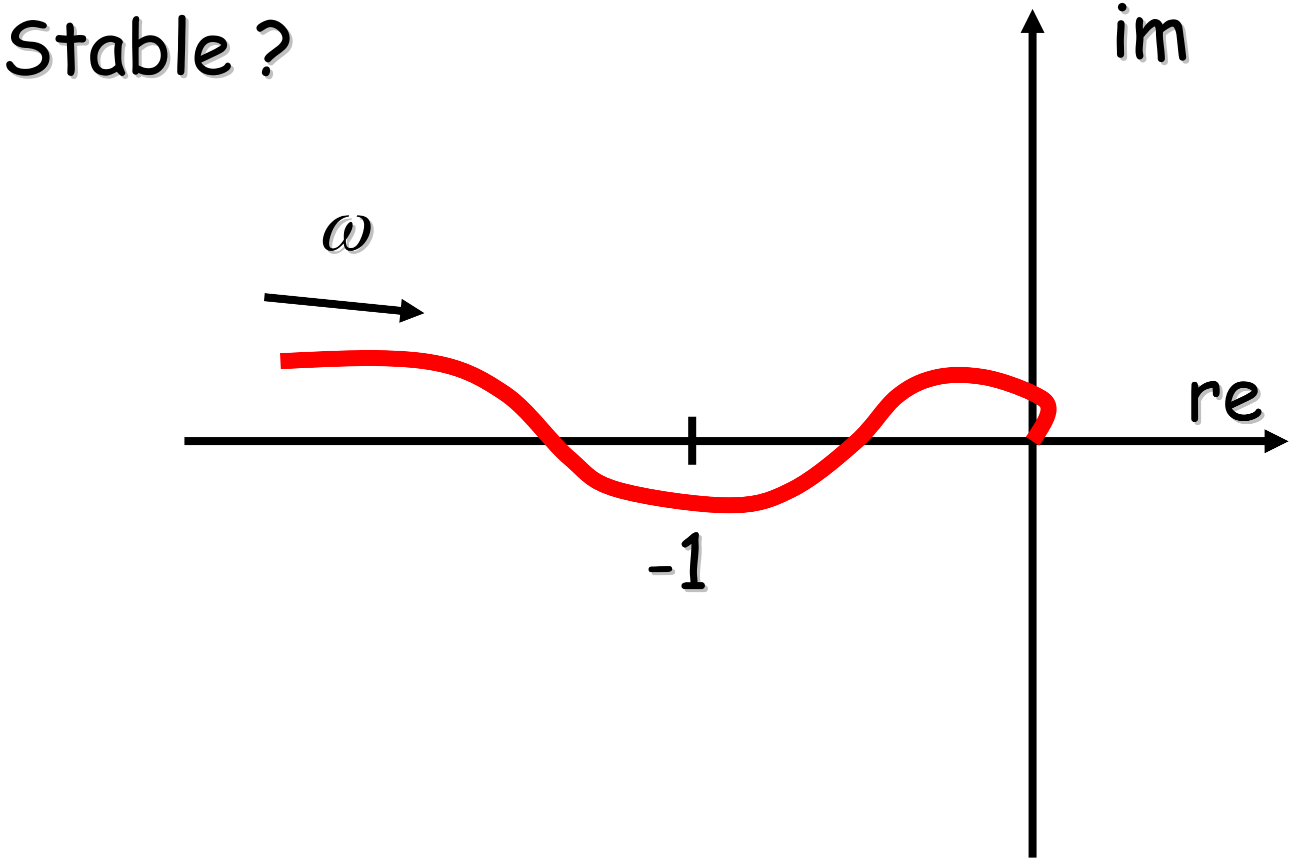
Shade the area at the right of the curve

