Design in the s-plane (root locus design)

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Contents

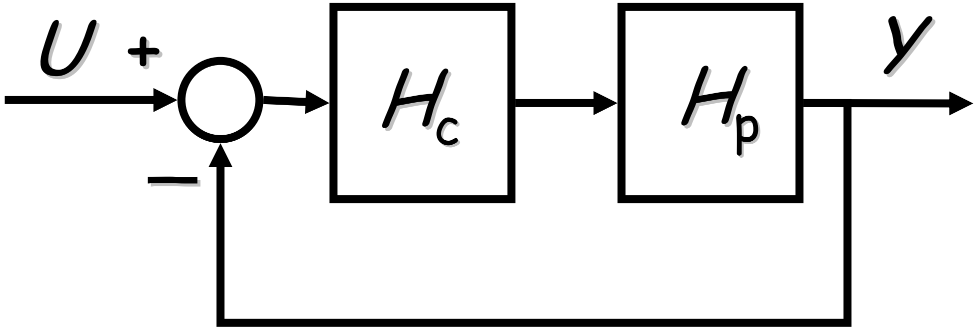
- Design of lag and lead networks
- tau-locus for a lead network
- systems with time delay
- non-minimum phase systems

Design

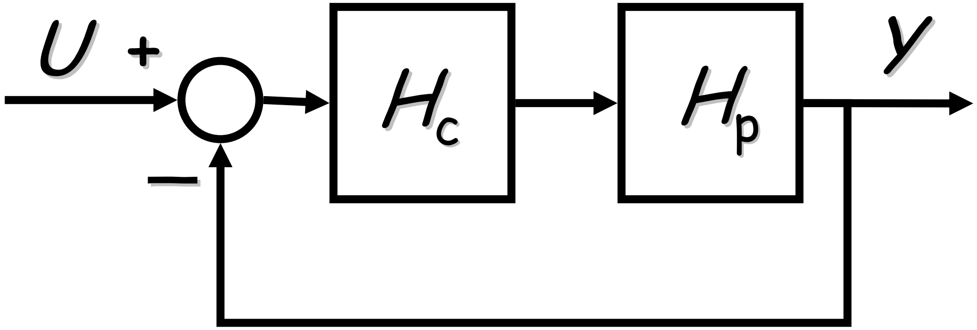
- Design a proportional controller such that the system has a damping ratio $z \approx 0.7$ (phase margin of 70 degrees) for the process:

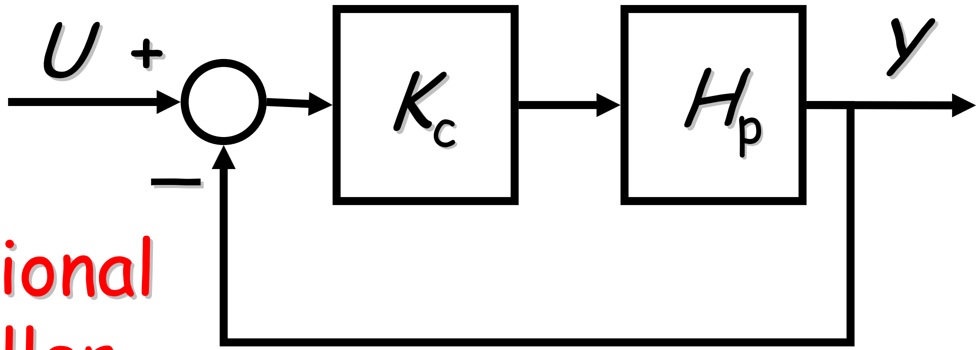
$$H_p(s) = \frac{10}{s(s+1)(s+10)}$$

Control system



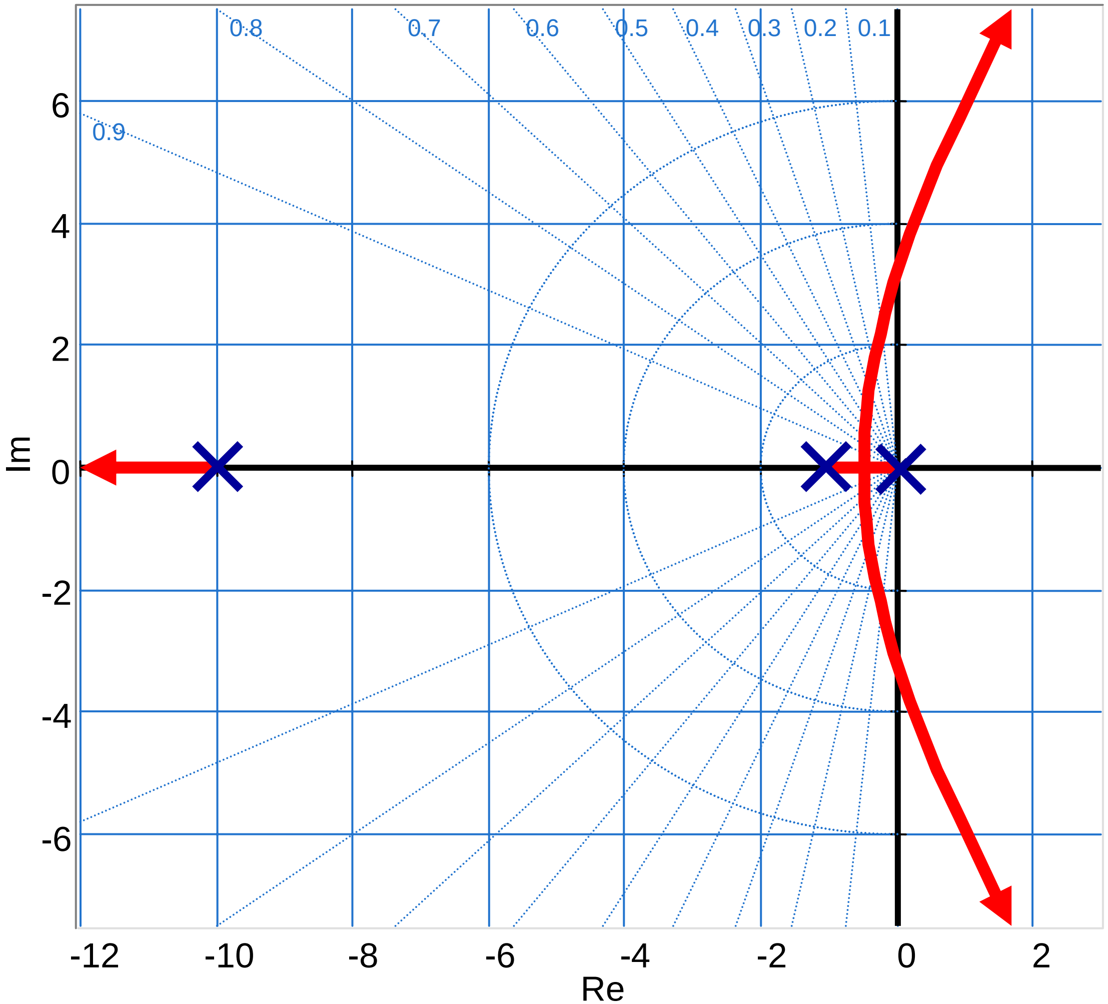
Control system



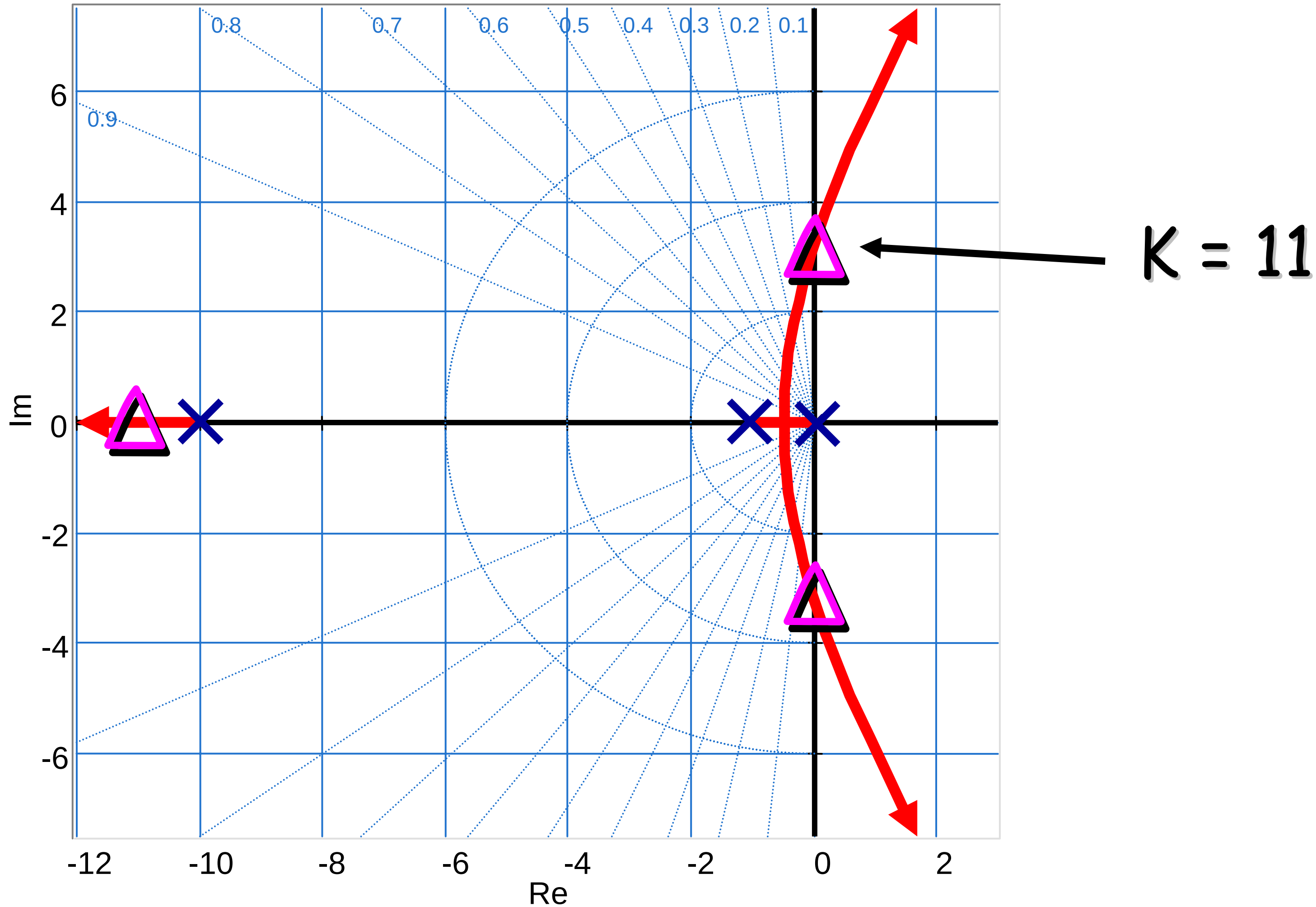


proportional controller

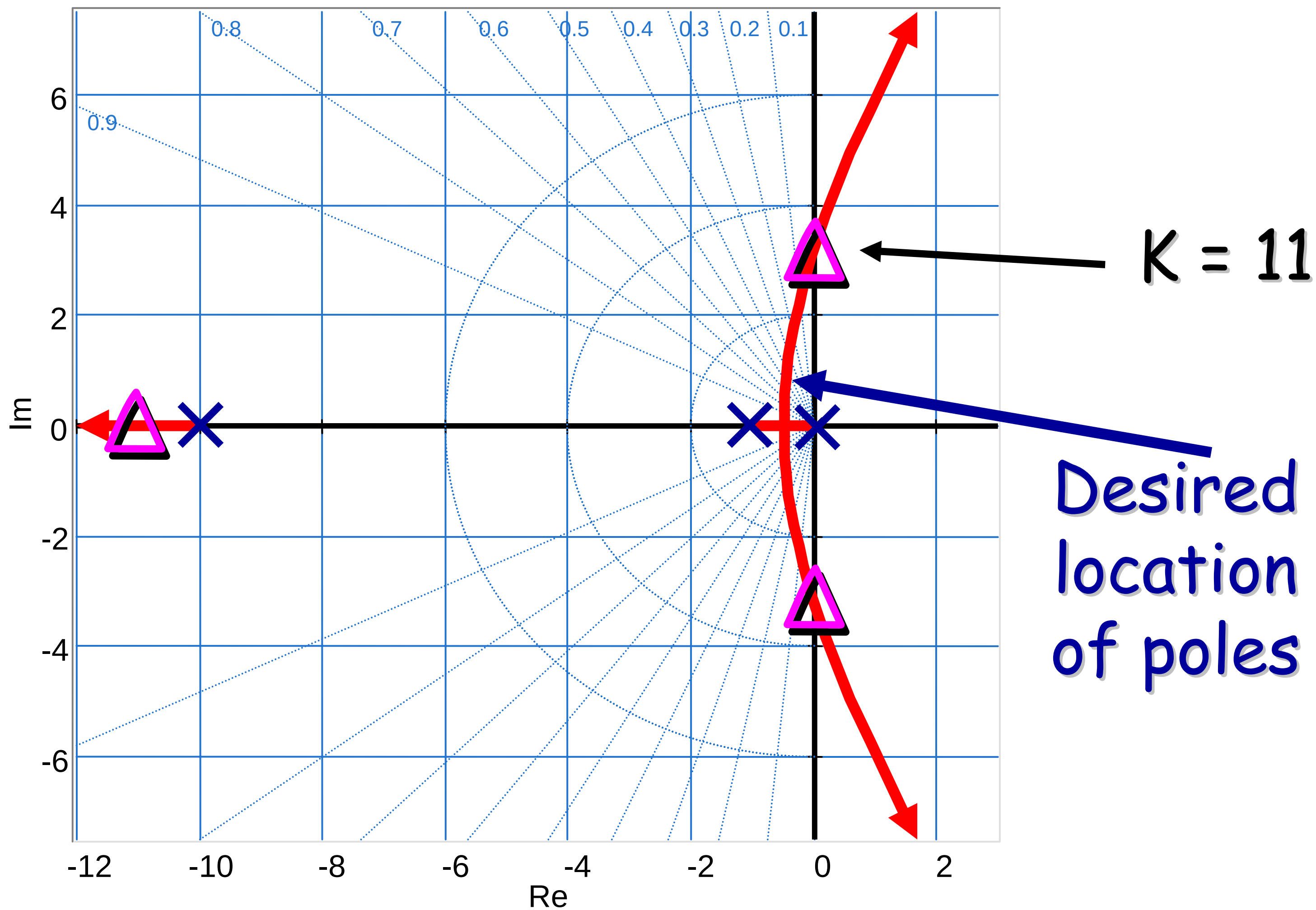
Root locus



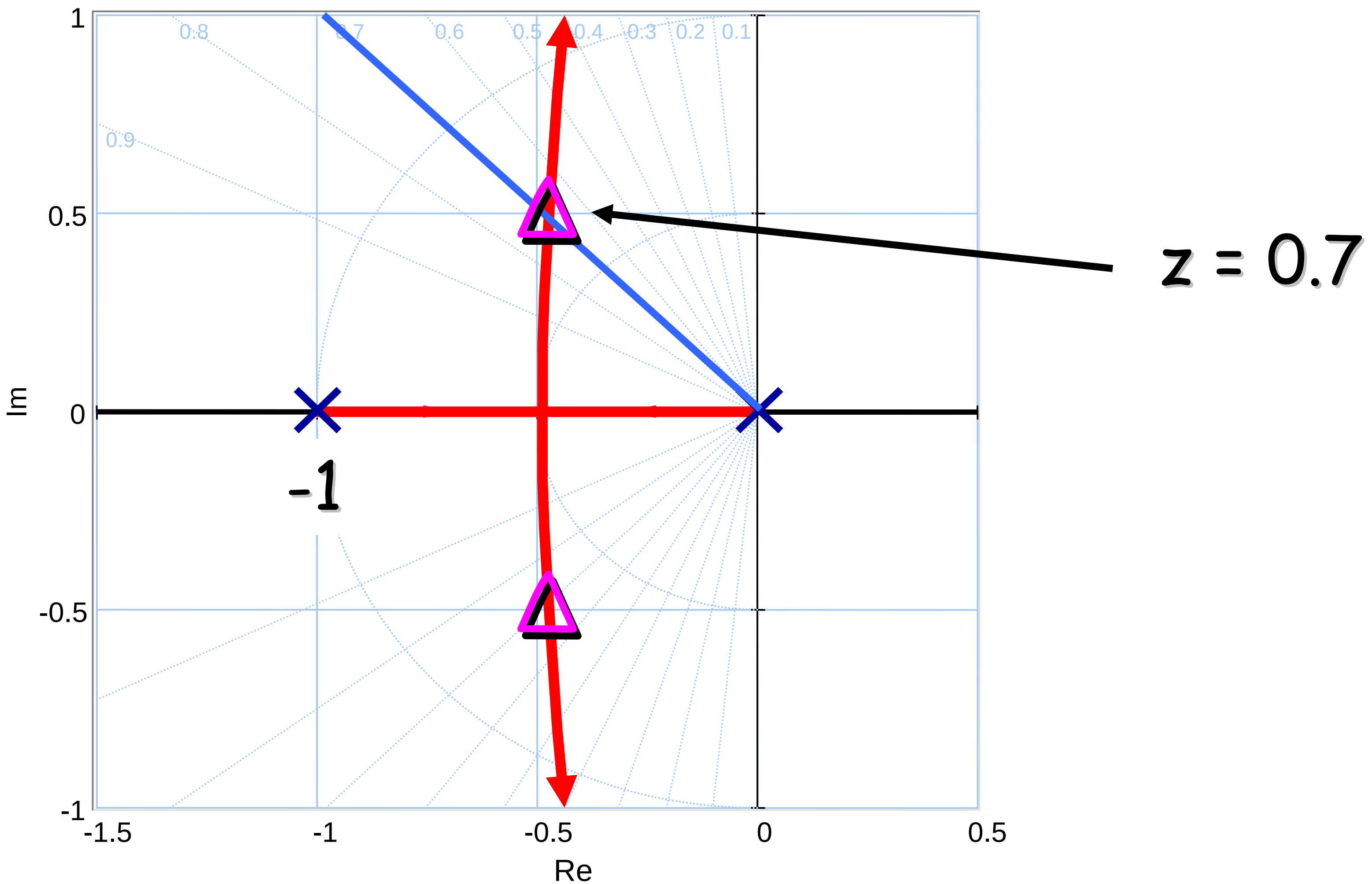
Root locus



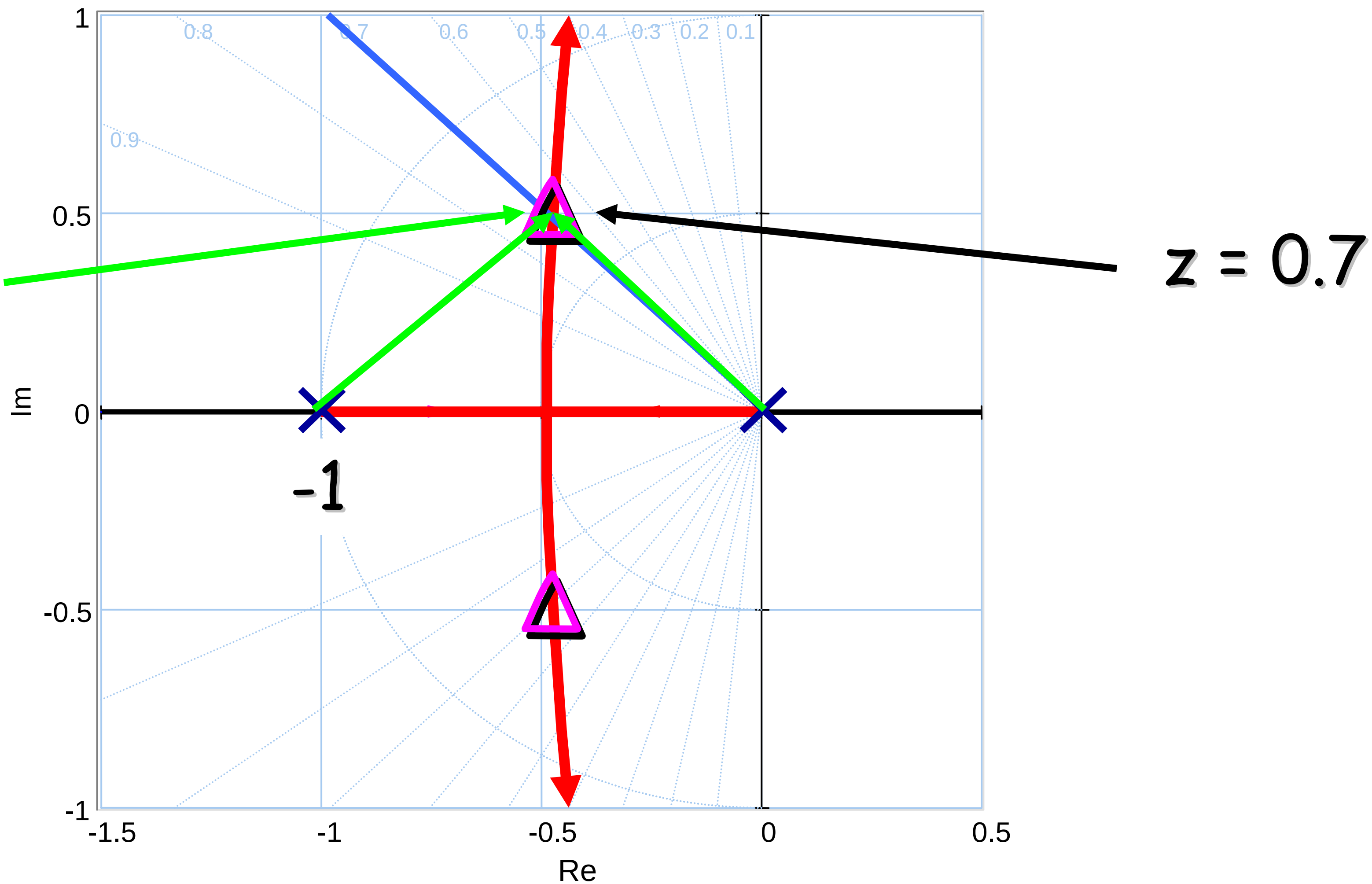
Root locus


0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.9
6
4
2
0
-2
-4
-6
Im
-12
-10
-8
-6
-4
-2
0
2
Re
K = 11
Desired location of poles

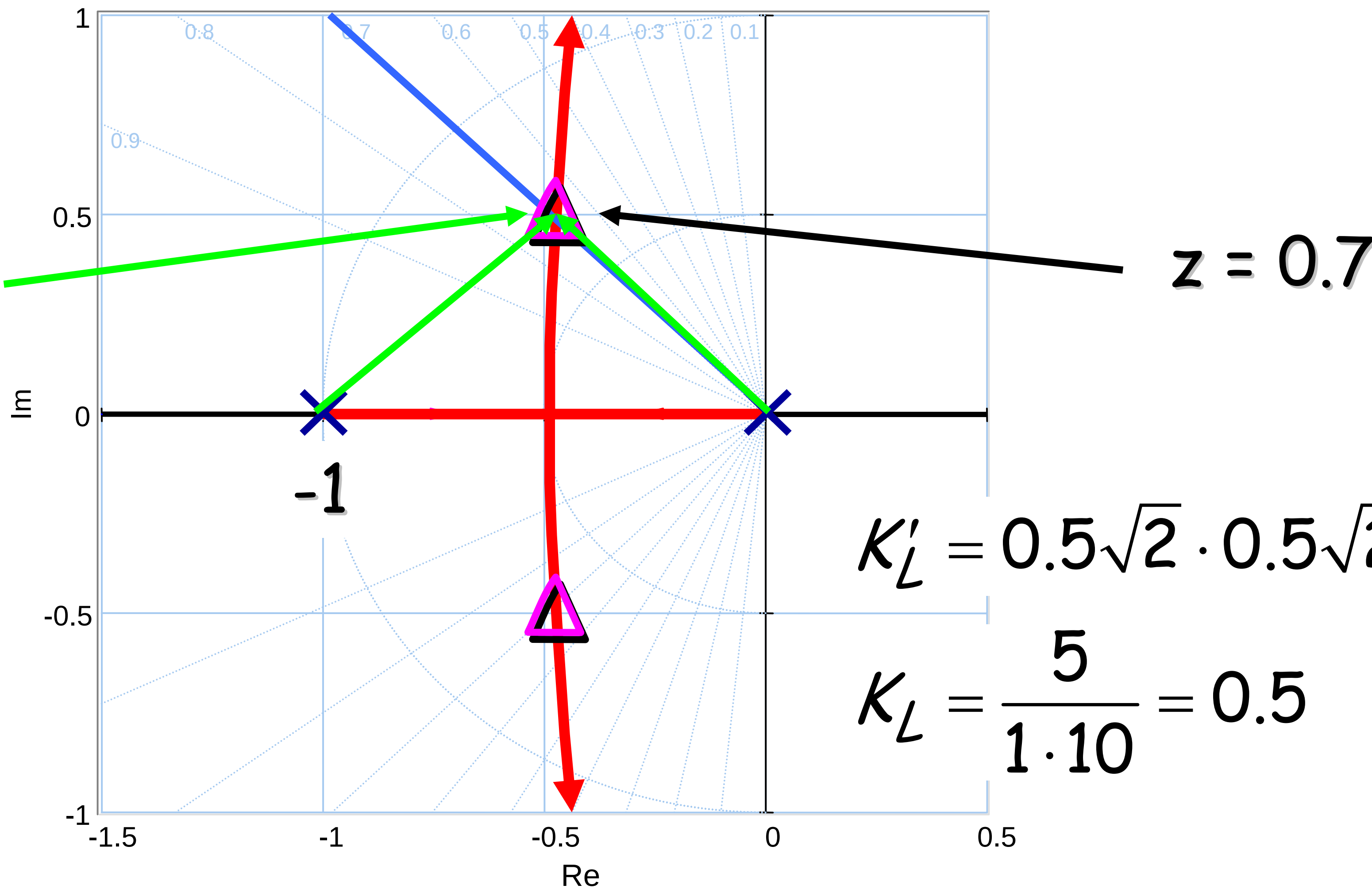
Design



Design



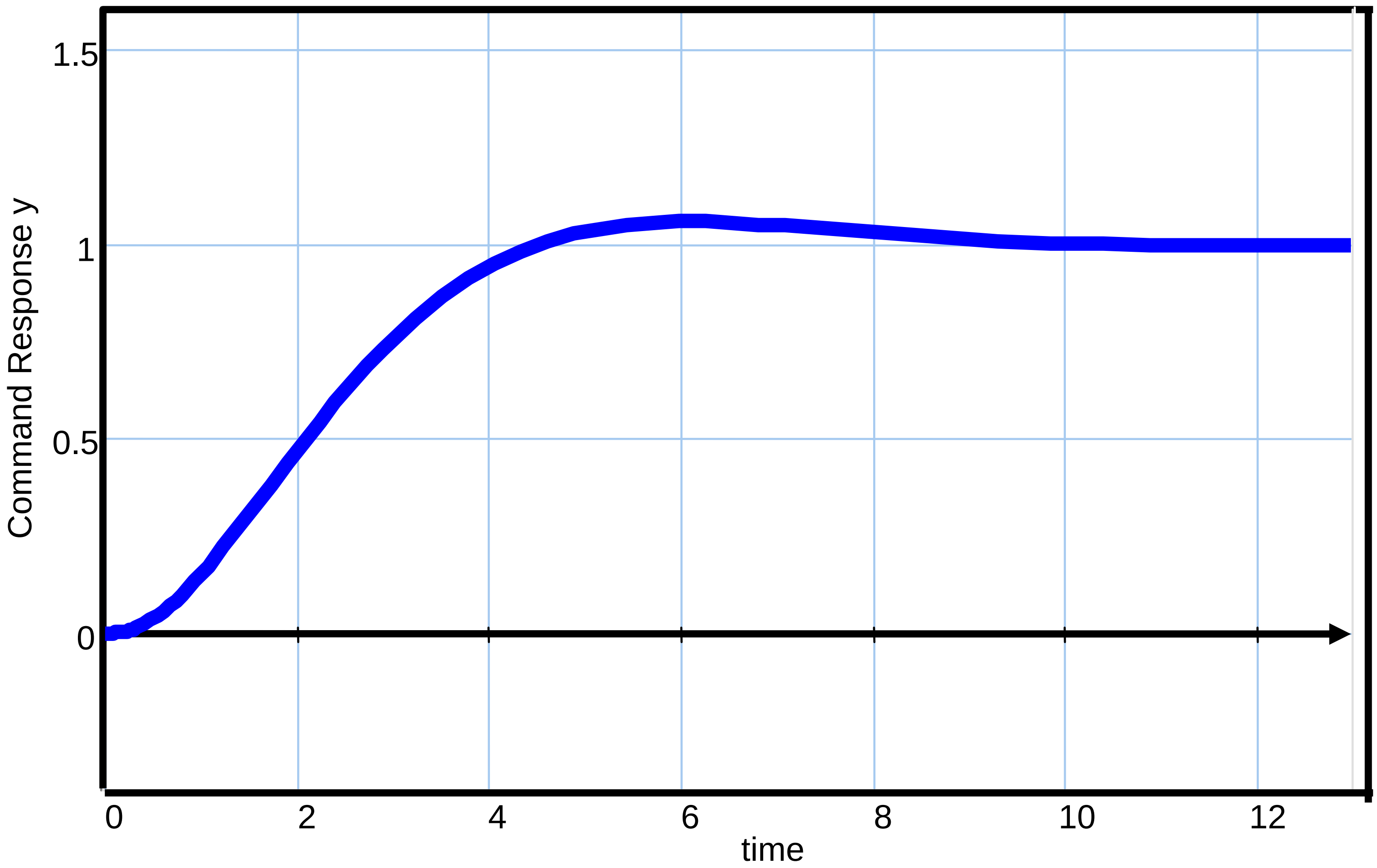
Design



$$K_L' = 0.5\sqrt{2} \cdot 0.5\sqrt{2} \cdot 10$$

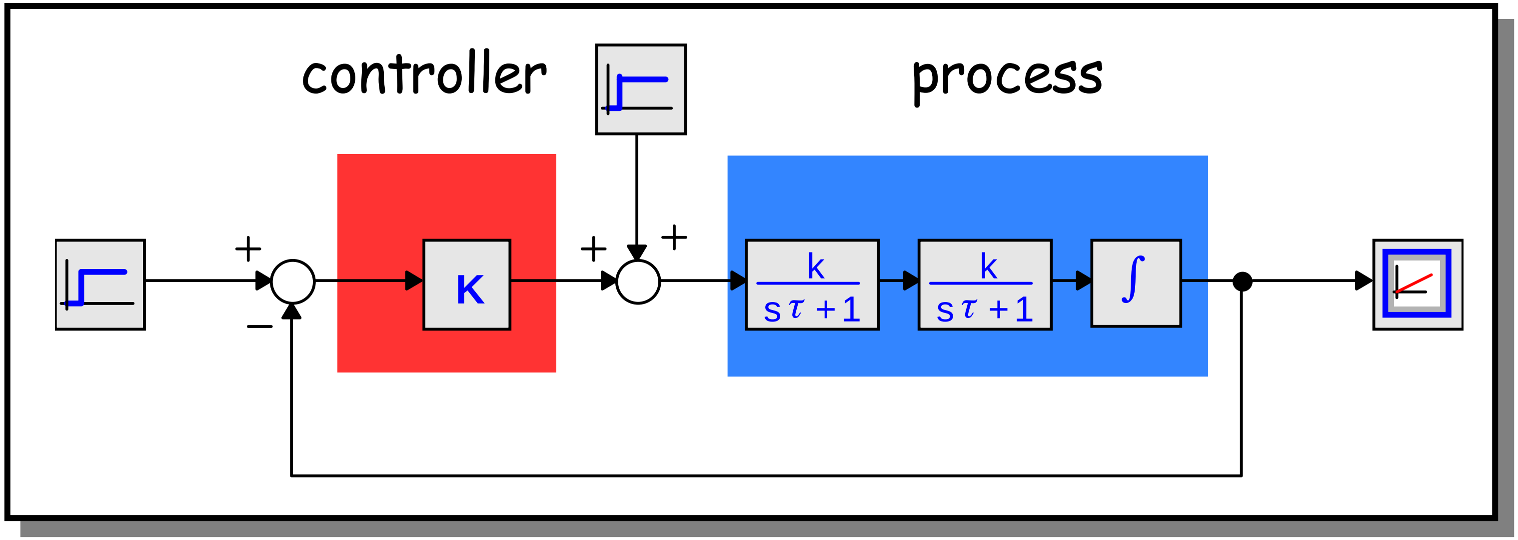
$$K_L = \frac{5}{1 \cdot 10} = 0.5$$

Response

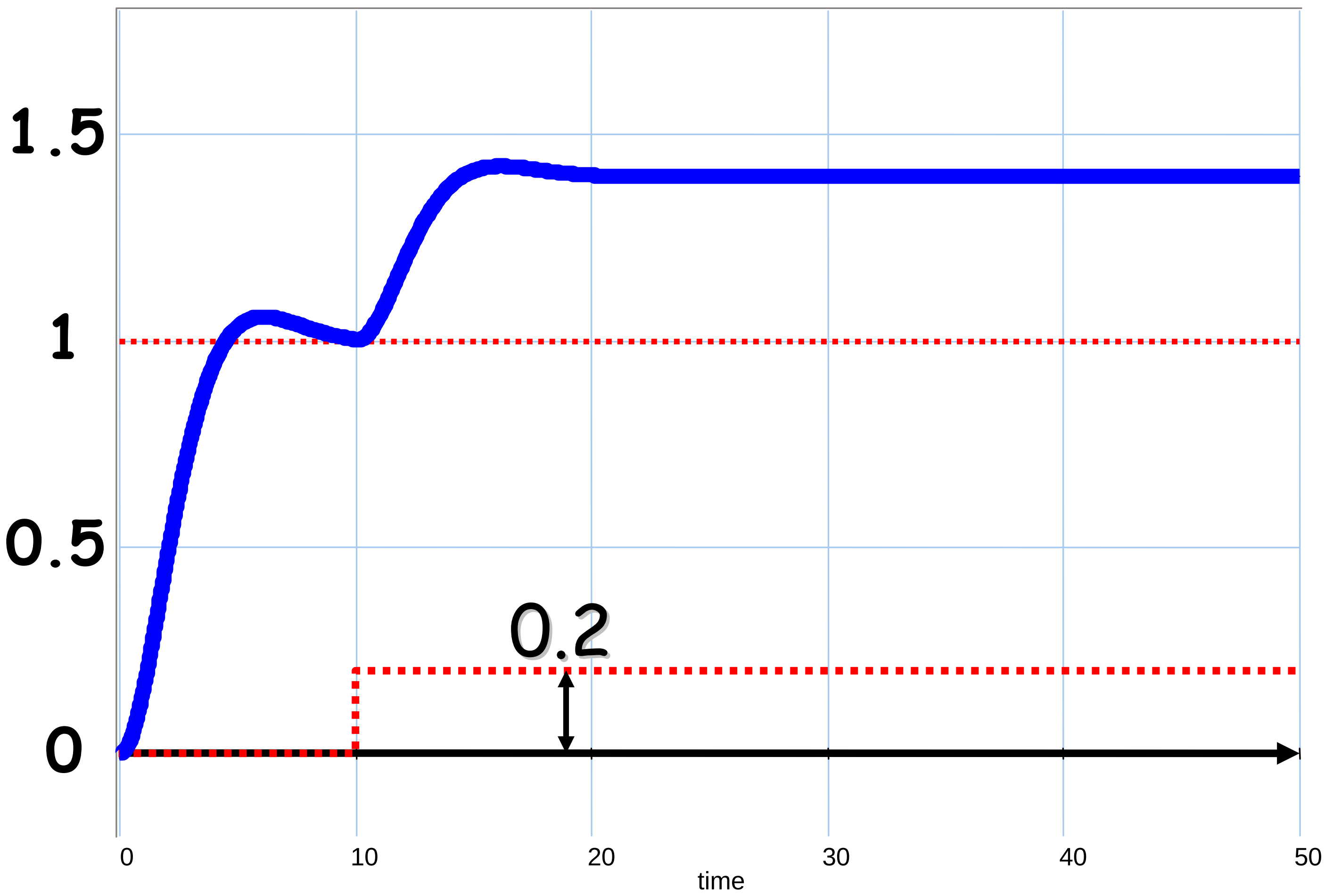


Disturbance suppression

Consider the following system



Response (K = 0.5)



Response (K = 0.5)

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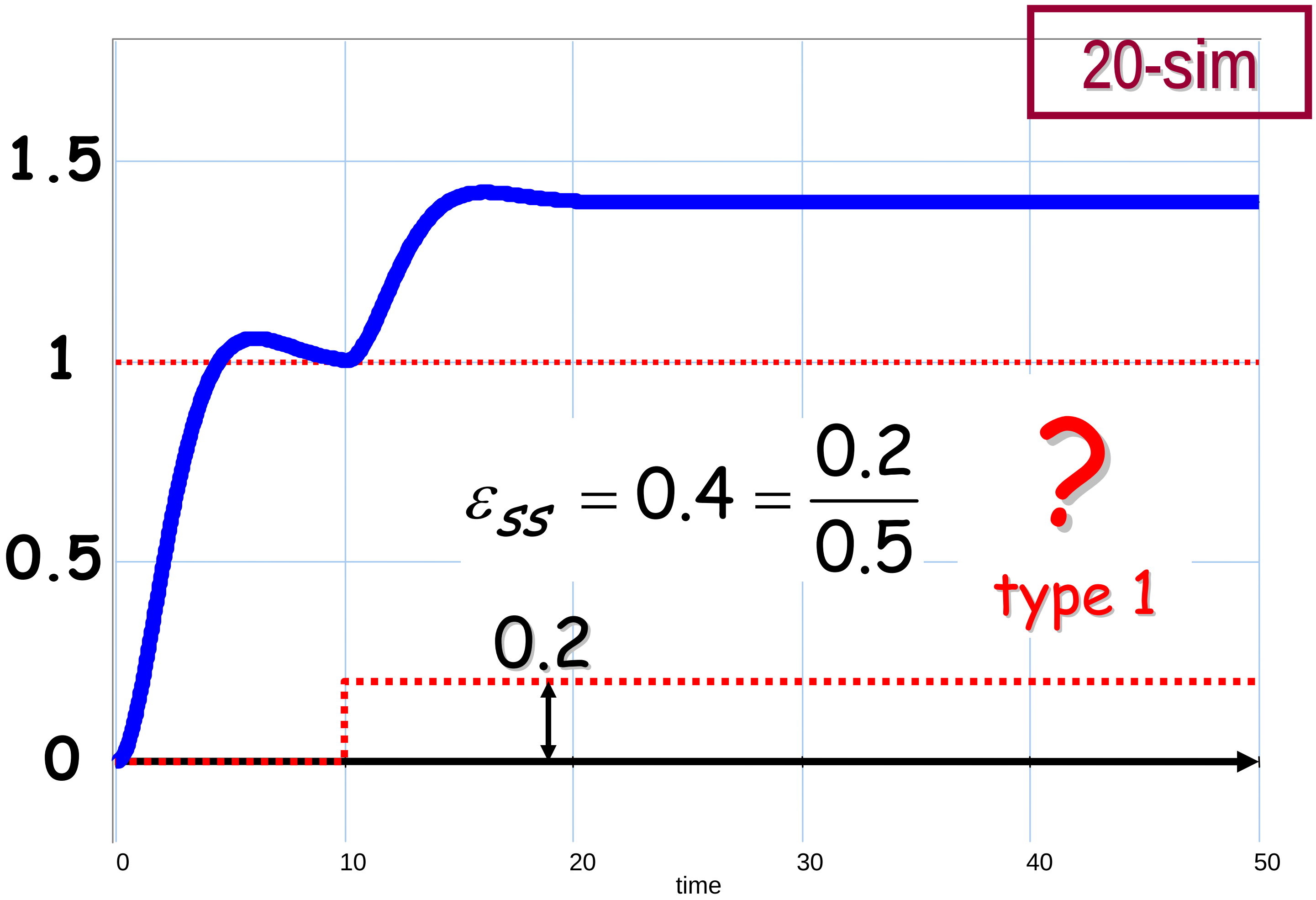
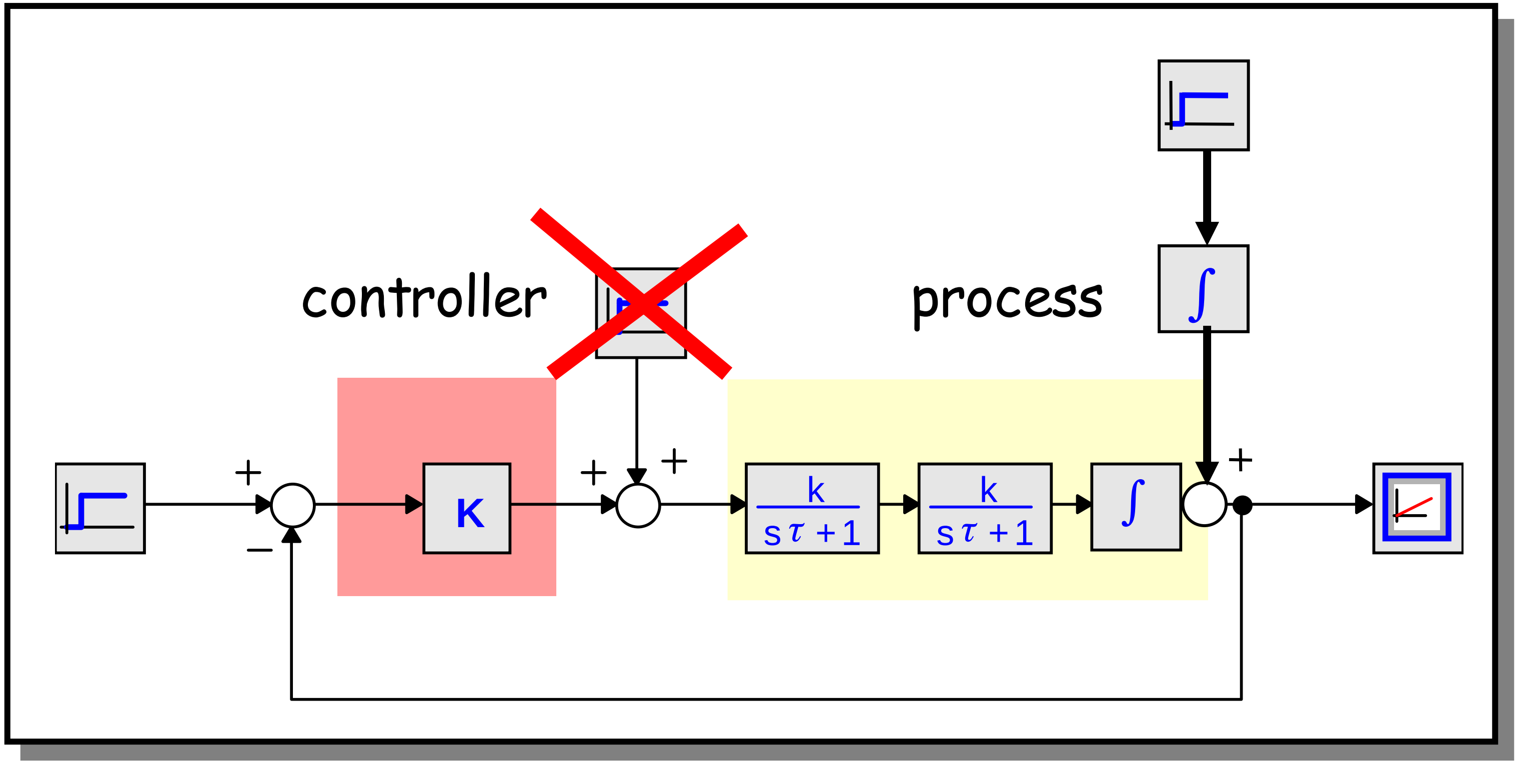
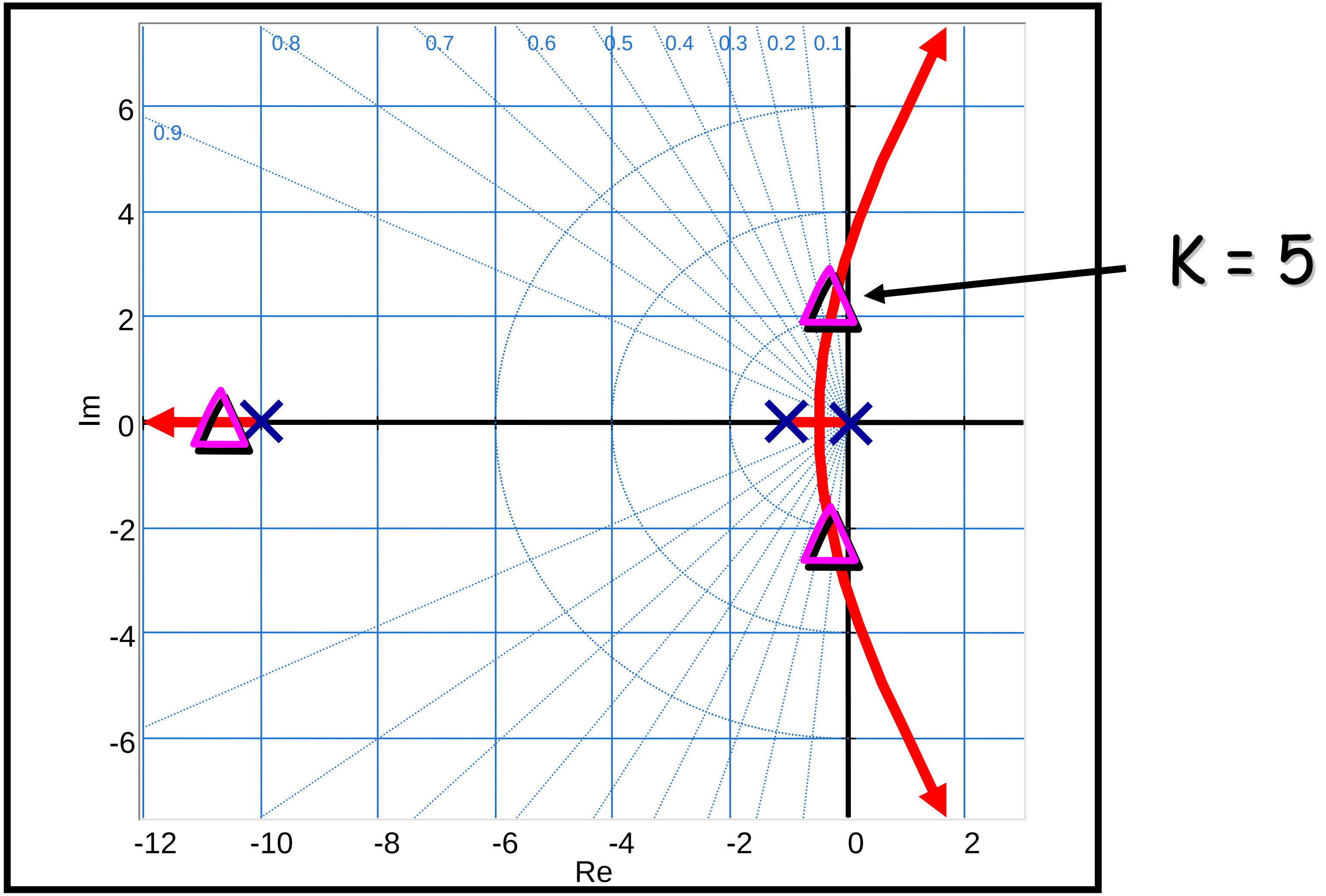