If -1 in the shaded area: closed loop system UNSTABLE



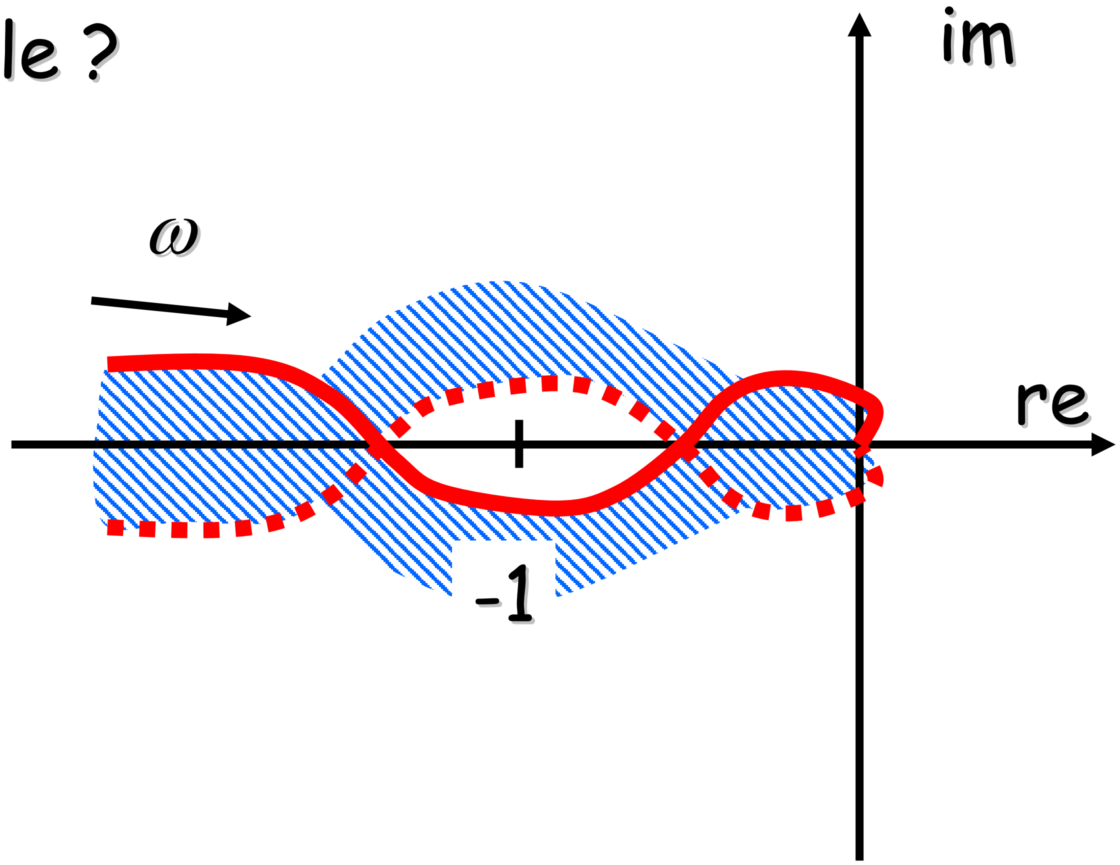
Linear System Re

Conditionally Stable



Conditionally Stable