Disturbance suppression

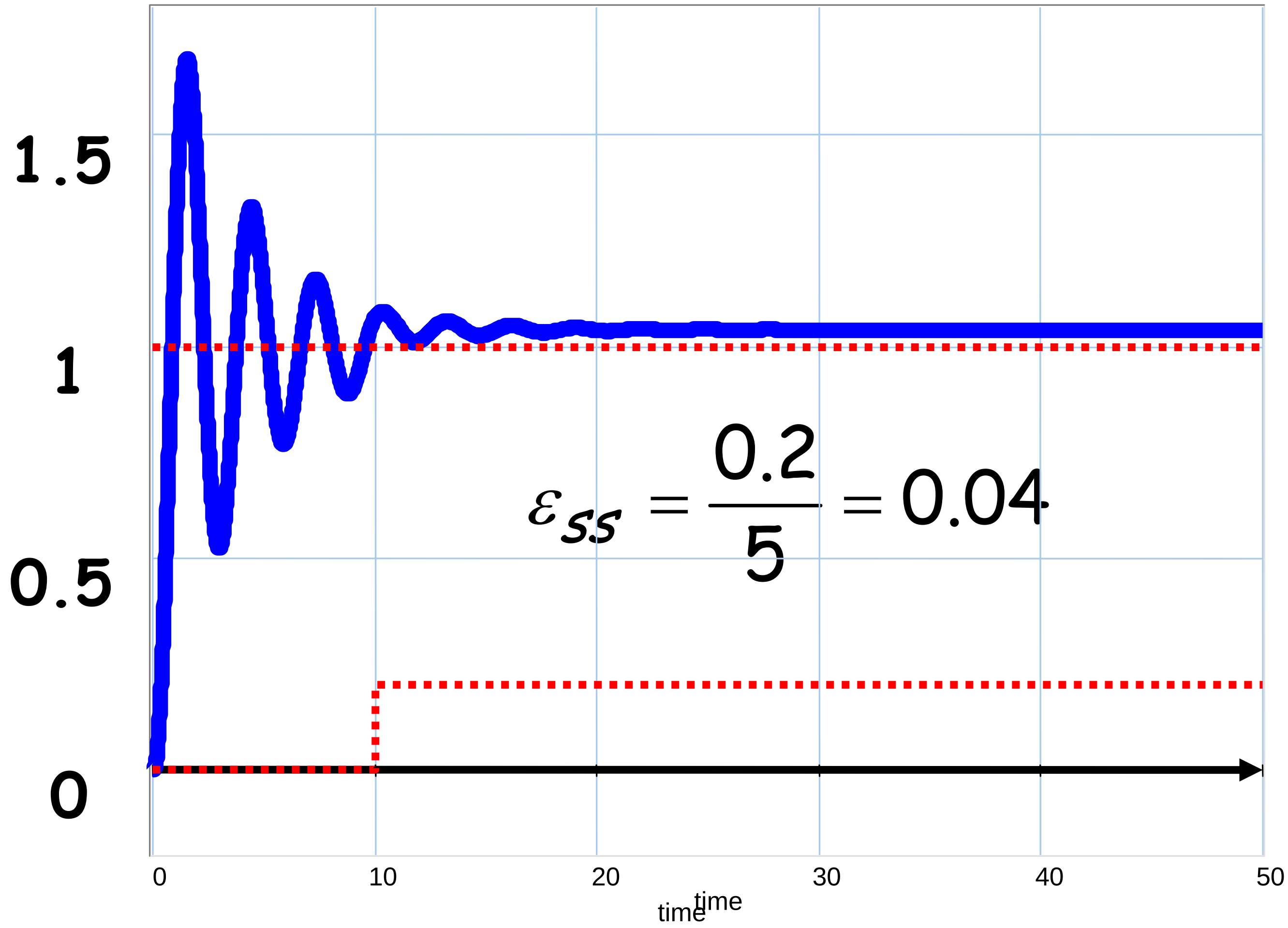
‘Equivalent’ disturbance



Root locus, K = 5

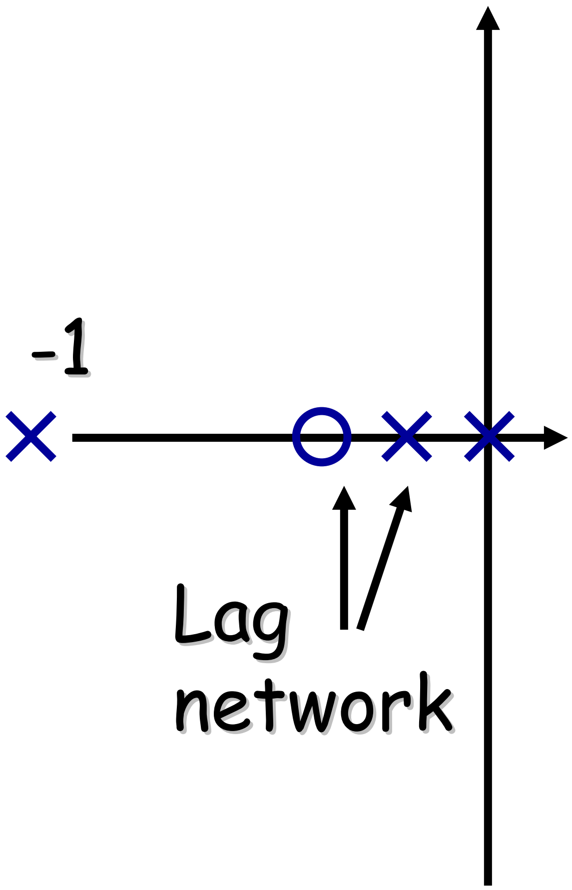


Response K = 5

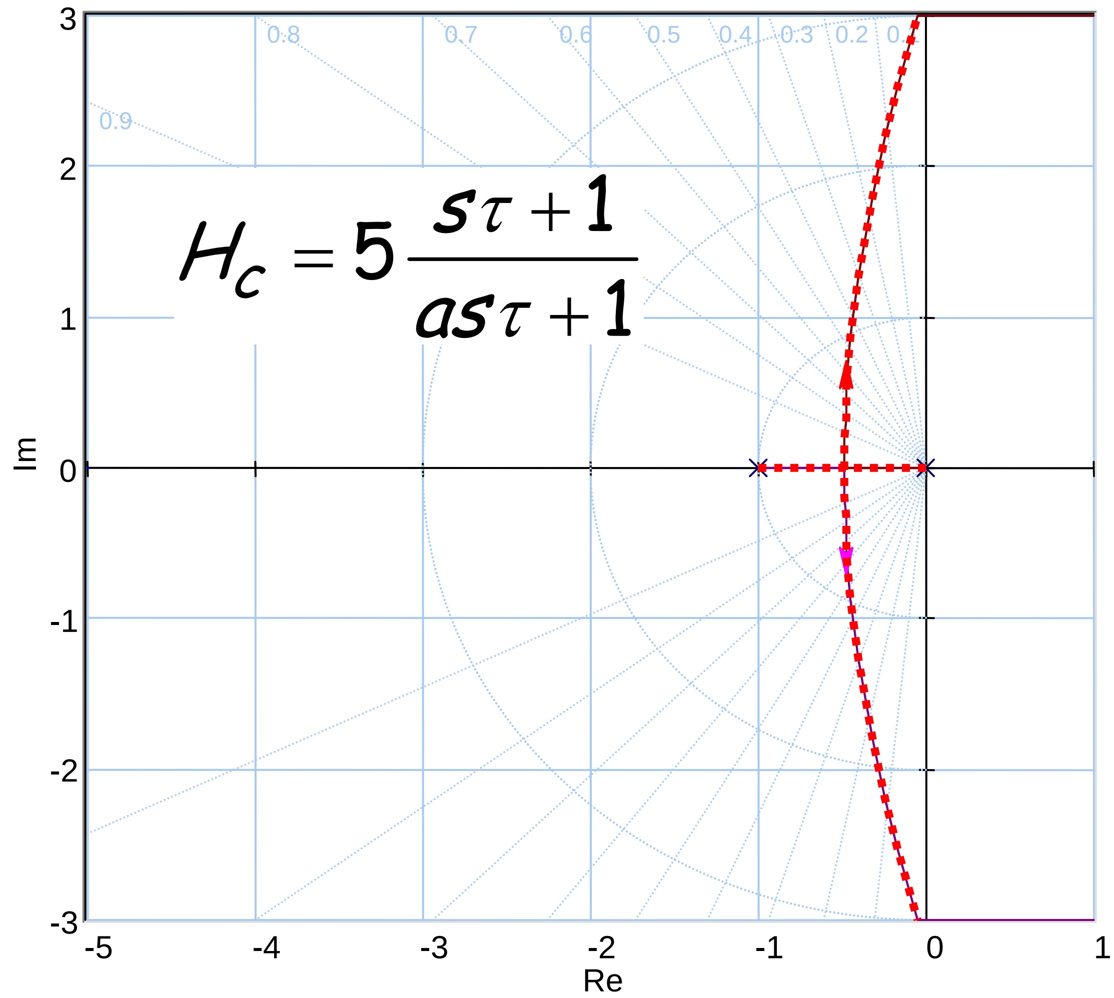


$$\varepsilon_{ss} = \frac{0.2}{5} = 0.04$$

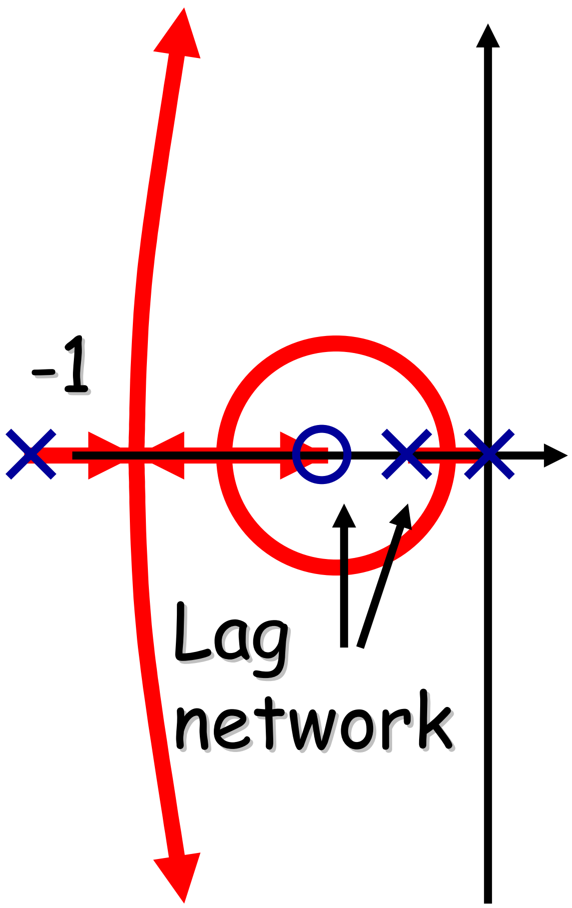
Lag network

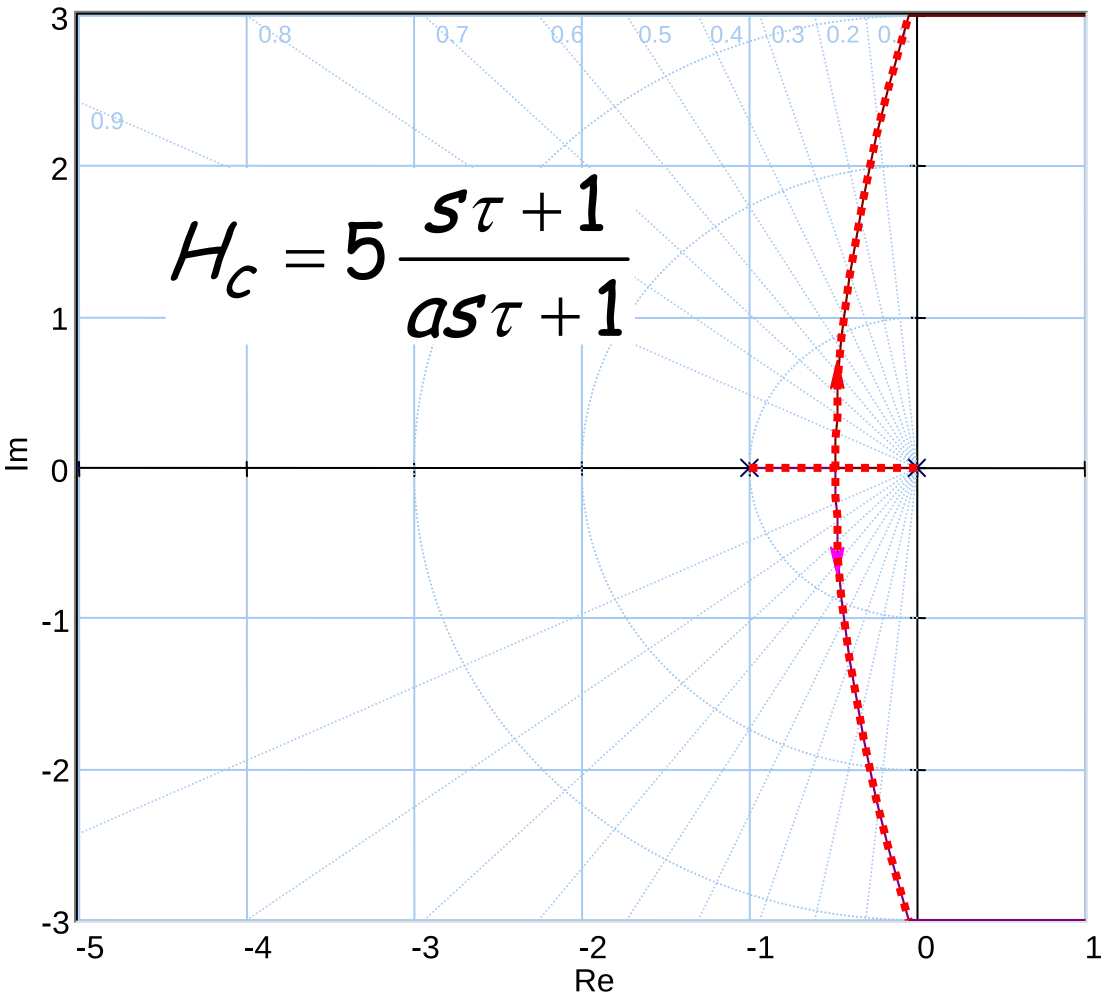


$$H_c = 5\,\frac{s\tau + 1}{as\tau + 1}$$

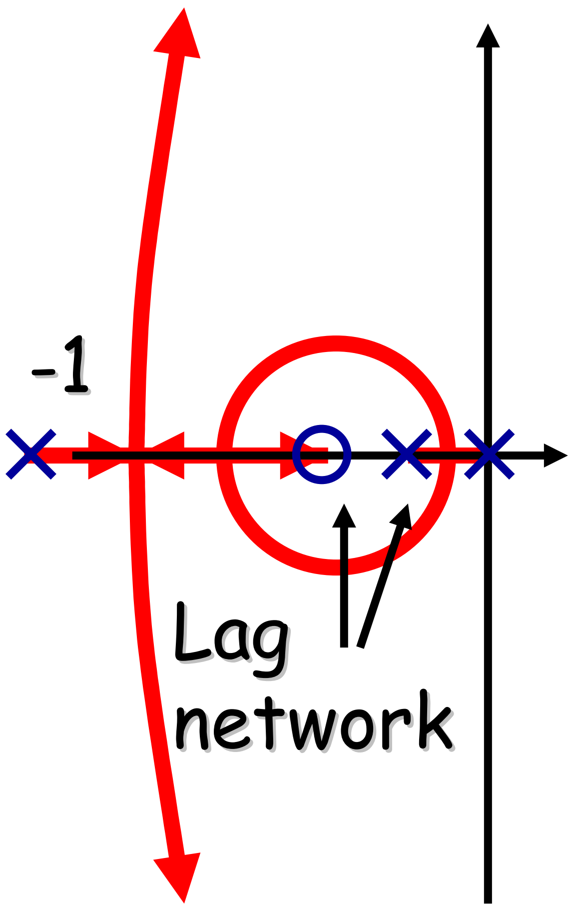


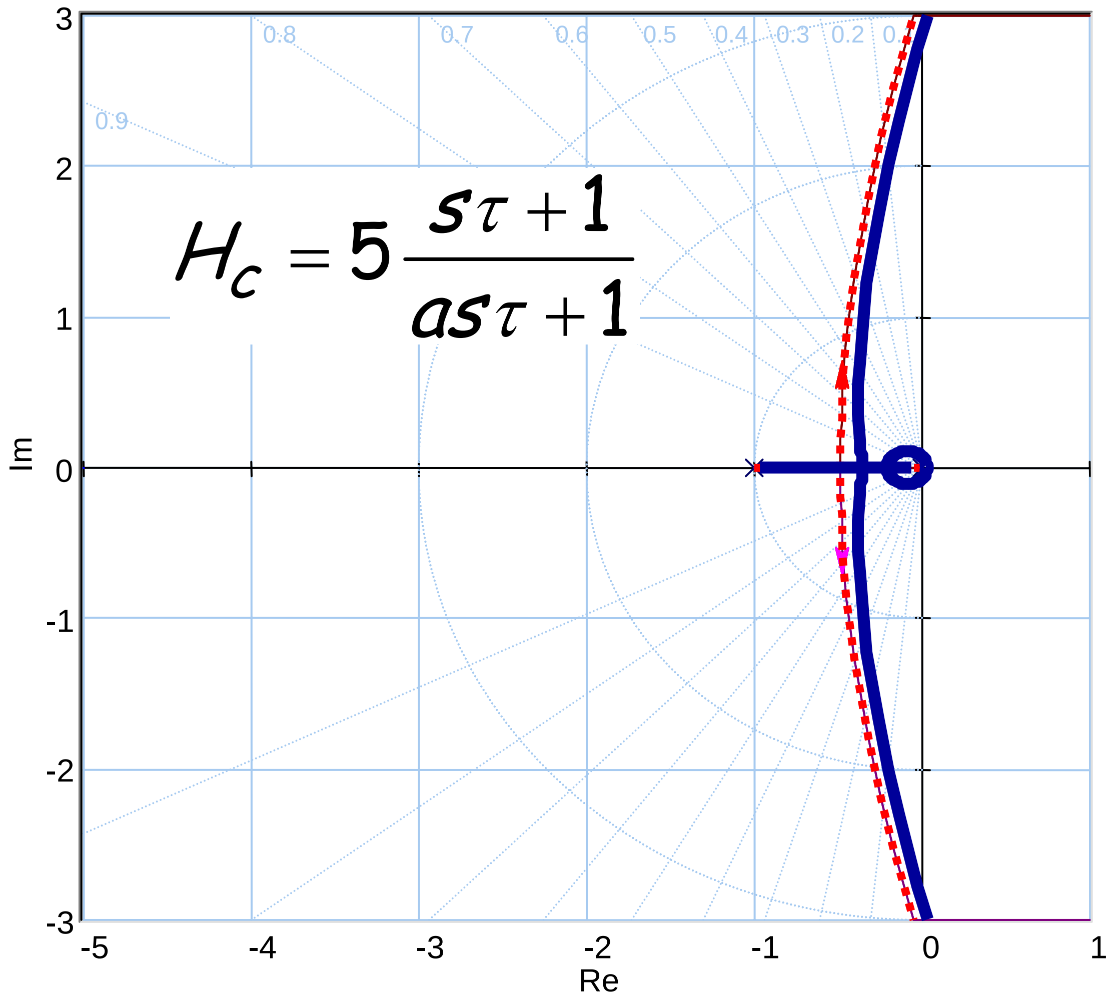
Lag network


-1
Lag
network


$H_c = 5\,\dfrac{s\tau + 1}{as\tau + 1}$
Im
Re
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.9
3
2
1
0
-1
-2
-3
-5
-4
-3
-2
-1
0
1

Lag network

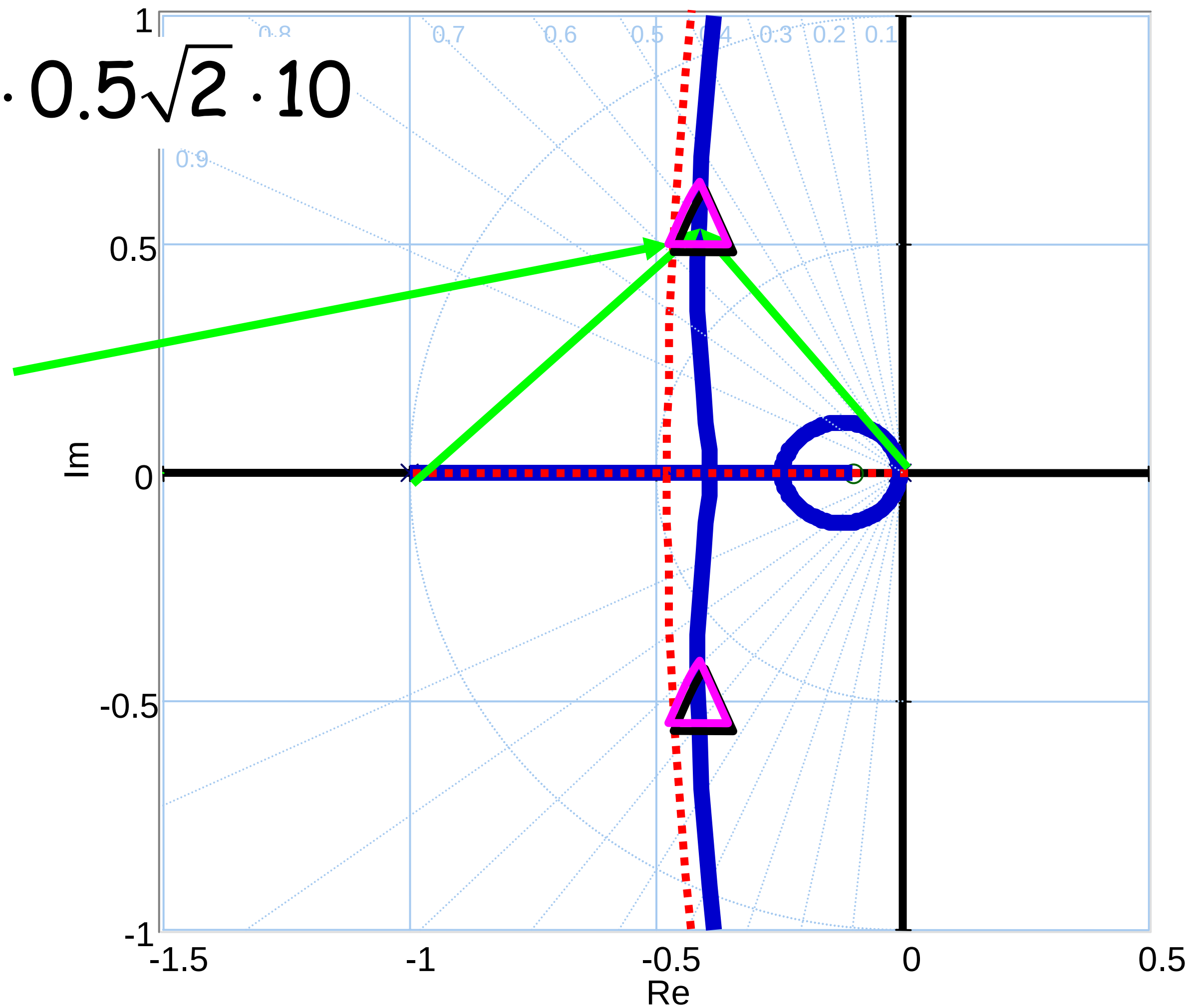




$$H_c = 5\frac{s\tau + 1}{as\tau + 1}$$

Gain for z = 0.7

$$K_L' = 0.5\sqrt{2} \cdot 0.5\sqrt{2} \cdot 10$$

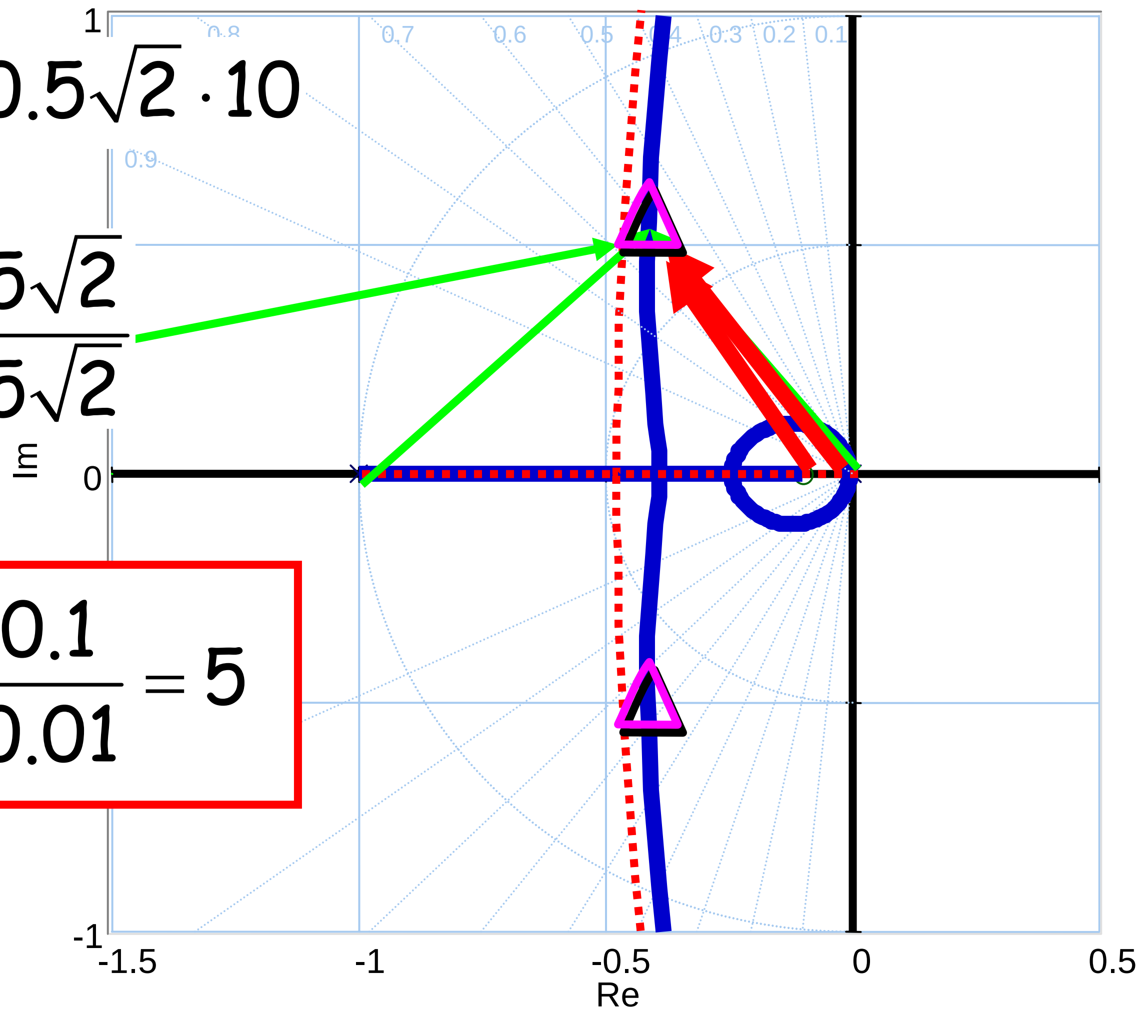


Gain for z = 0.7

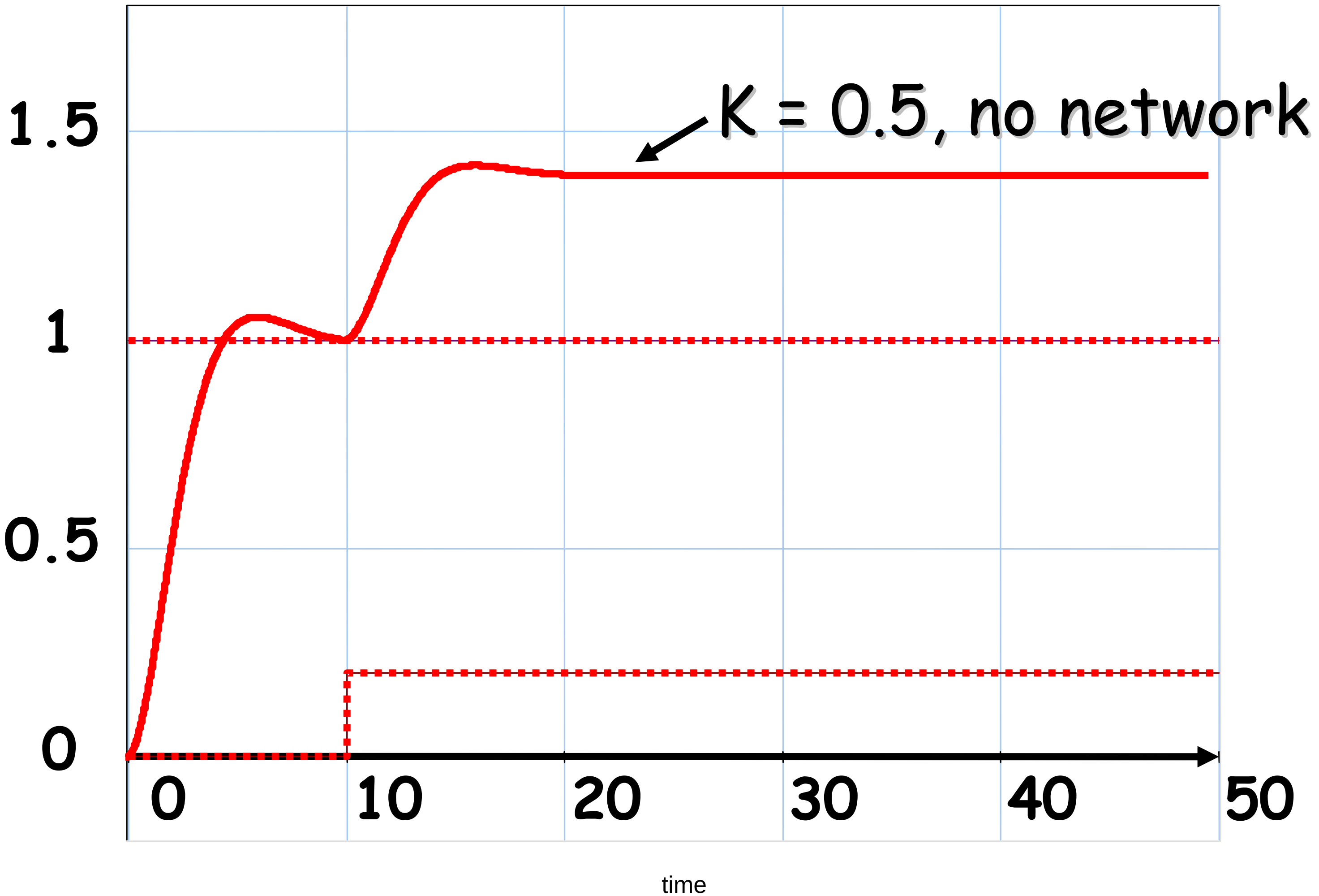
$$K_L' = 0.5\sqrt{2}\cdot 0.5\sqrt{2}\cdot 10$$

$$K_{L,lag}' = K_L' \frac{0.5\sqrt{2}}{0.5\sqrt{2}}$$

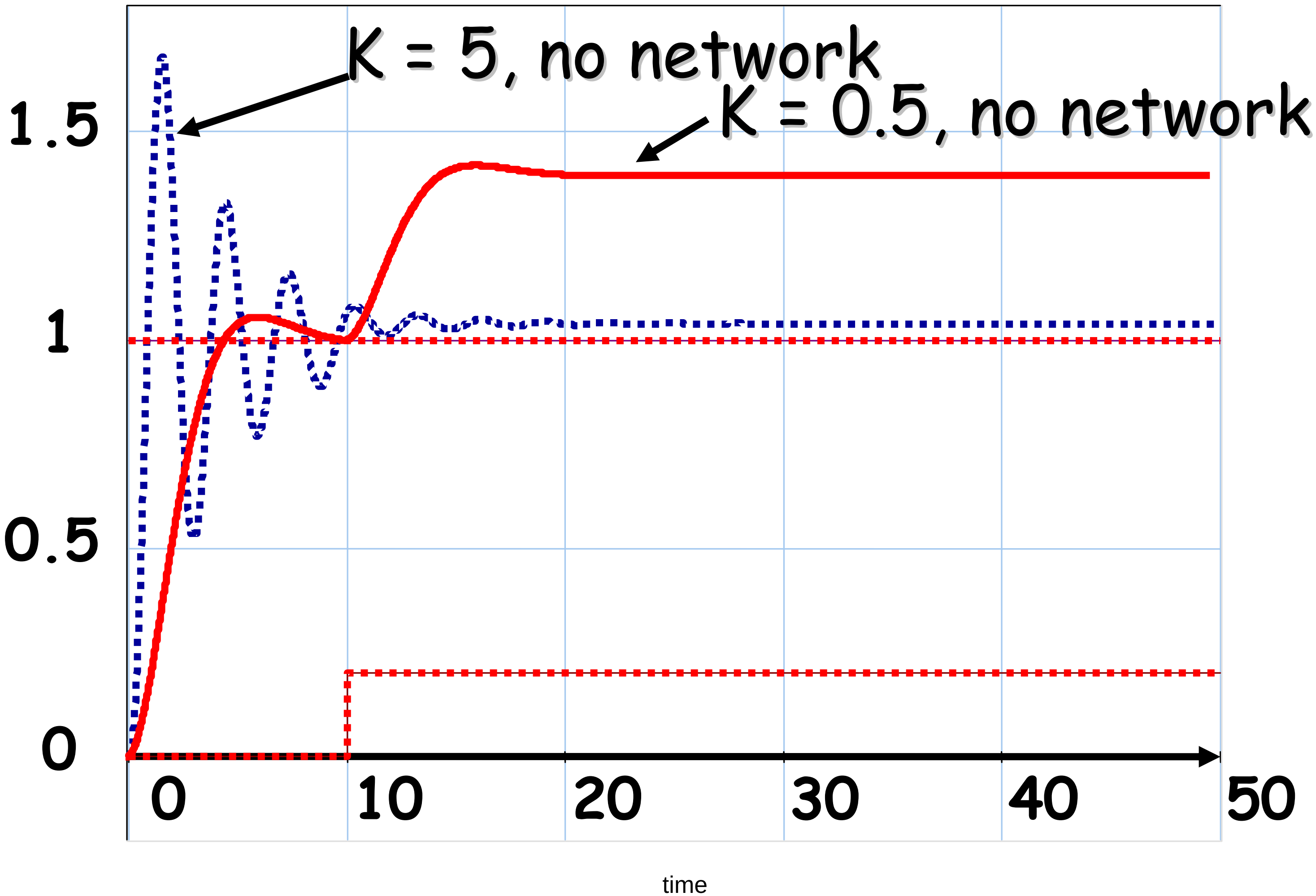
$$K_L = \frac{5}{1\cdot 10}\frac{0.1}{0.01} = 5$$



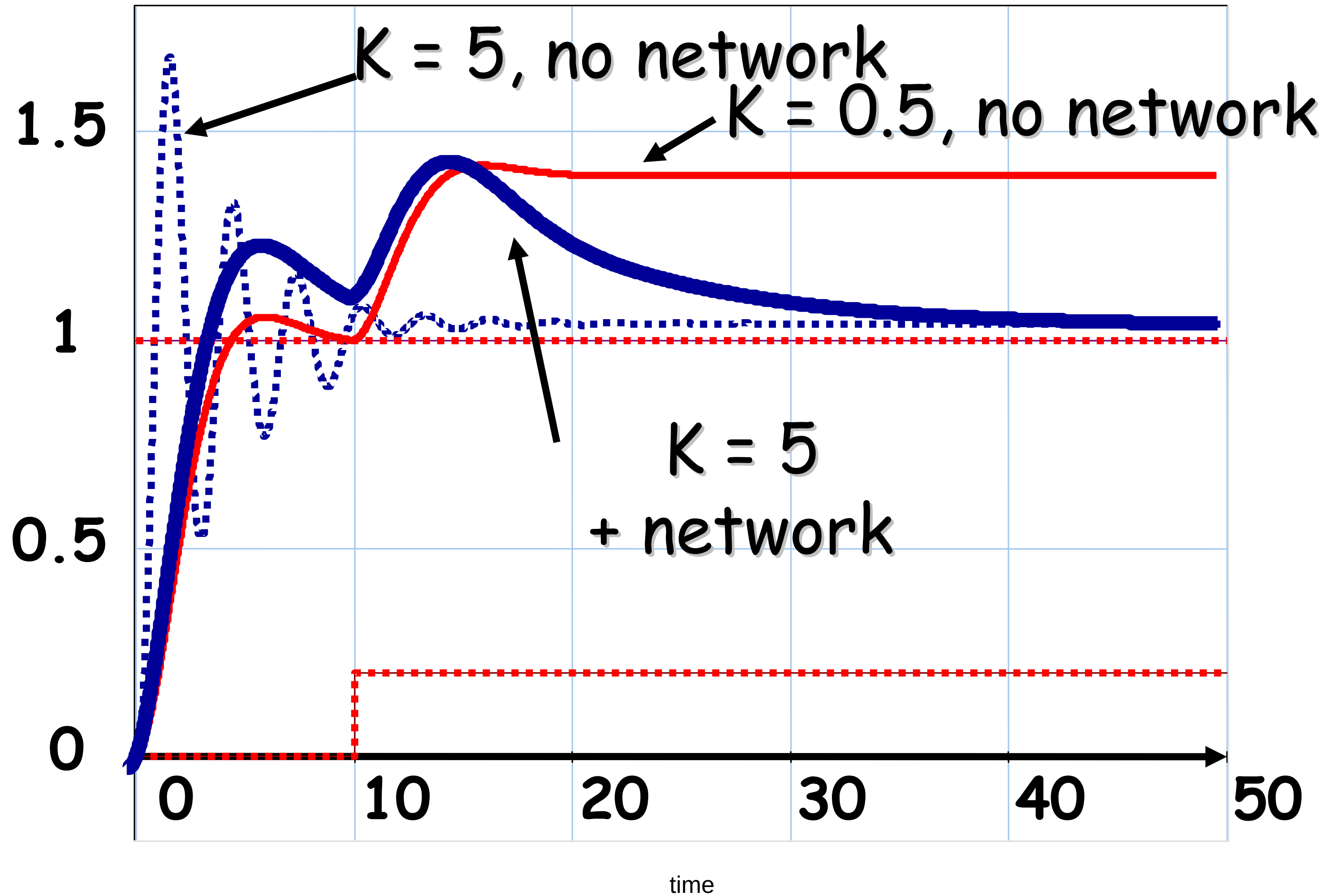
Response



Response



Response

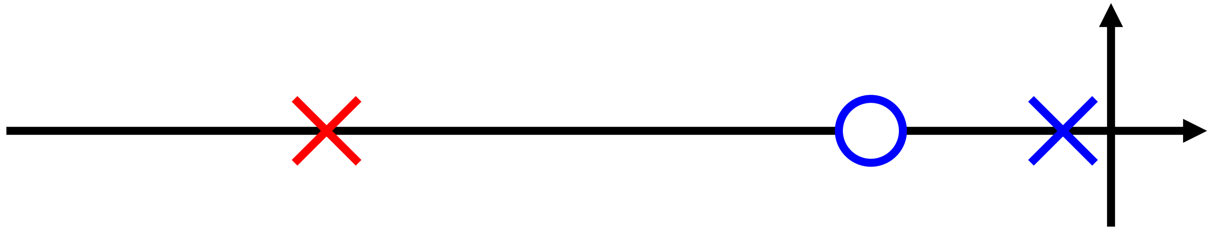

K = 5, no network
K = 0.5, no network
K = 5
+ network
1.5
1
0.5
0
0
10
20
30
40
50
time

Conclusions (1)

- Lag network
  - almost no influence on shape of the root locus at the desired location of the closed loop poles
  - dynamics similar to low-gain system
  - almost no influence on K'_L
  - K_L increases with a factor a (e.g. 10)
  - accuracy increases