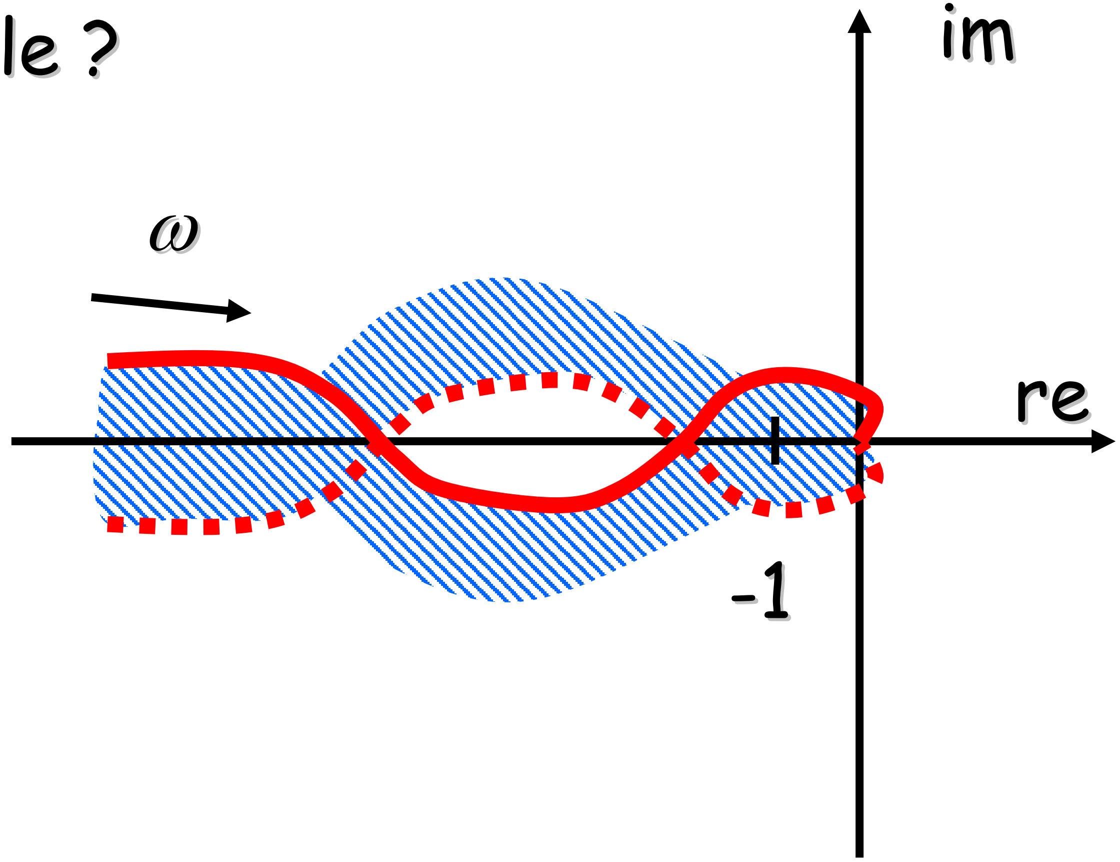
Stable ?



YES

Conditionally Stable

Stable ?

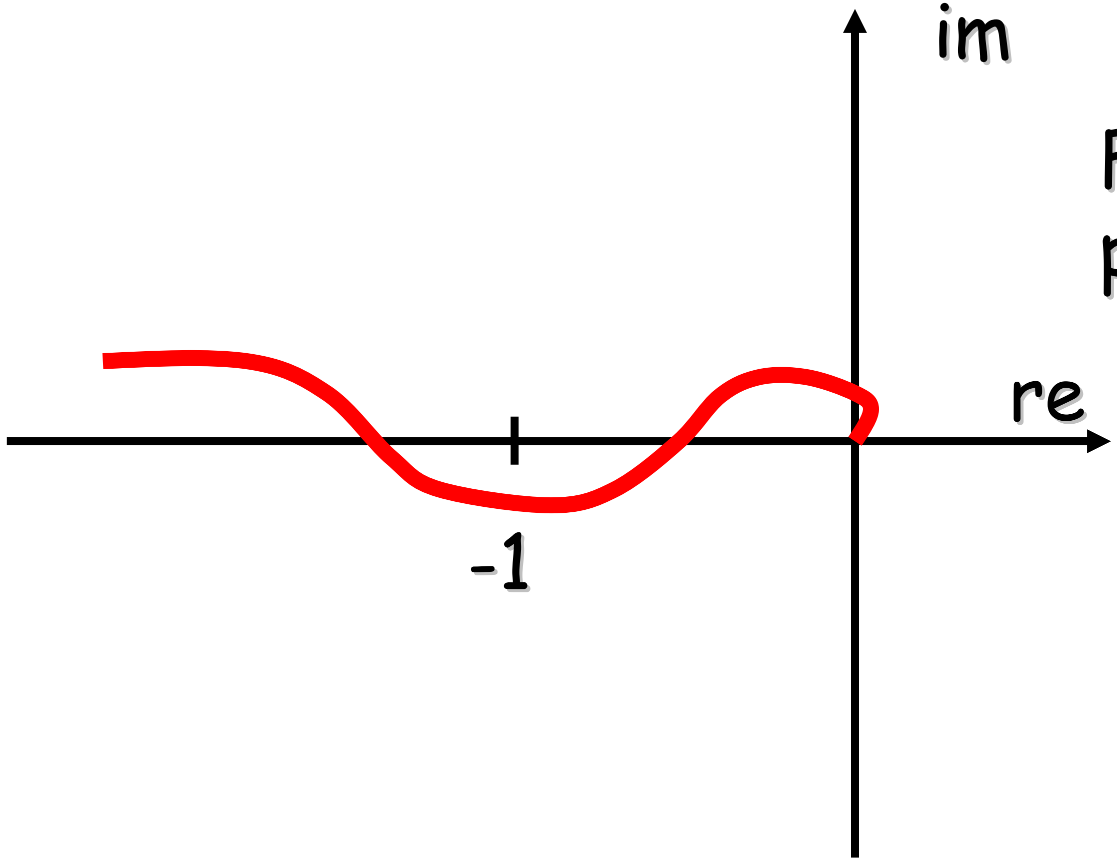


NO

Important !