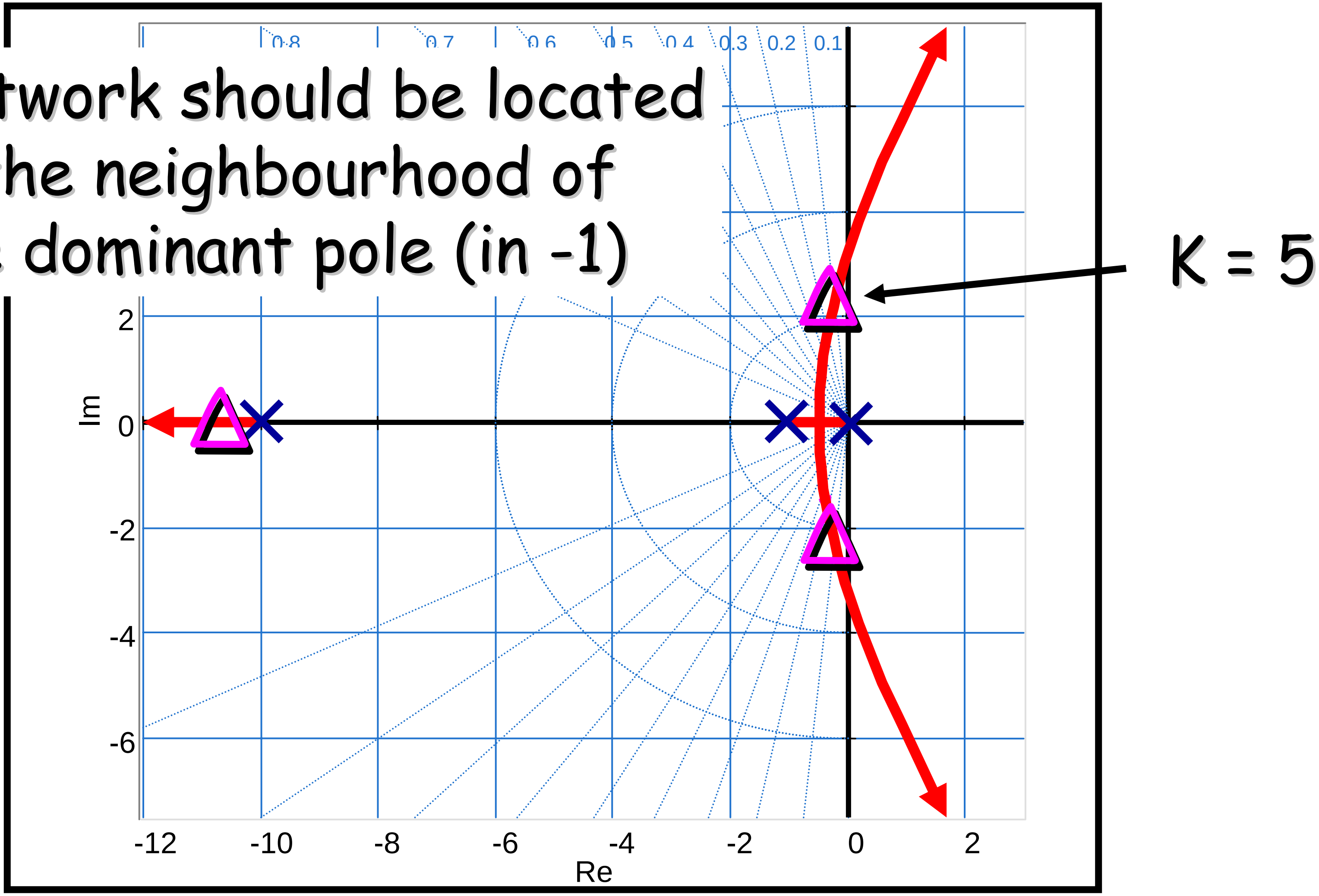
Conclusions (2)

- Lag network
  - located close to the origin
  - a kind of 'dipole': "no" influence on the shape of the root locus
  - zero a factor 10 right of the dominant pole
  - pole a factor *a* (e.g. 10) right of the zero



Lead network (phase lead)

Network should be located in the neighbourhood of the dominant pole (in -1)



Lead network (phase lead)

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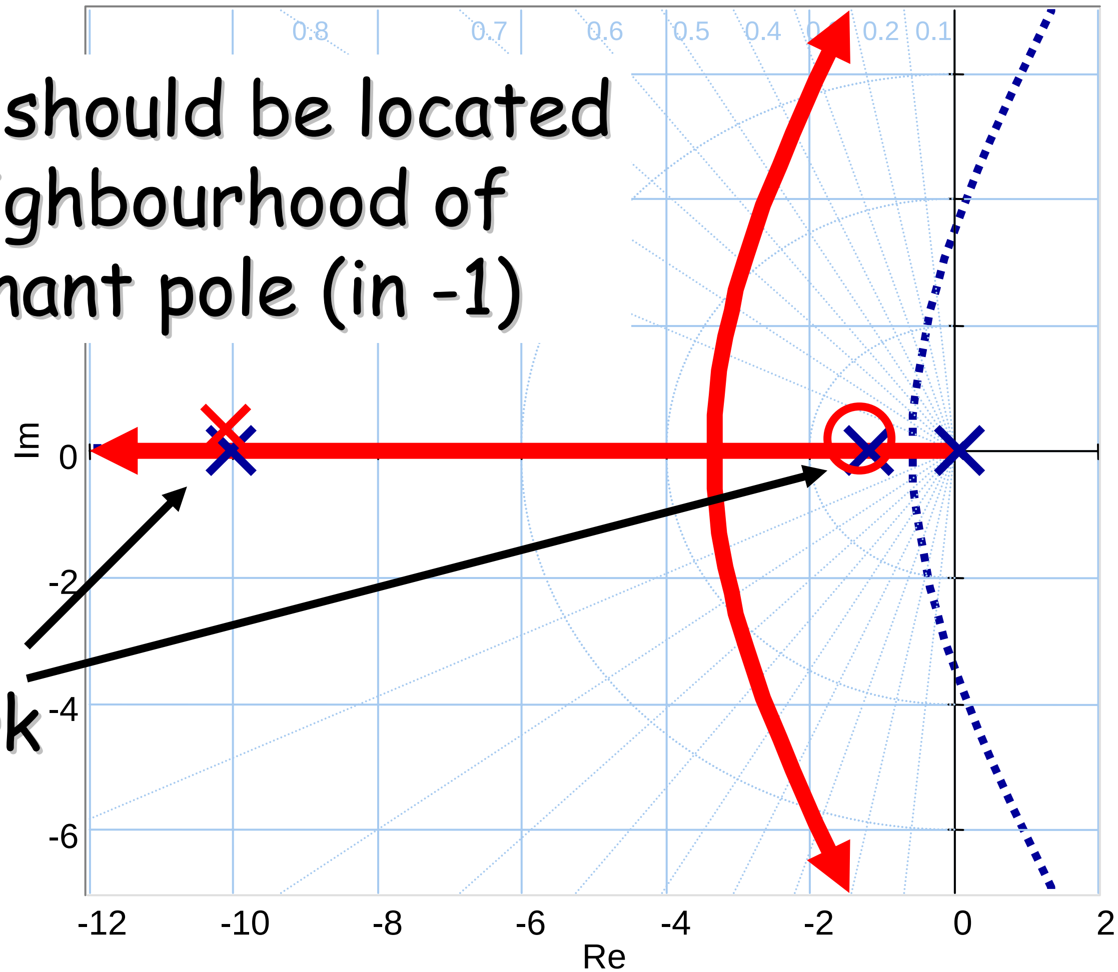
Network should be located in the neighbourhood of the dominant pole (in -1)



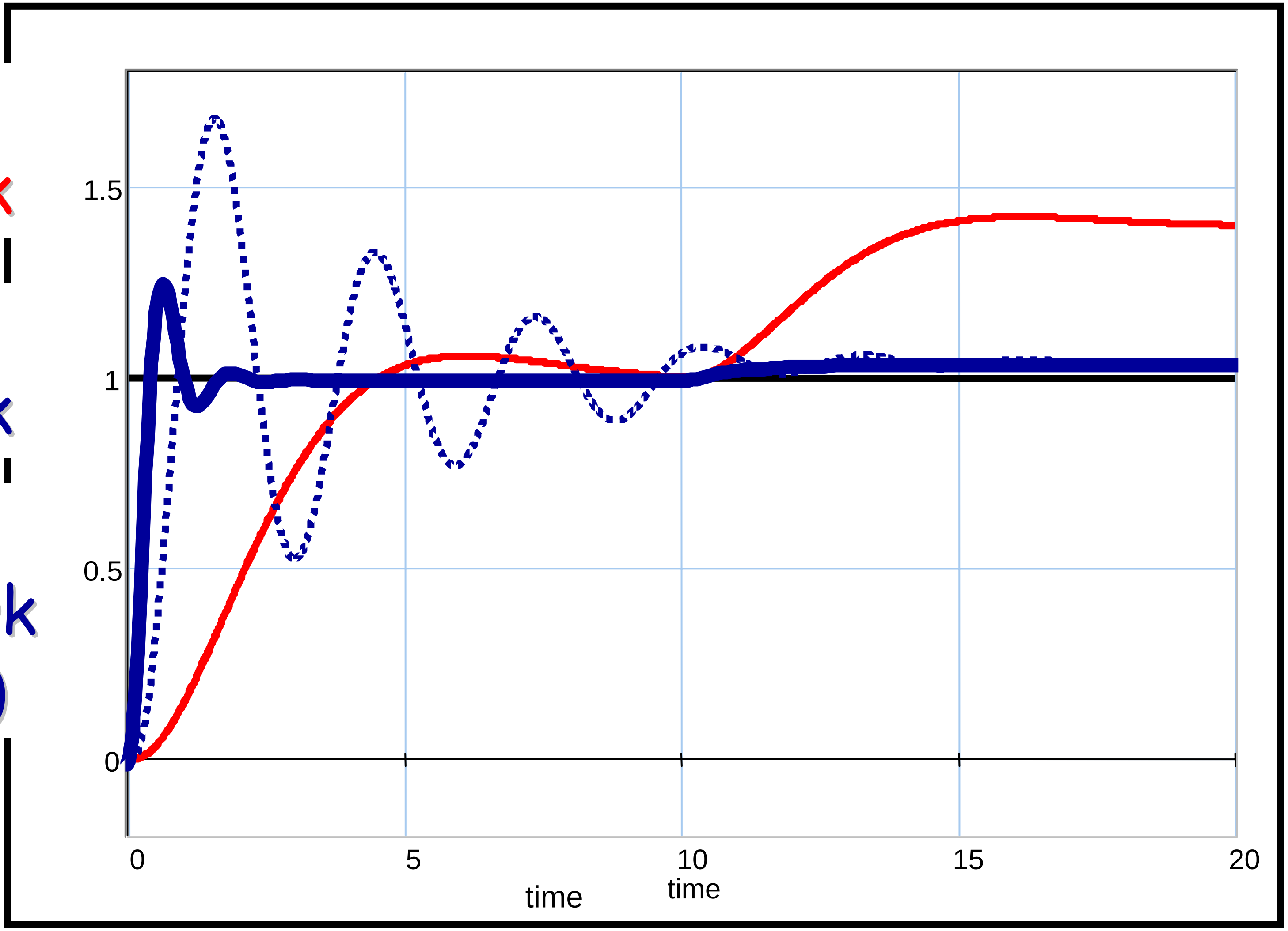
Lead network

Responses

K = 0.5
no network

K = 5
no network

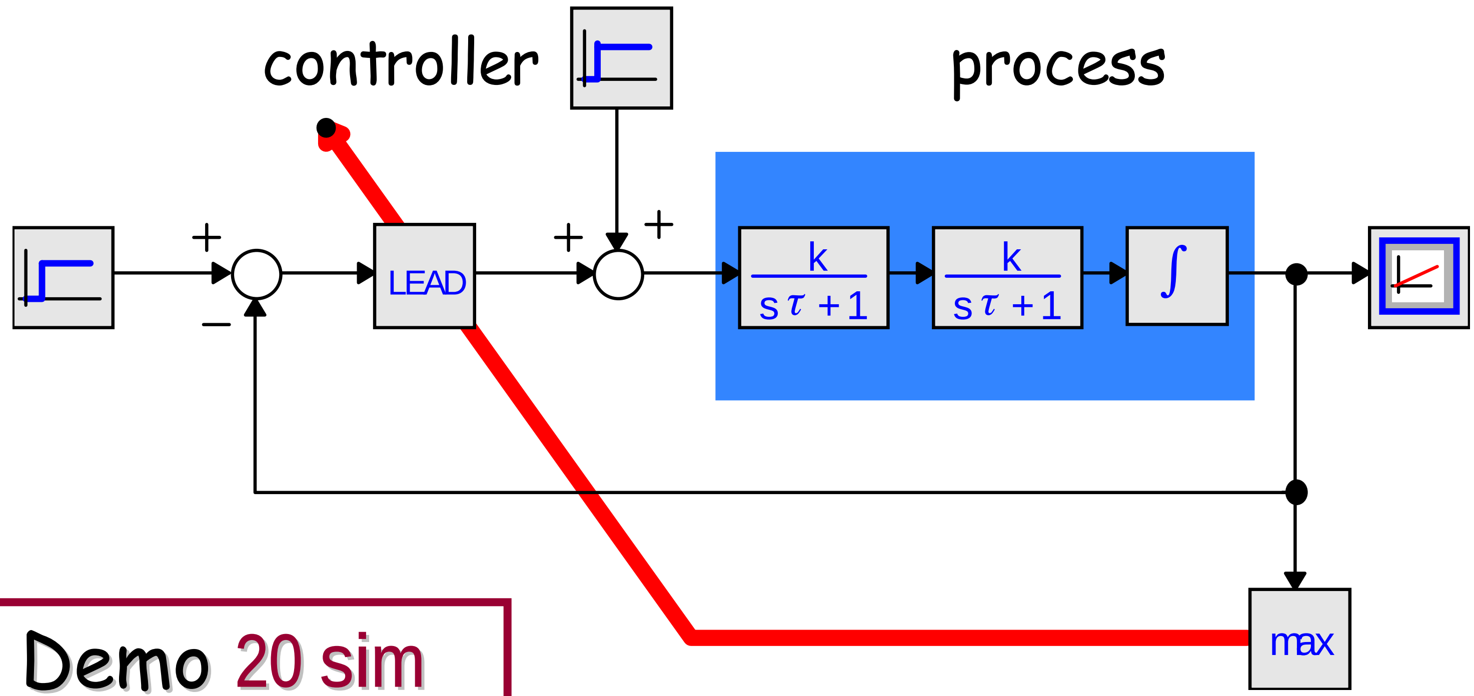
K = 5
lead network
(zero in -1)



How to find optimal τ?

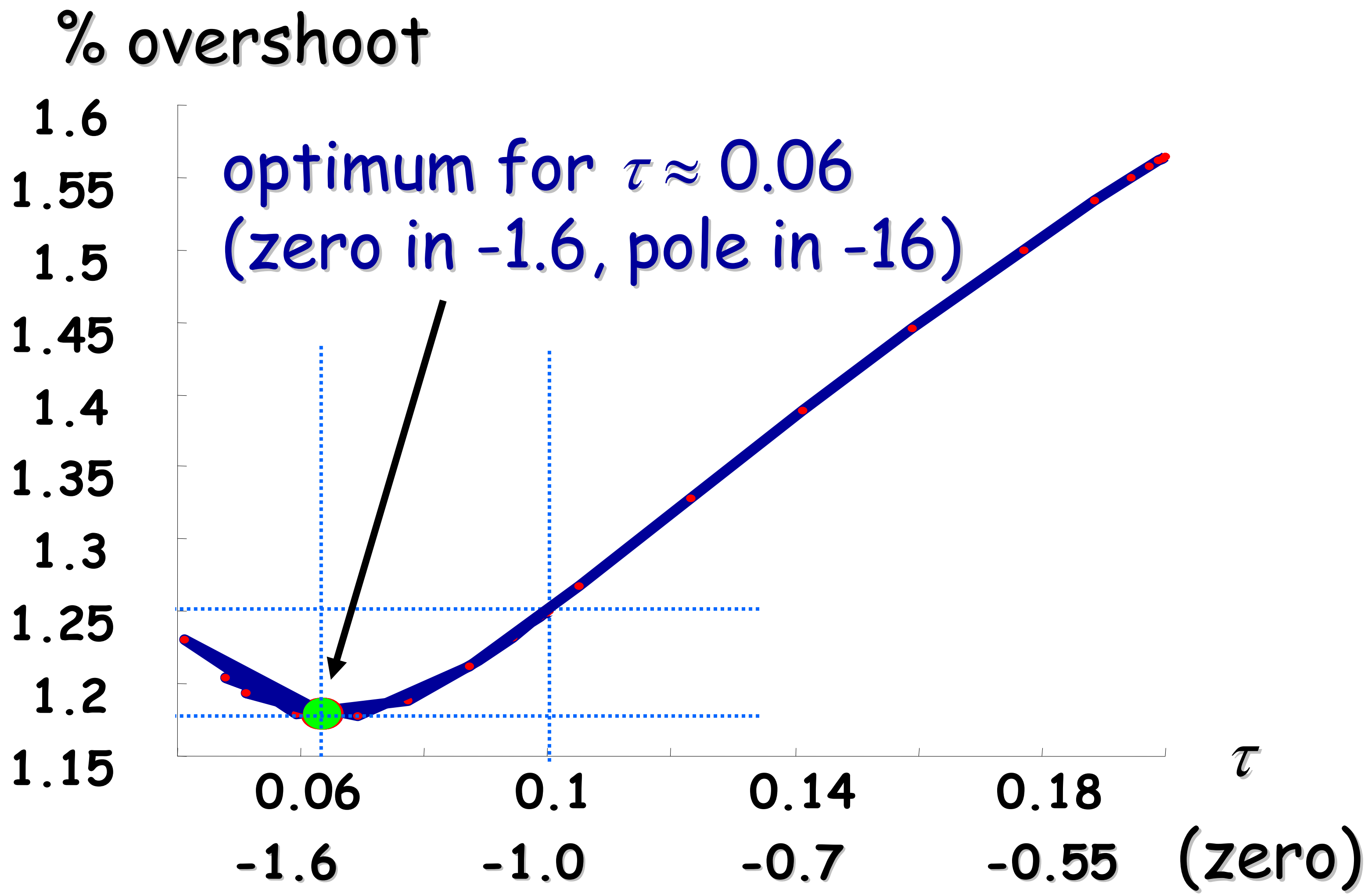
- Trial and error
- Optimisation in 20-sim
- tau-locus

Optimisation in 20-sim

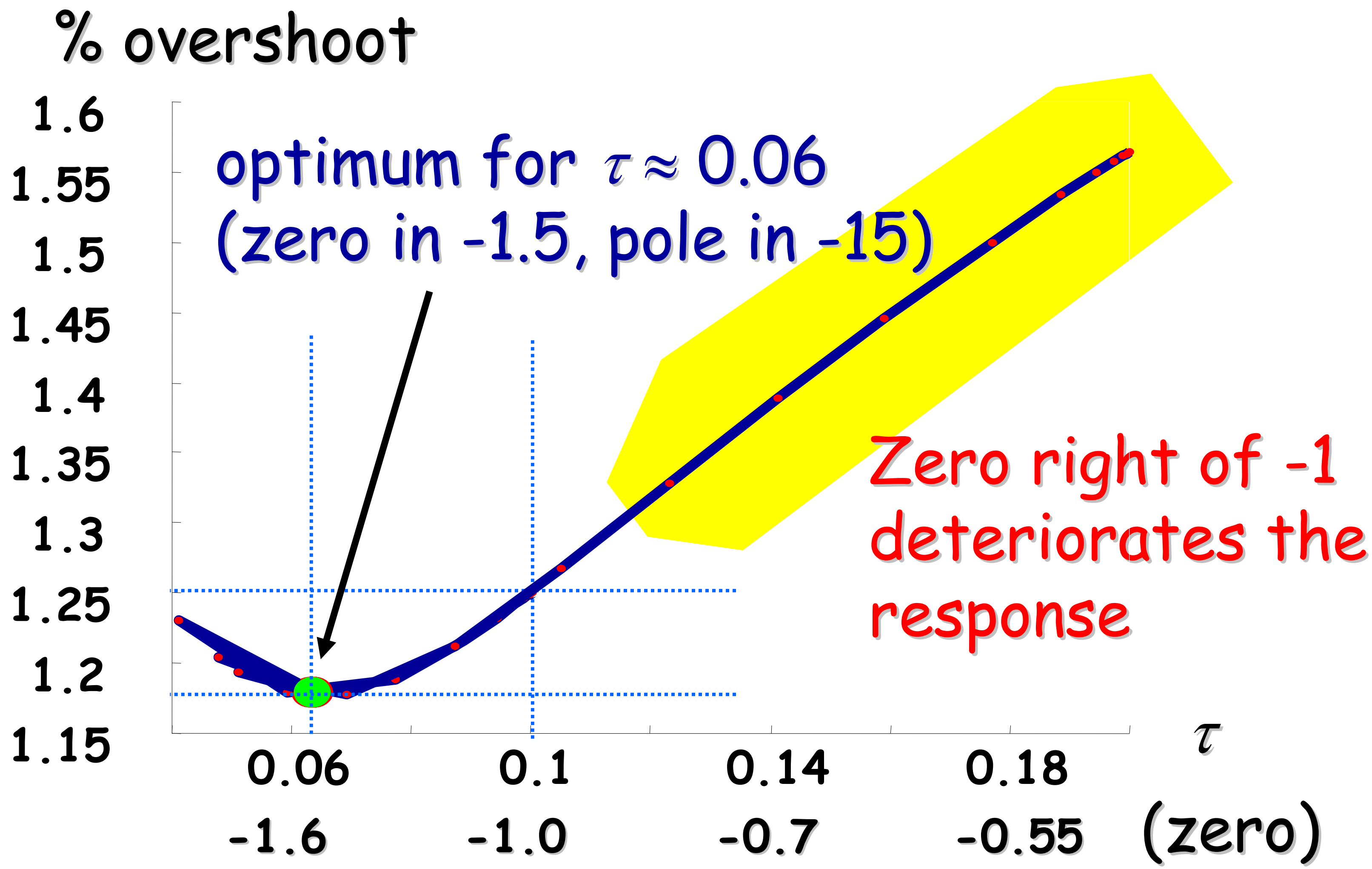


Demo 20 sim optimisation of lead network

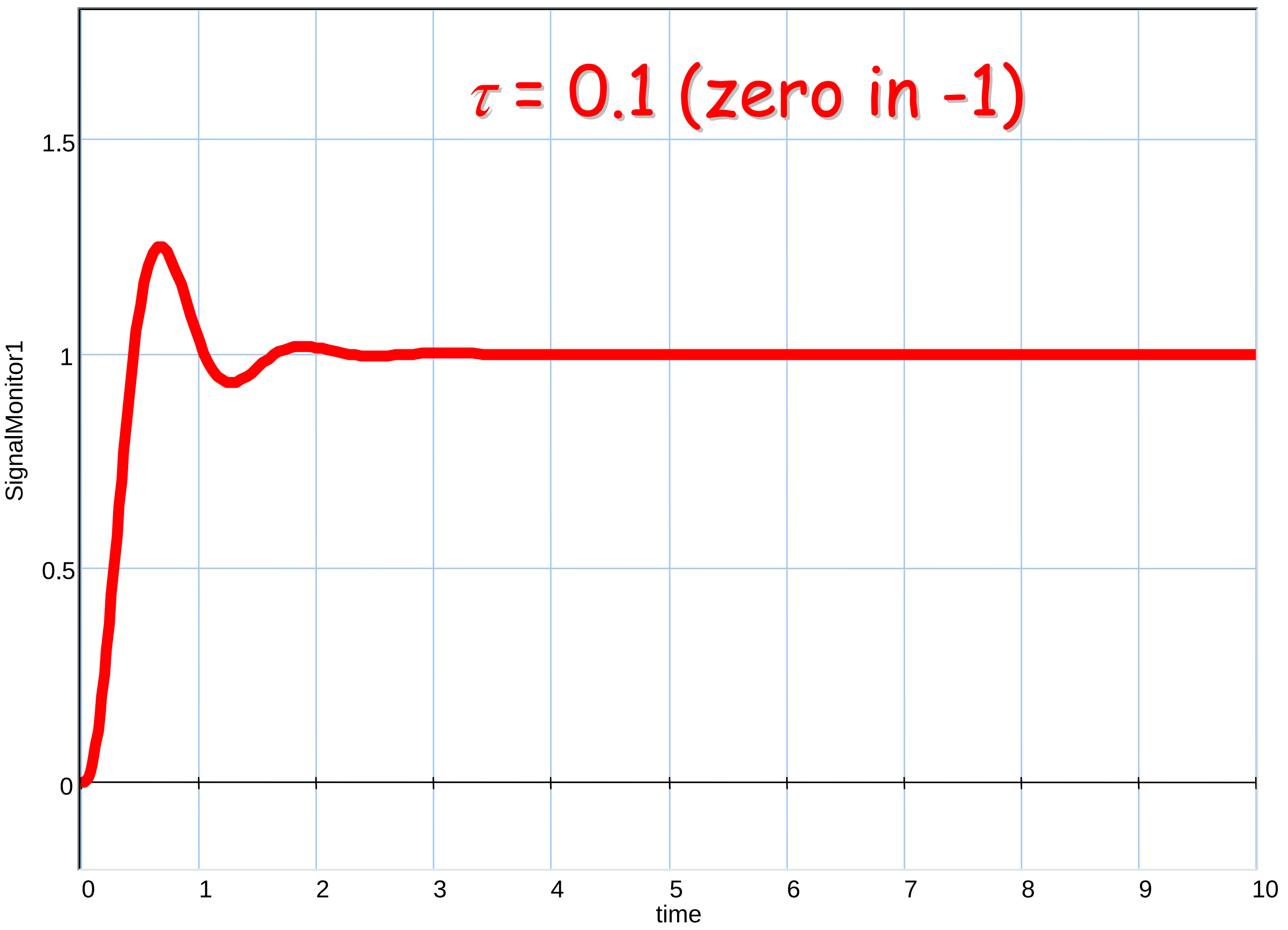
Optimum (sensitivity)



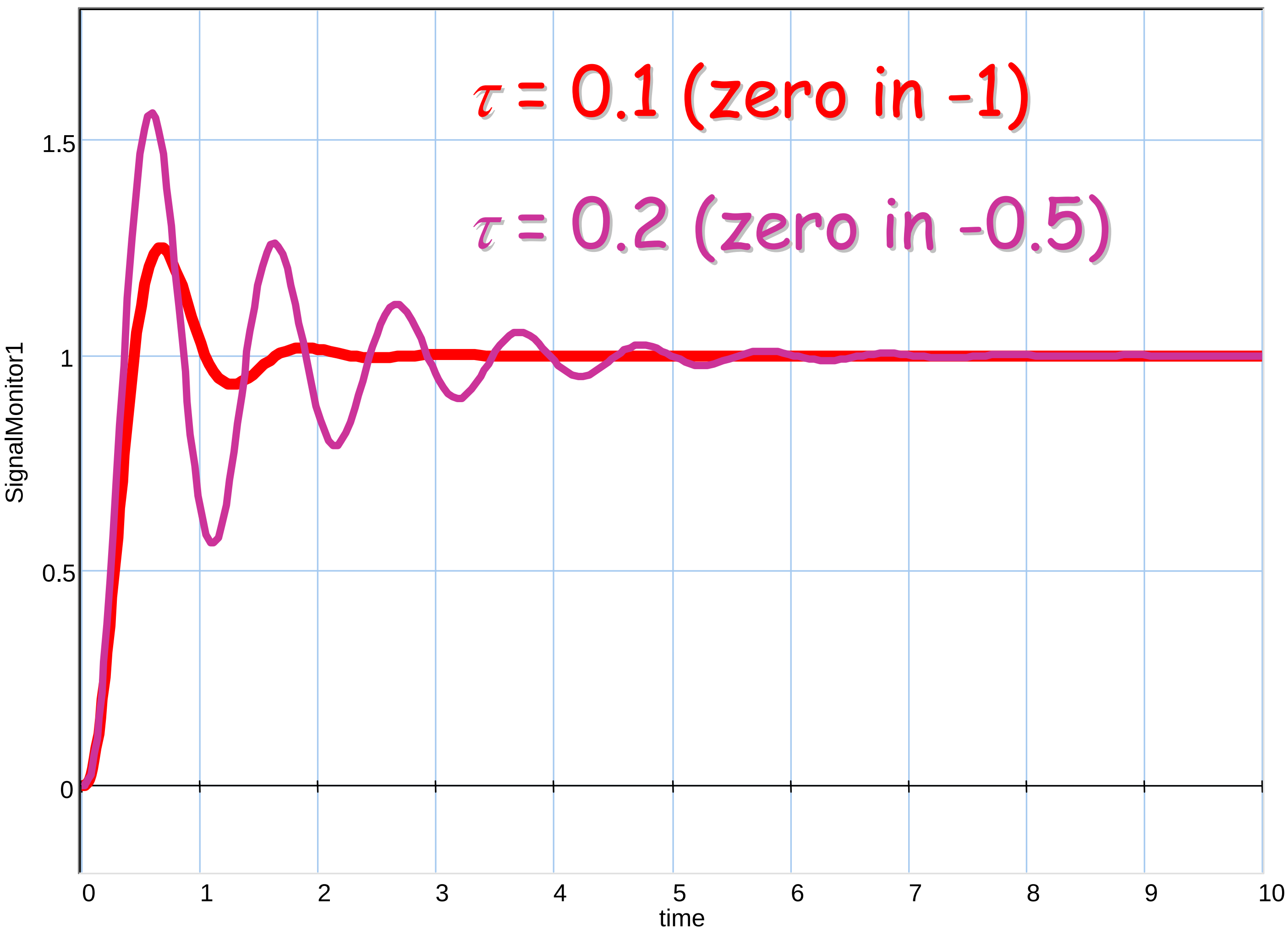
Optimum (sensitivity)



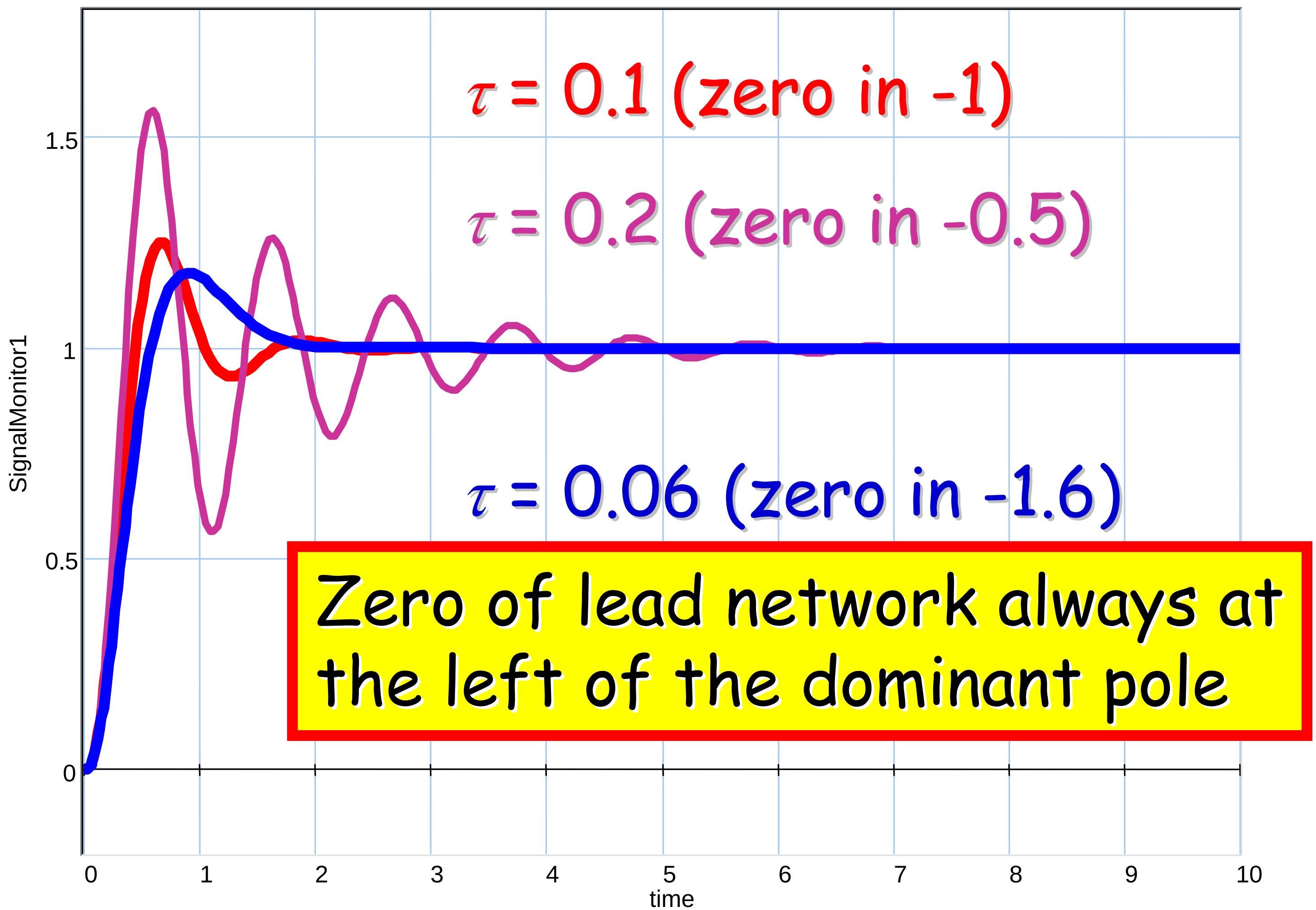
Responses



Responses


τ = 0.1 (zero in -1)
τ = 0.2 (zero in -0.5)
SignalMonitor1
1.5
1
0.5
0
0
1
2
3
4
5
6
7
8
9
10
time

Responses


τ = 0.1 (zero in -1)
τ = 0.2 (zero in -0.5)
τ = 0.06 (zero in -1.6)
Zero of lead network always at the left of the dominant pole
SignalMonitor1
time
1.5
1
0.5
0
0
1
2
3
4
5
6
7
8
9
10

τ-locus (1)

$$H_L = \frac{K_L'\left(10s\tau + 1\right)}{s(s+1)(s+10)(s\tau+1)}$$

root locus equation: $1 + H_L = 0$

$$s(s+1)(s+10)(s\tau+1) + K_L' 10 s\tau + K_L' = 0$$

$$s(s+1)(s+10)s\tau + K_L' 10 s\tau +$$
$$+ s(s+1)(s+10) + K_L' = 0$$

τ-locus (2)

$$s(s+1)(s+10)s\tau + K_L' 10 s\tau + \\ + s(s+1)(s+10) + K_L' = 0$$

$$s\tau\left[s(s+1)(s+10) + K_L' 10\right] + \\ + \left[s(s+1)(s+10) + K_L'\right] = 0$$

with $\tau = \dfrac{1}{b}$

Equation for τ-locus

$$-\frac{1}{b} = \frac{s(s+1)(s+10) + K_L'}{s\left[s(s+1)(s+10) + K_L' 10\right]}$$

τ-locus (3)

$$-\frac{1}{b} = \frac{s(s+1)(s+10)+K_L'}{s\left[s(s+1)(s+10)+K_L'10\right]}$$

Zeros are found by solving the numerator:

$$s(s+1)(s+10)+K_L' = 0$$

$$\frac{1}{s(s+1)(s+10)} = -\frac{1}{K_L'}$$

Root locus equation for K_L' = 50 (K_L = 5)

τ-locus (4)

$$-\frac{1}{b} = \frac{s(s+1)(s+10)+K_L'}{s\left[s(s+1)(s+10)+K_L' 10\right]}$$

Poles are found by solving the denominator:

$$s\left[s(s+1)(s+10)+K_L' 10\right] = 0$$

$$\frac{1}{s(s+1)(s+10)} = -\frac{1}{10K_L'}$$, plus pole in $s = 0$

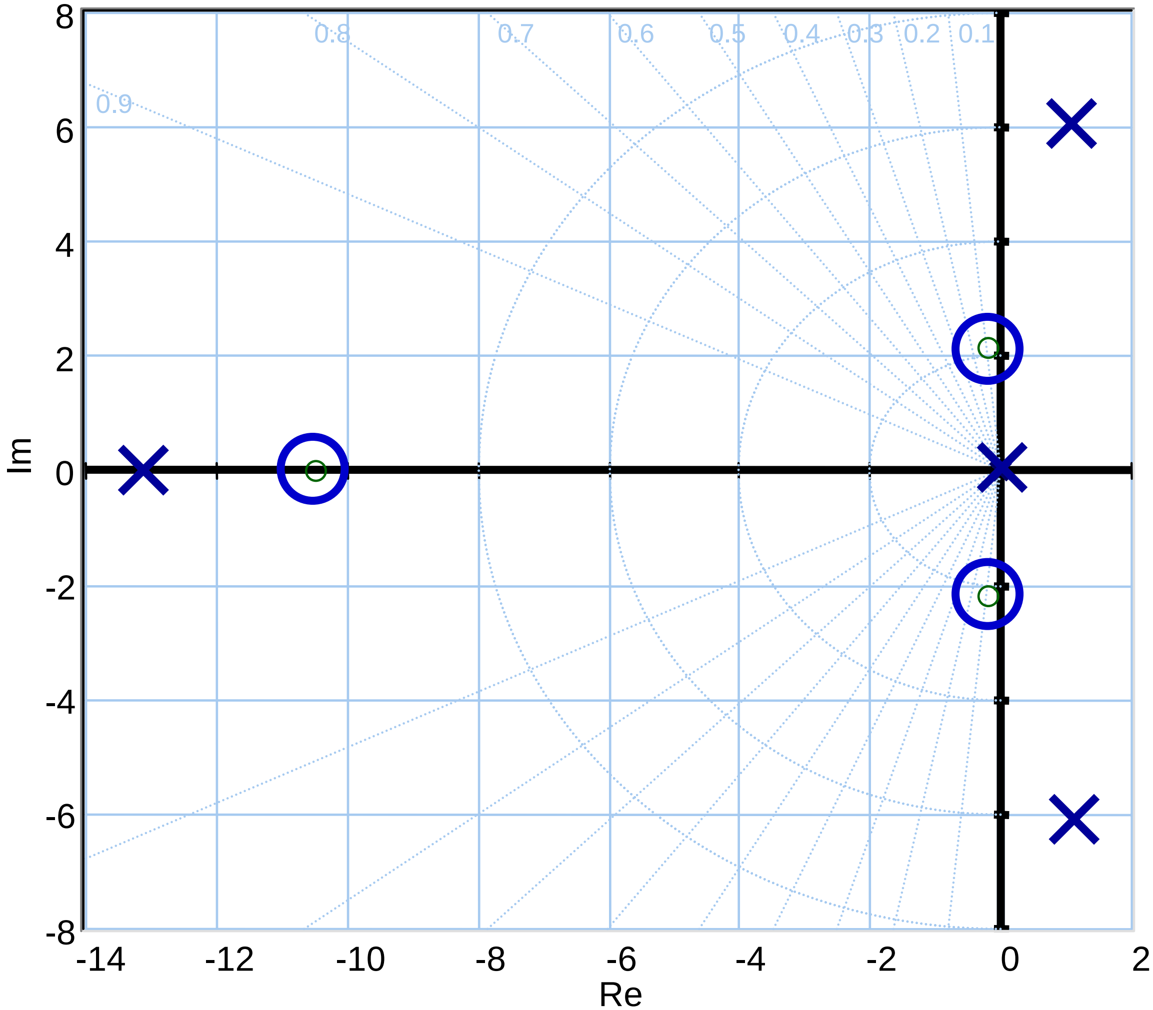
Root locus equation for $K_L' = 500$ ($K_L = 50$)

Steps

- Draw root locus of the uncompensated system
- Determine the roots for K = 5 → zero's
- Determine the roots for K = 50 → poles
- Draw the tau-locus

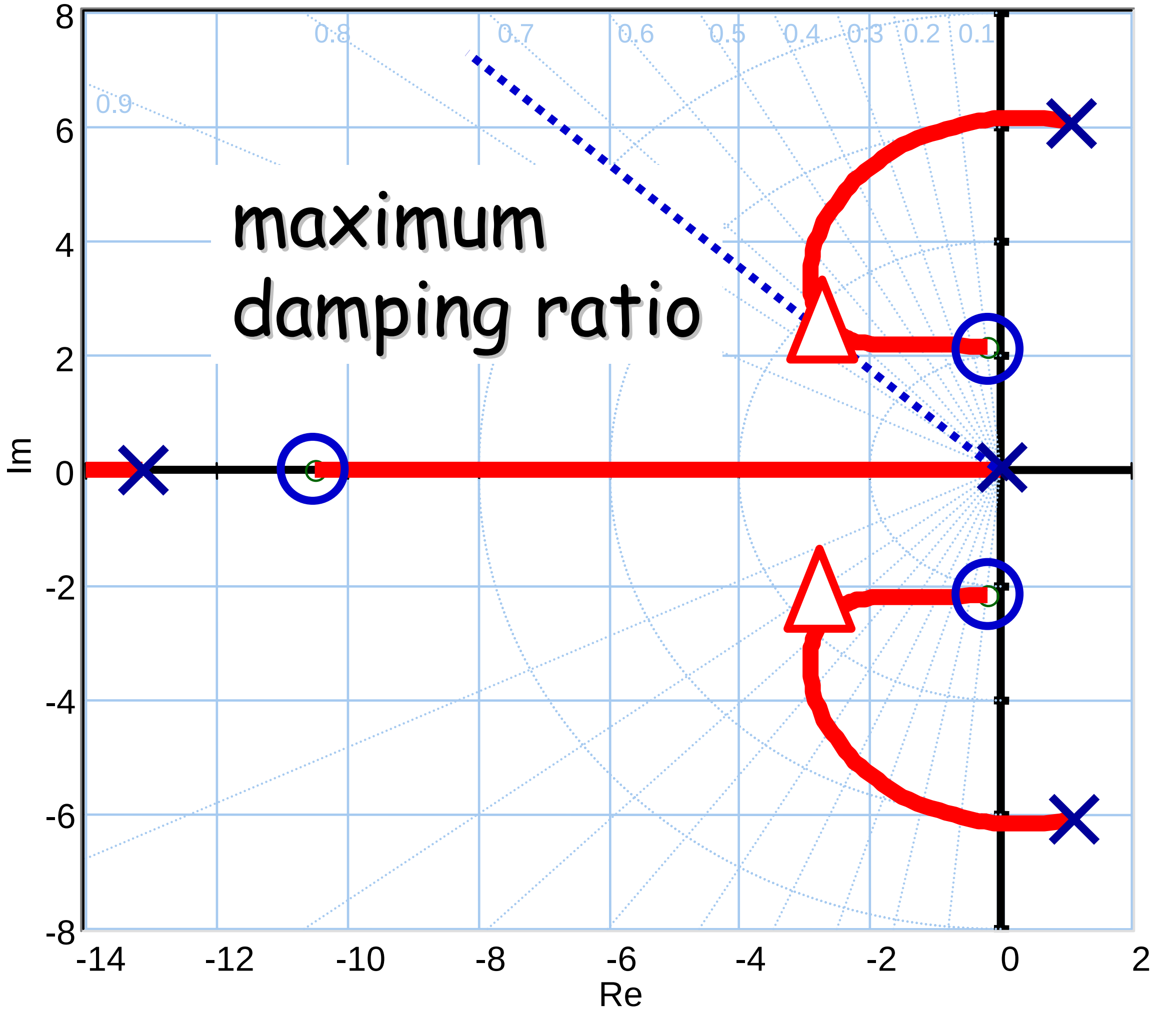
20 sim

τ-locus (5)



Set root locus gain to 1

τ-locus (5)