- The stability of the **CLOSED** system depends on the fact whether the Nyquist plot of the **OPEN** system encircles -1
- If the Nyquist plot of the **CLOSED** system encircles -1, this tells nothing about the stability of the system !

Exercise



Find the poles and zero's

Draw the root locus

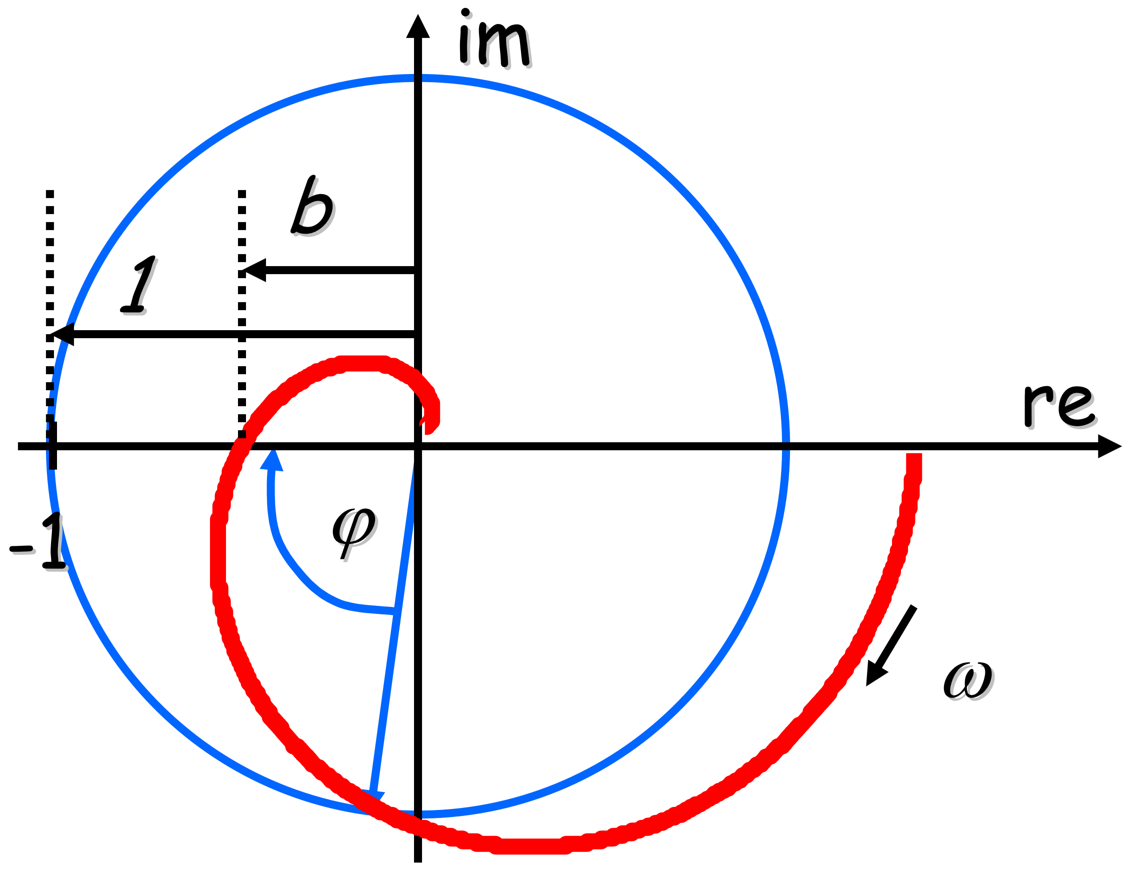
Gain & Phase margins

Gain margin:

1/b

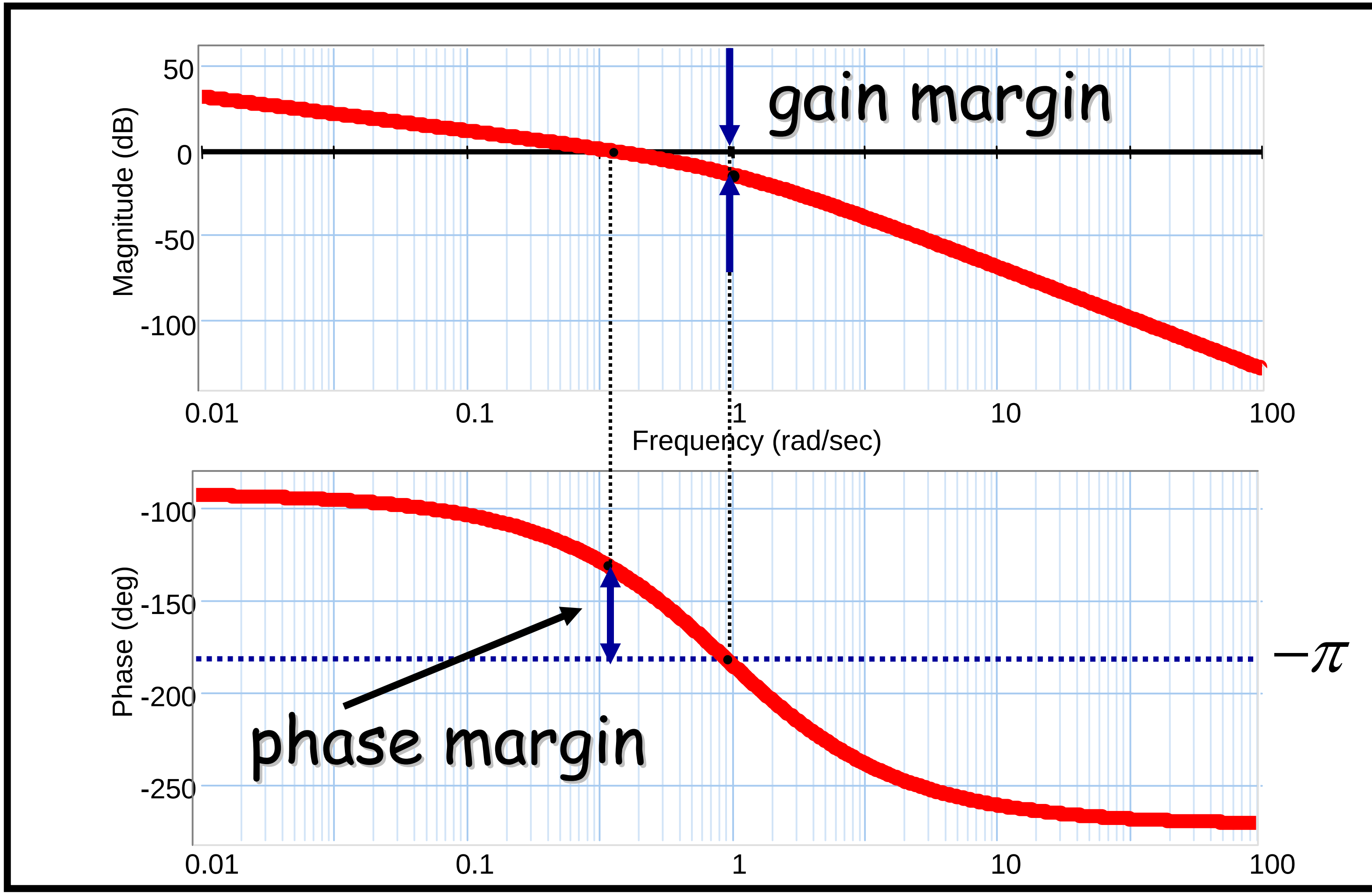
Phase margin:

φ



Linear System Re

Gain & Phase margins (Bode)



Gain & phase margins

- Gain margin determines how much the gains may vary, before the system becomes unstable
- Phase margin influences transient behaviour (damping ratio, overshoot)
- Second order system:
- z ≈ phase margin (in degrees) / 100