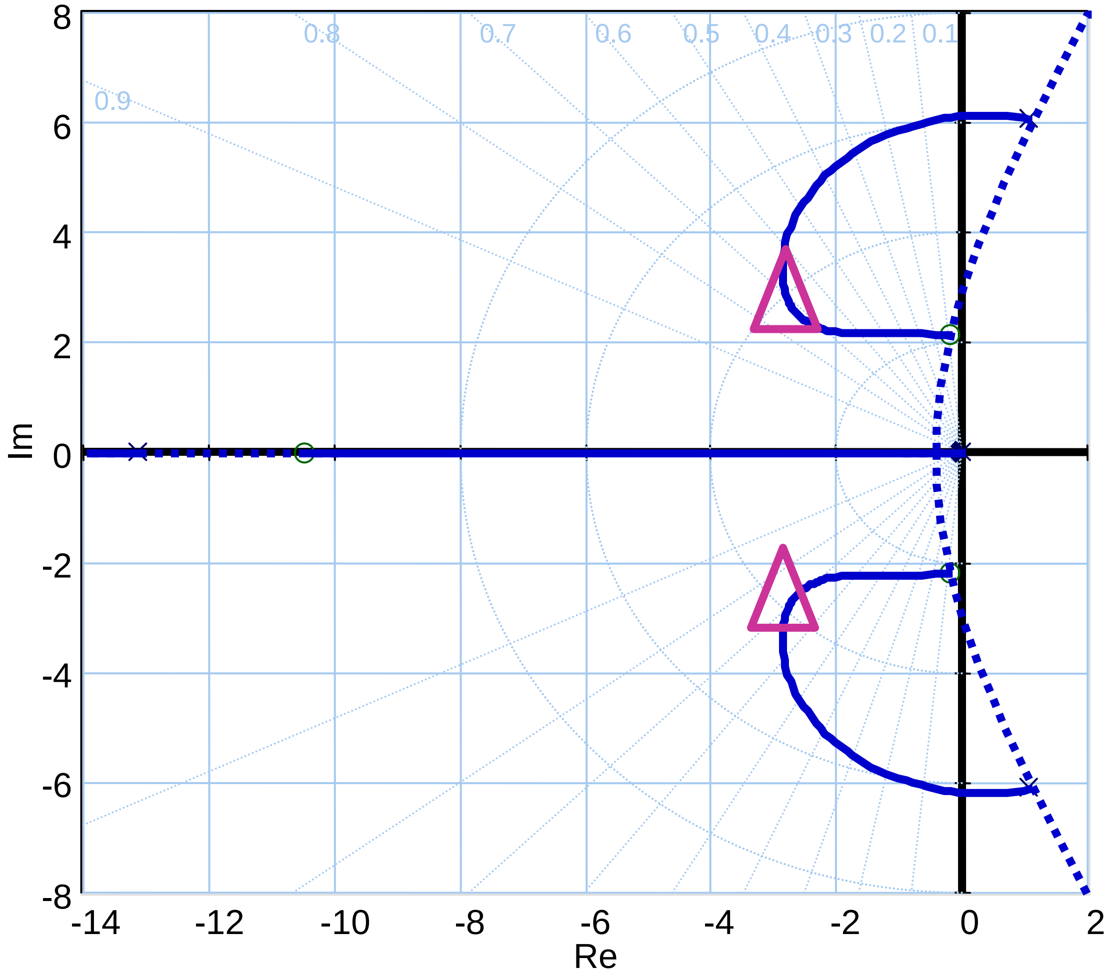
Set root locus gain to 1

$b = 17.5$

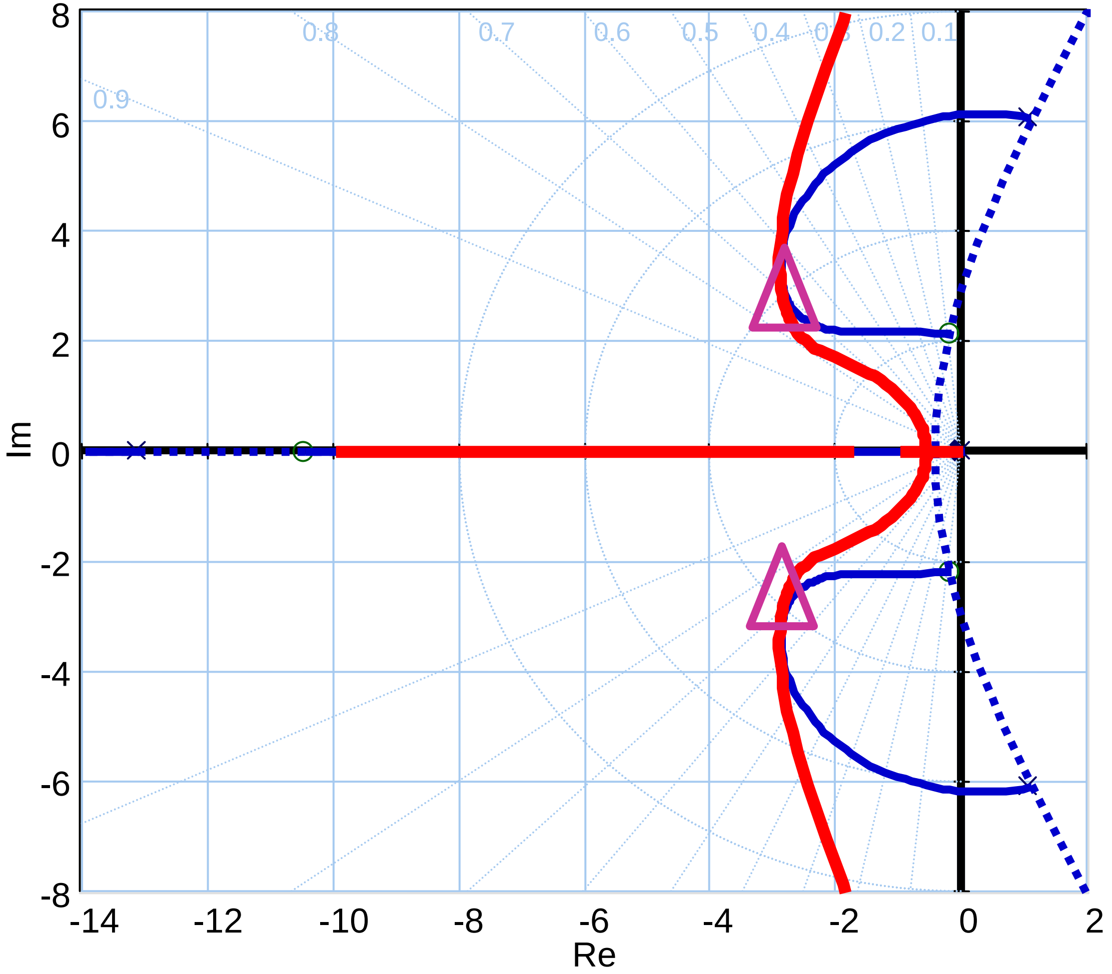
$\tau = 0.057$

20-sim

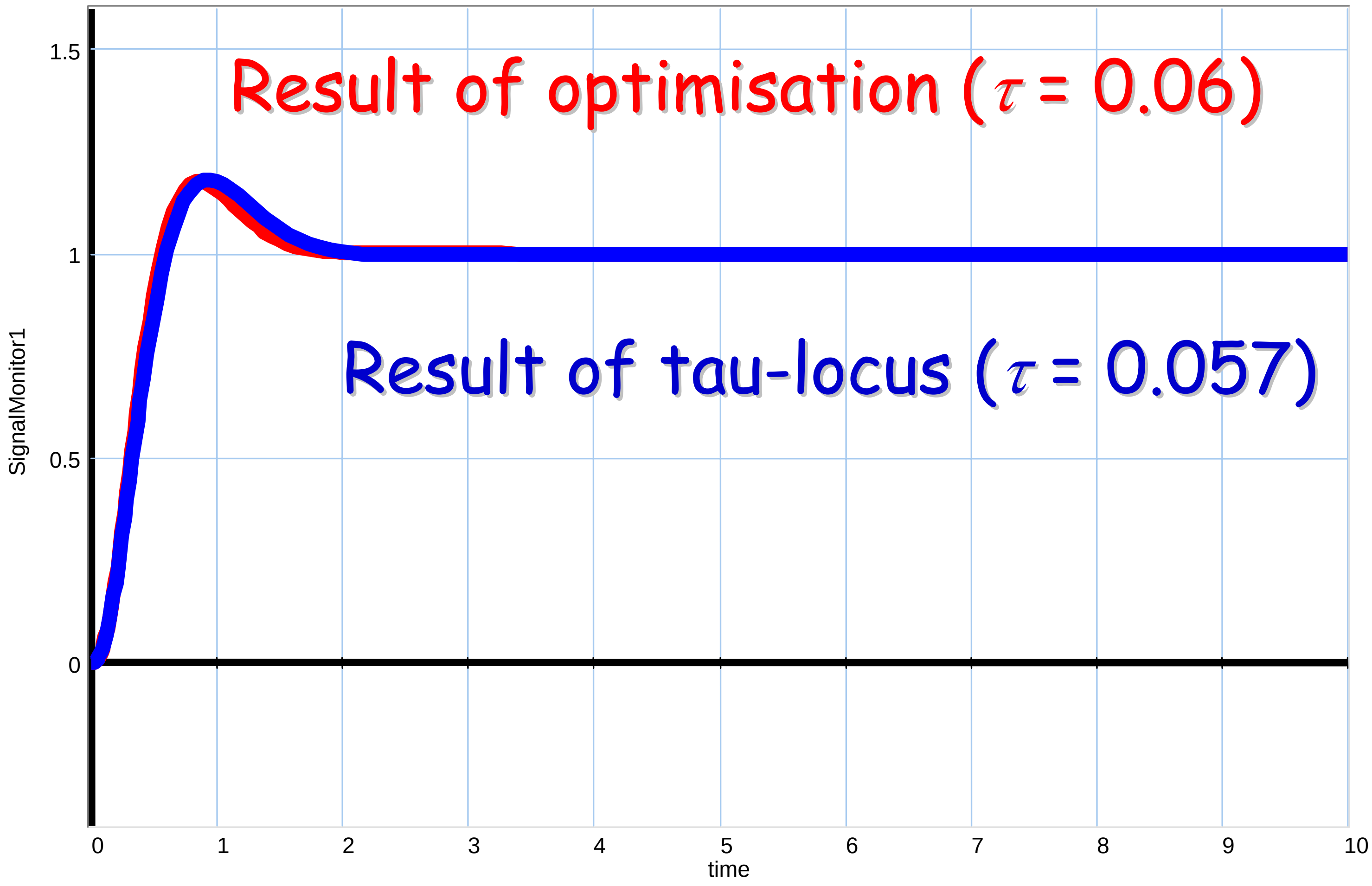
Resulting root locus



Resulting root locus



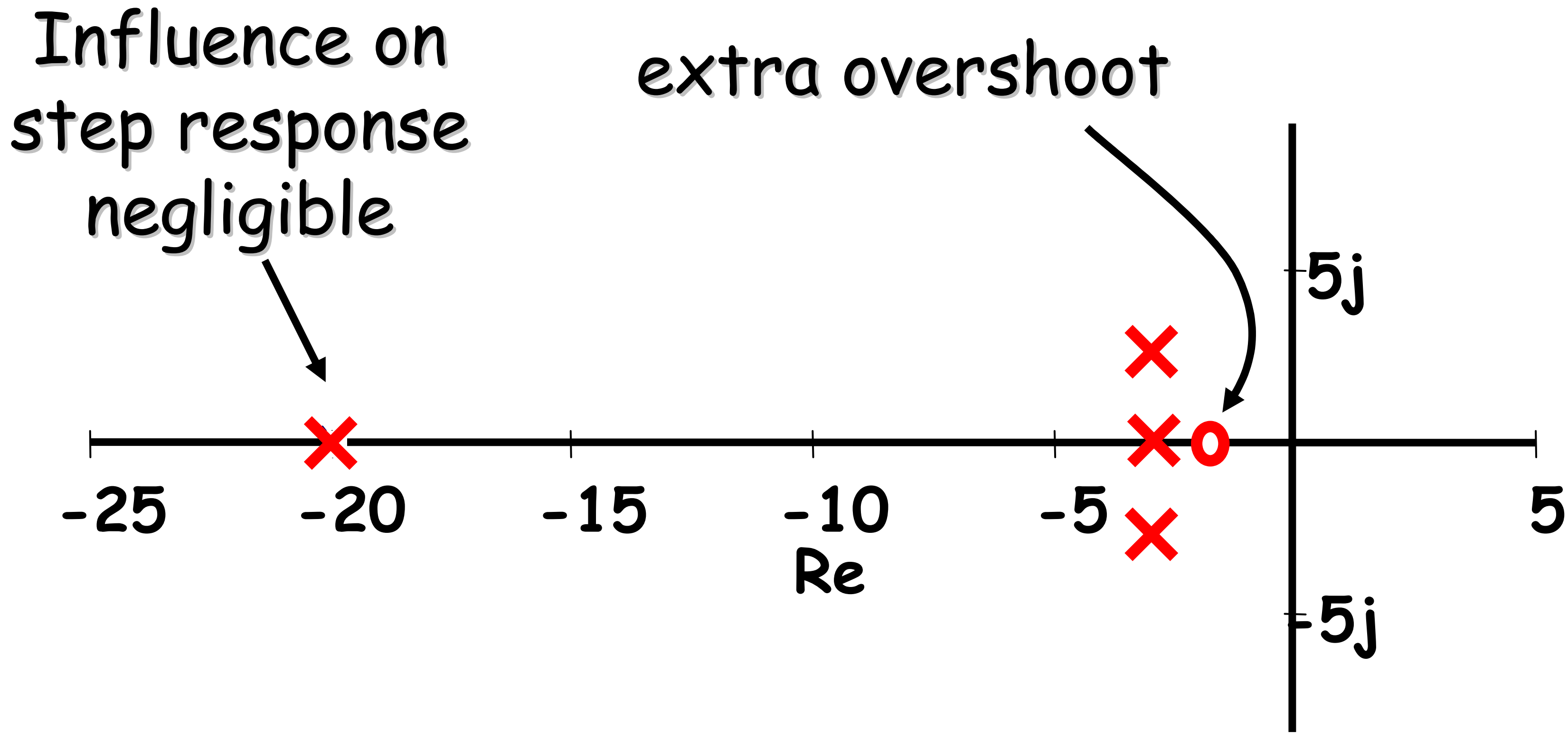
Responses


Result of optimisation (τ = 0.06)
Result of tau-locus (τ = 0.057)
SignalMonitor1
1.5
1
0.5
0
0
1
2
3
4
5
6
7
8
9
10
time

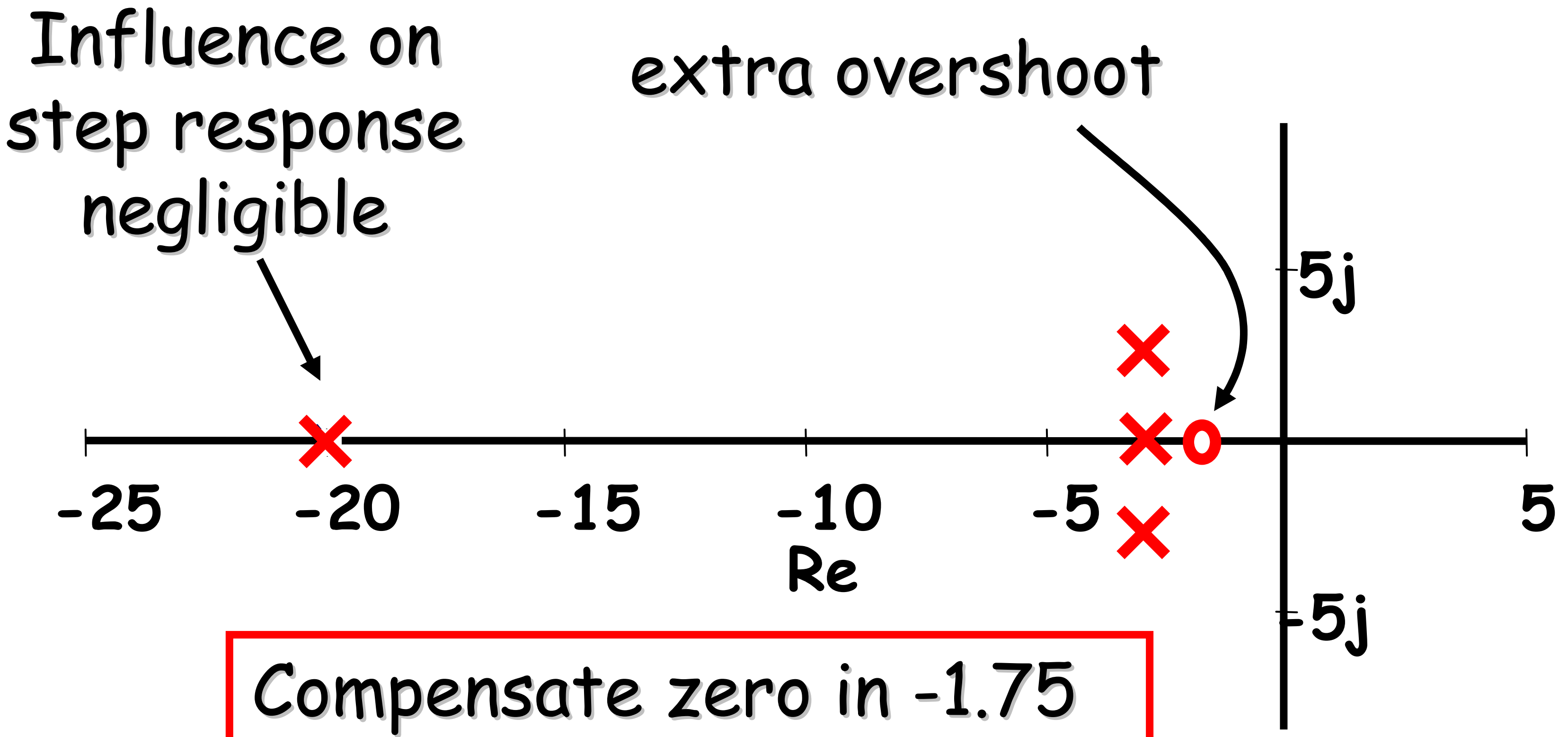
Evaluation

- Why is overshoot much larger than the 4% corresponding with $z = 0.7$?
- Examine closed-loop poles and zeros.