Gain & Phase margins (Nichols)

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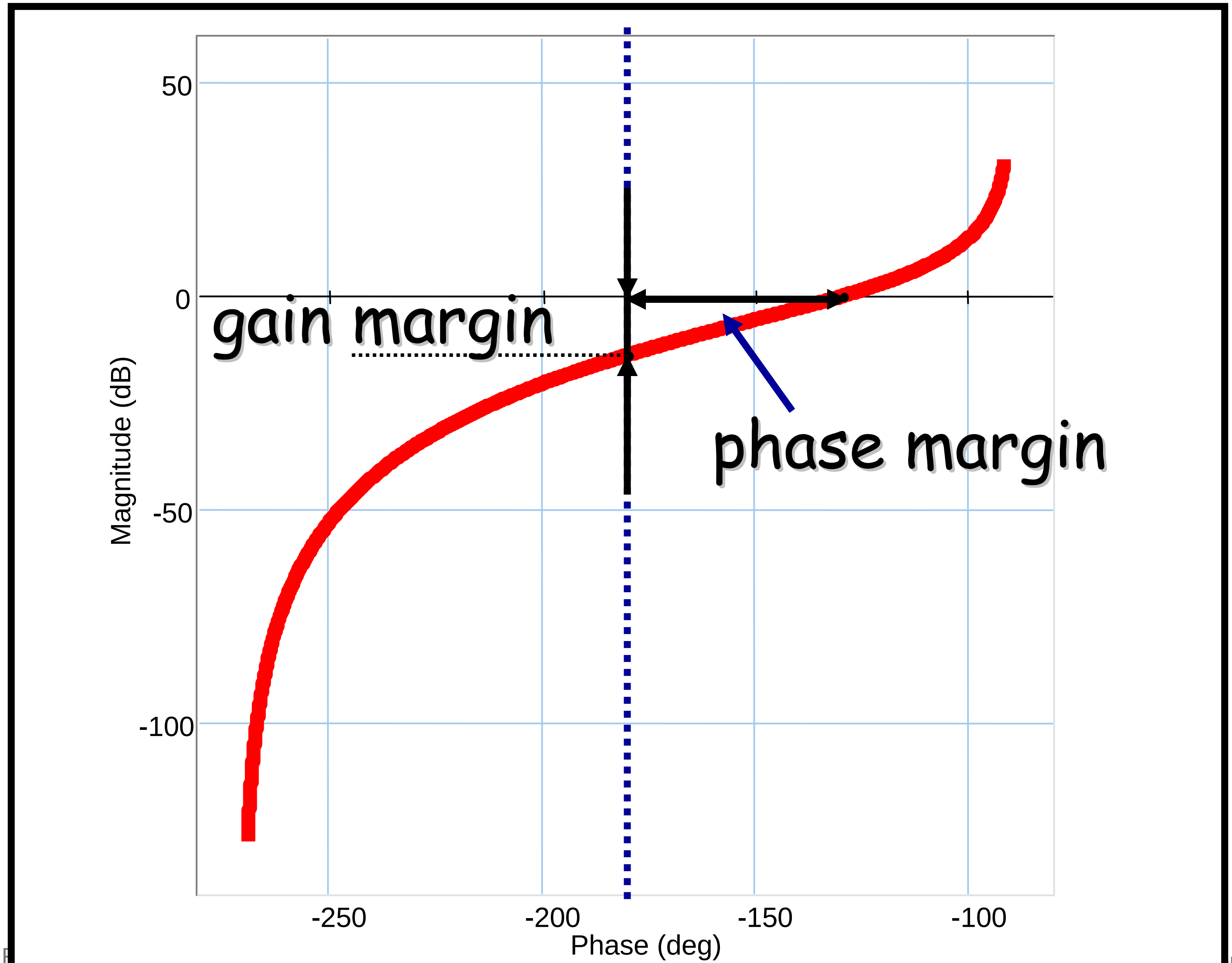

Excercises

- Investigate the influence of gain and phase margins on the step response of the close loop system for various second- and third-order systems

Warning

Don't mix up the

- s-plane with its real (α) and imaginary ($j\omega$) axes and
- The complex plane used to draw the Nyquist (polar) plot of $H(j\omega)$