Close-loop poles

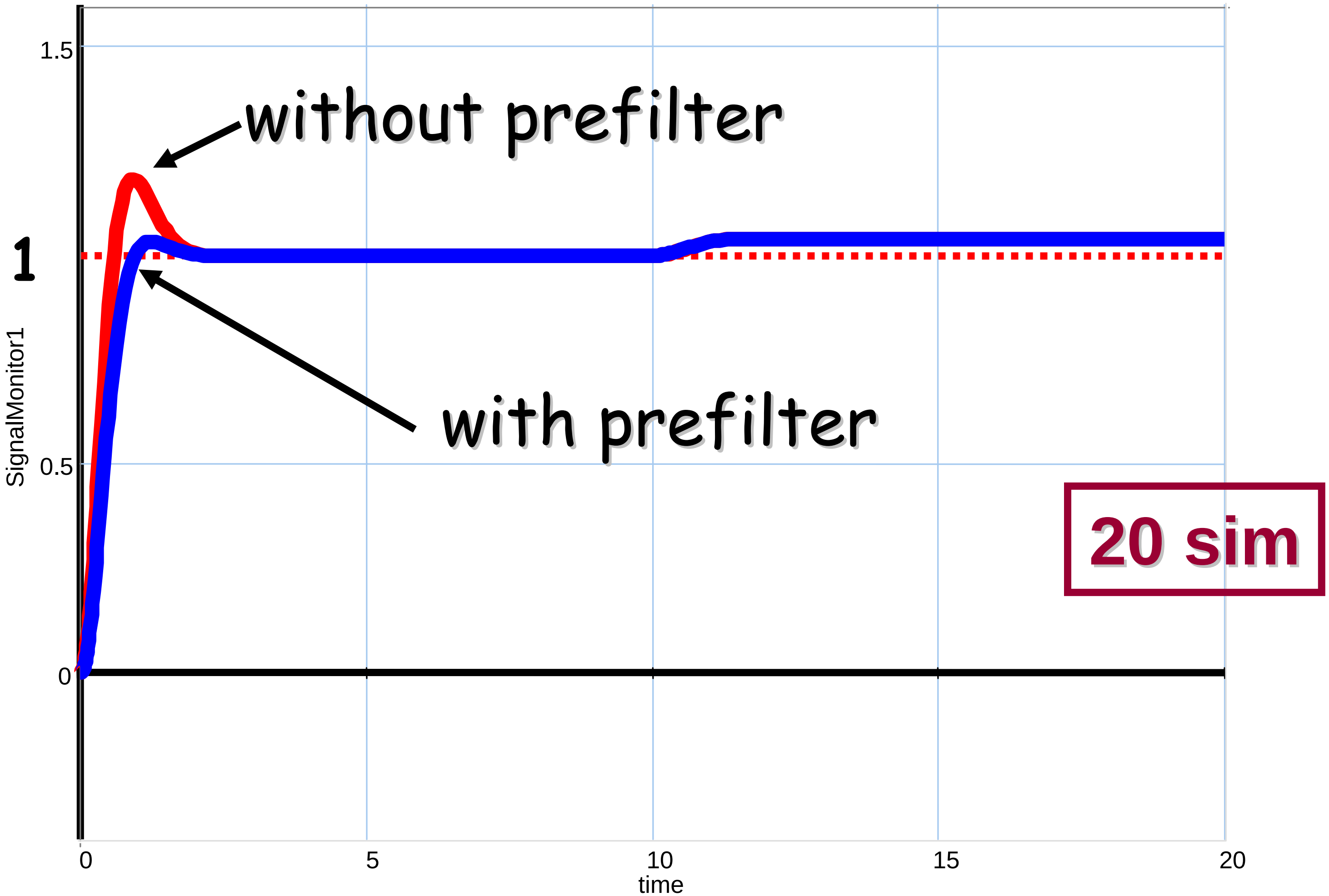


Close-loop poles

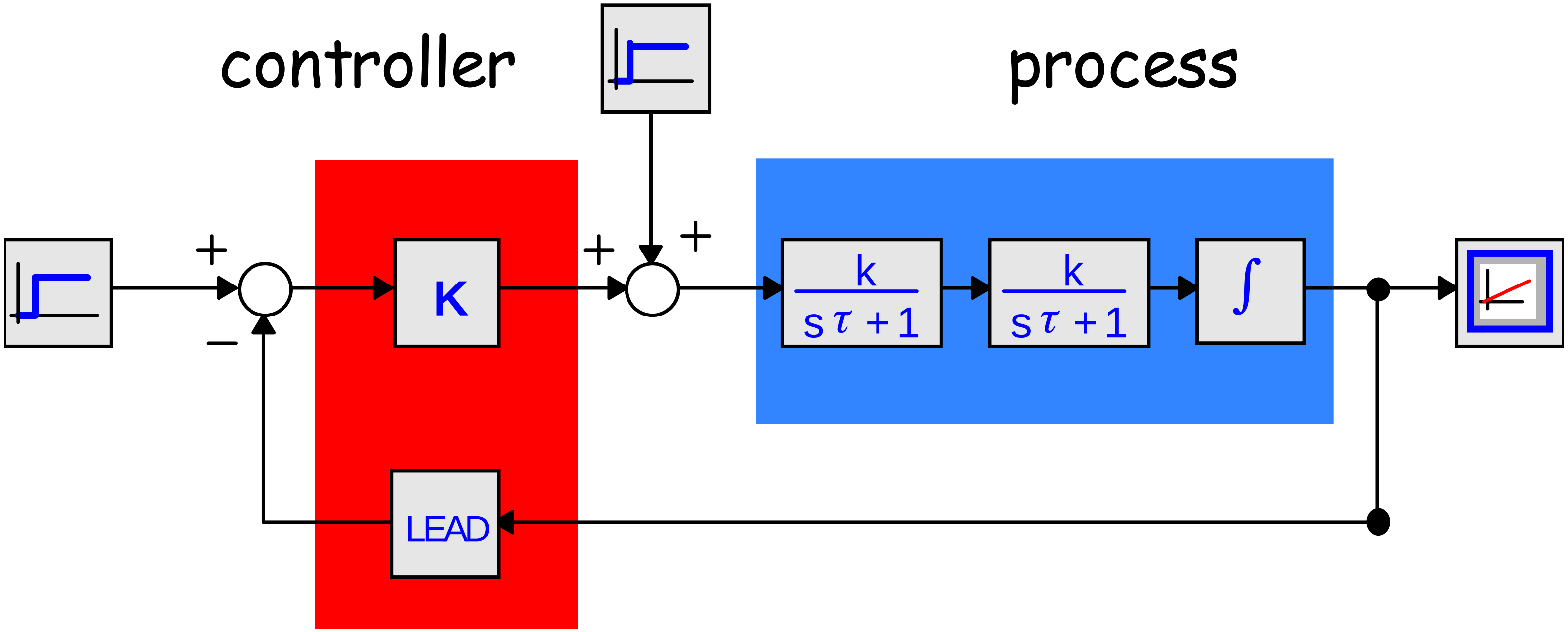


Compensate zero in -1.75 (and pole in -2.9) by means of a prefilter

Response + prefilter

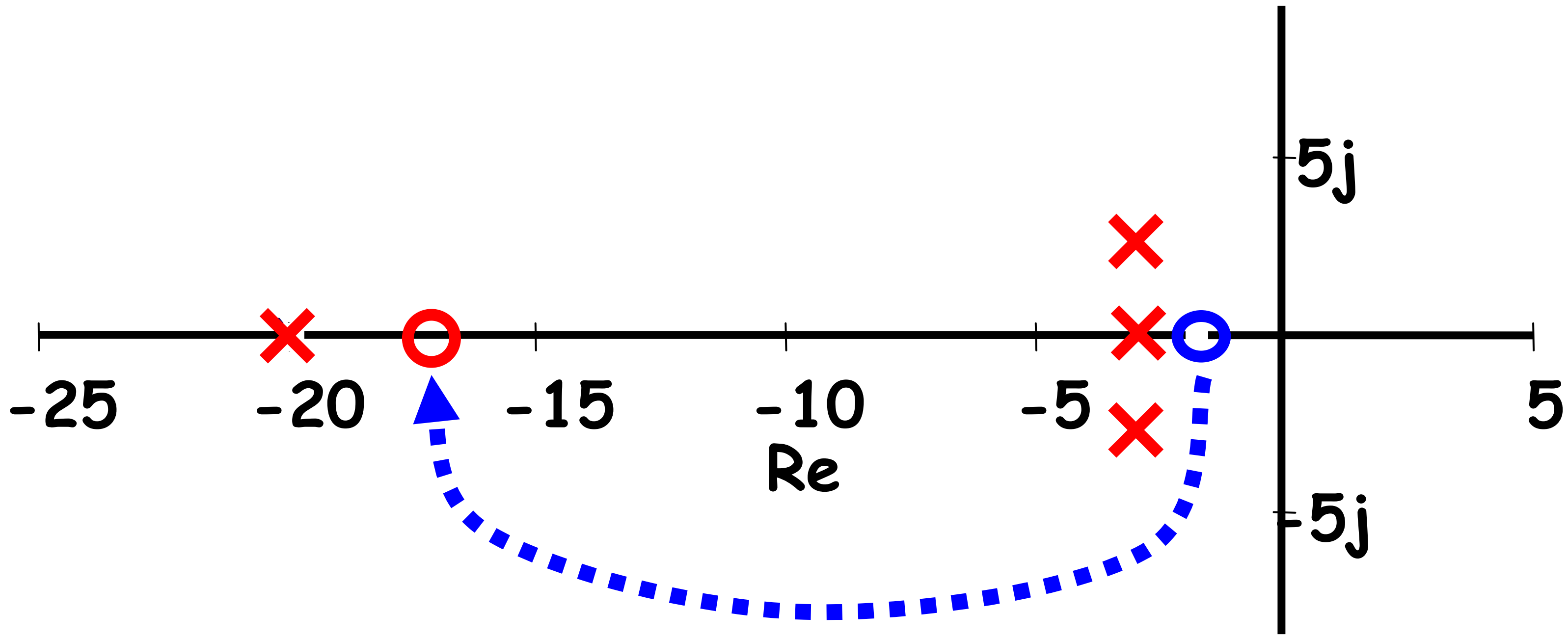


Lead network in feedback



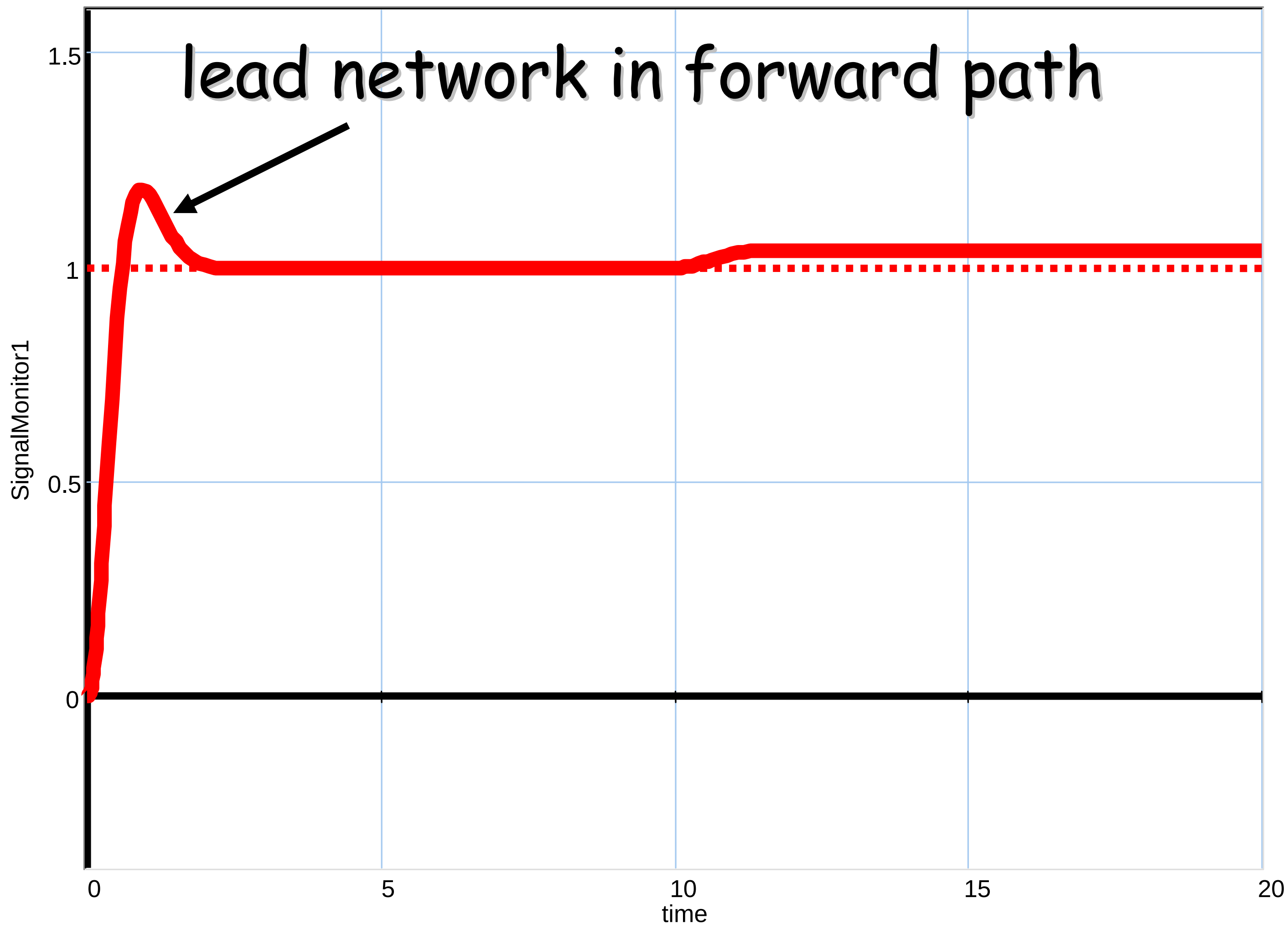
For C/R this implies that pole of lead network (in -17.5) is zero in C/R

Poles and zeros

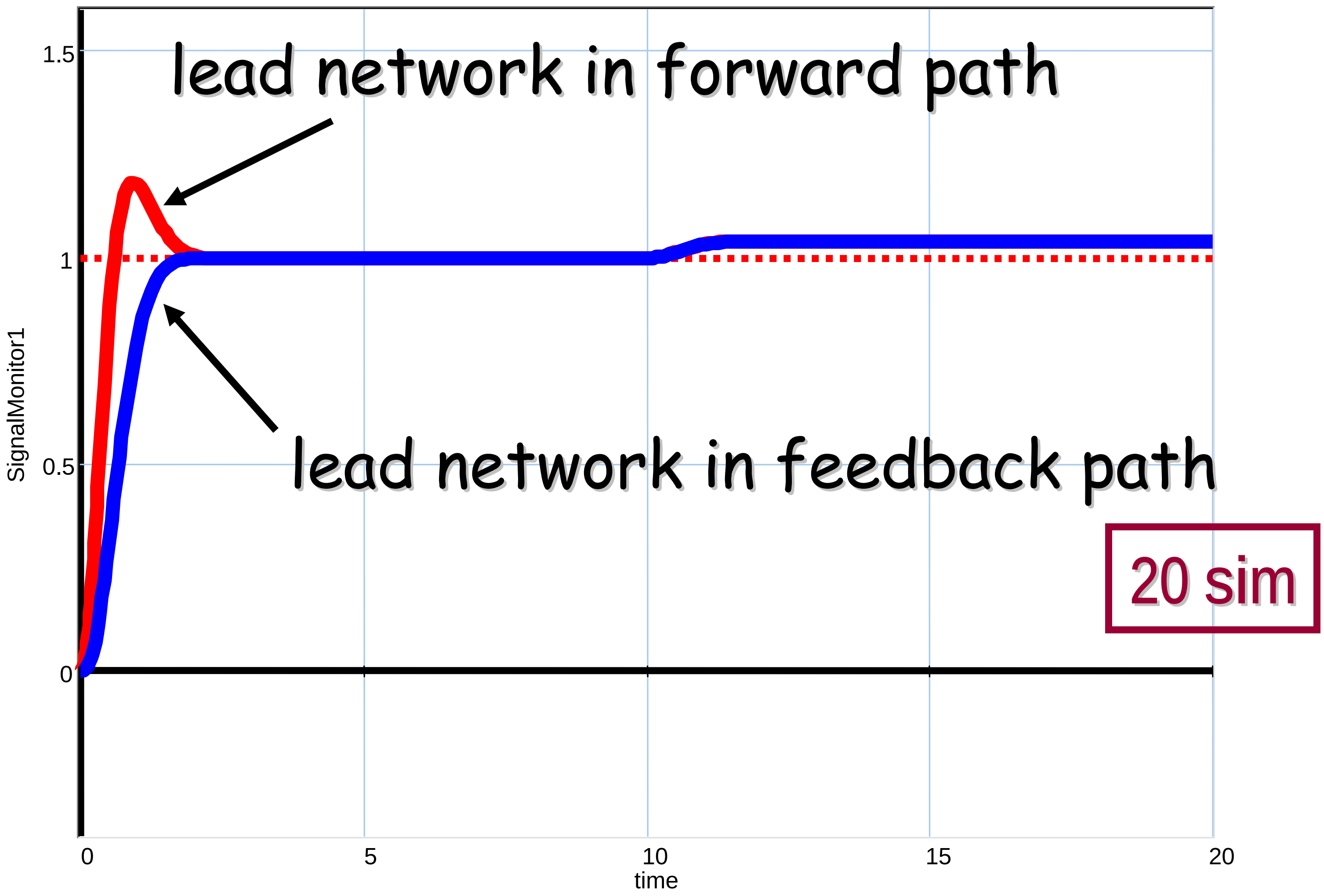


zero now in -17.5

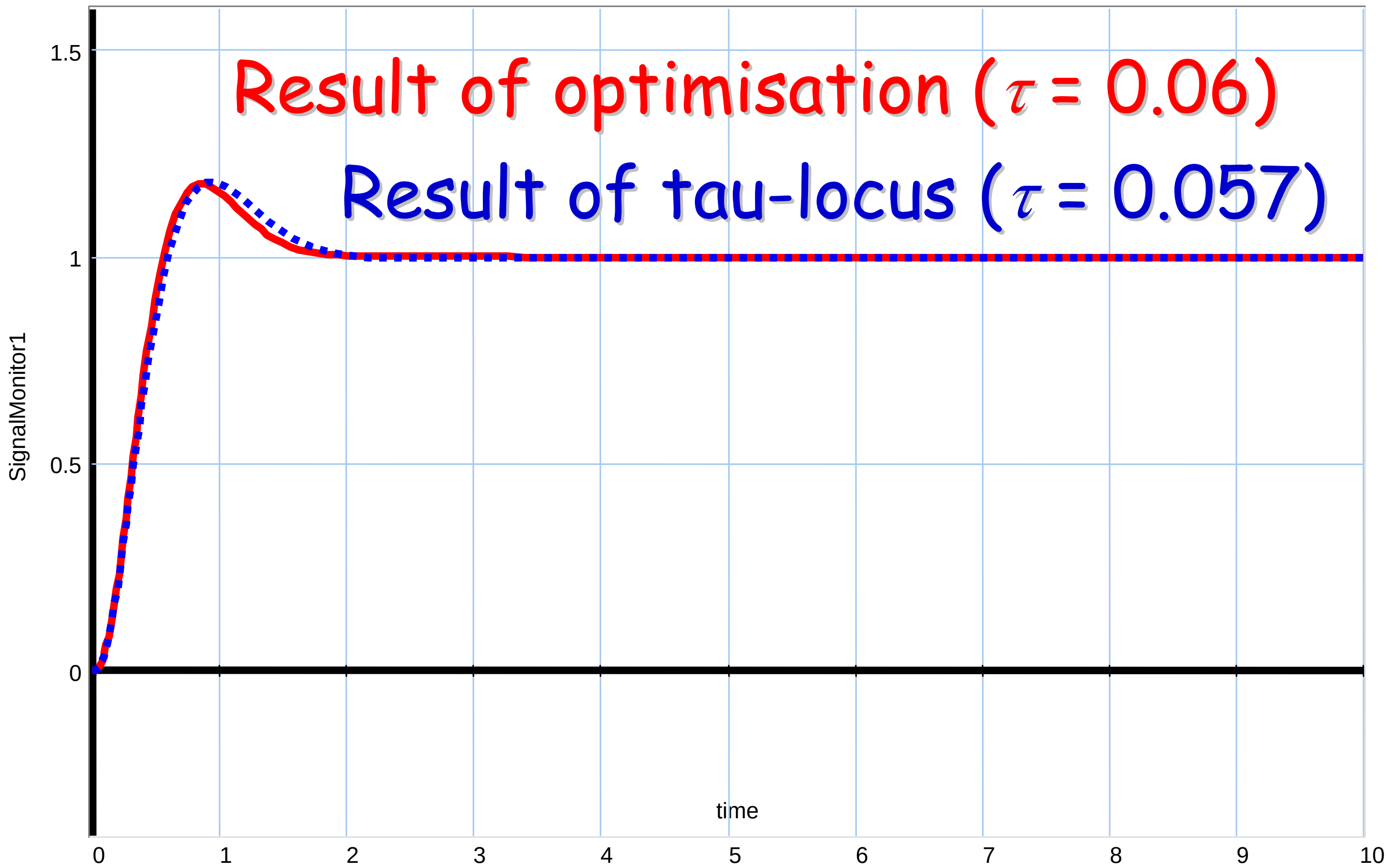
Responses


lead network in forward path
1.5
1
0.5
0
SignalMonitor1
0
5
10
15
20
time

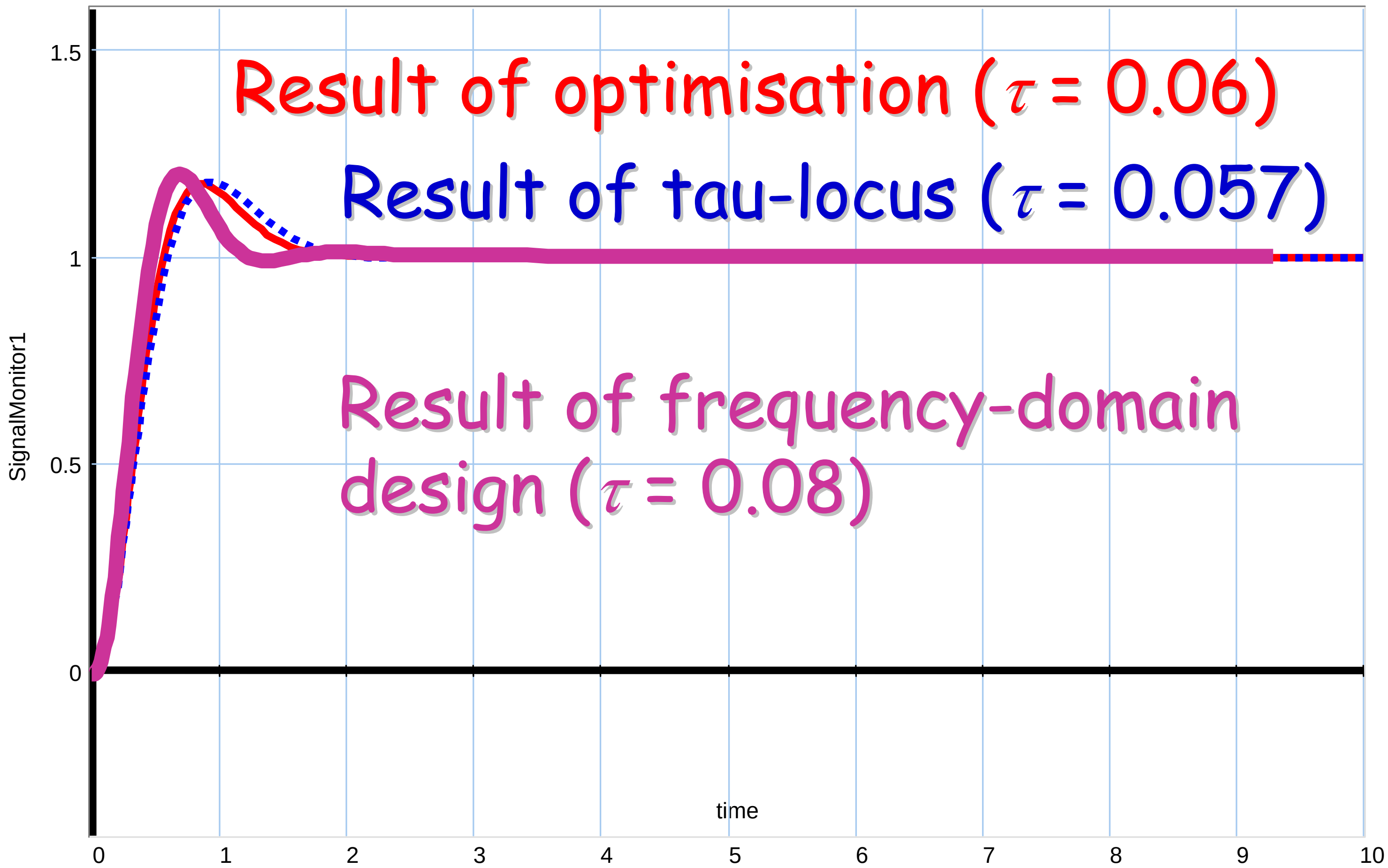
Responses


1.5
1
0.5
0
SignalMonitor1
0
5
10
15
20
time
lead network in forward path
lead network in feedback path
20 sim

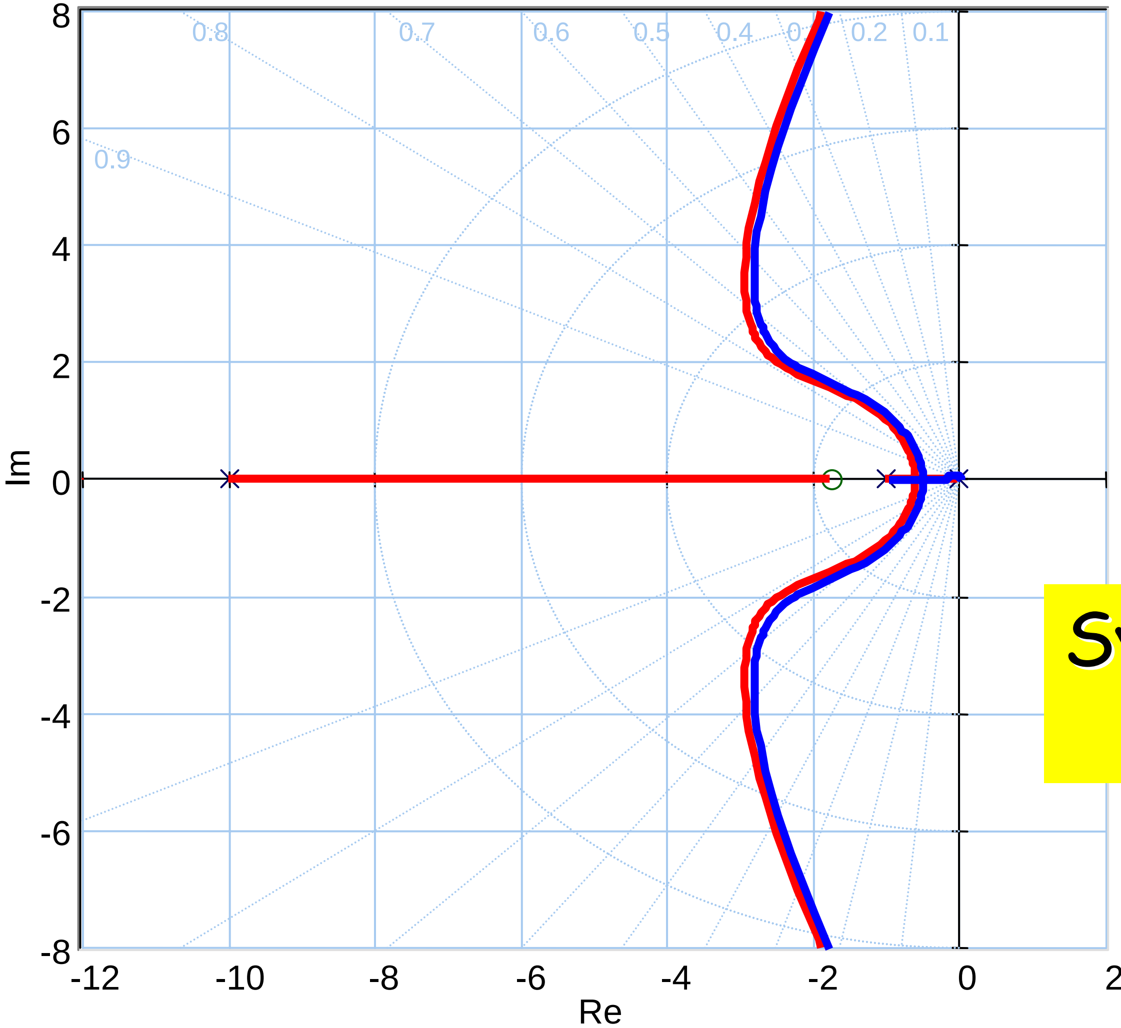
Comparison with frequency- domain design



Comparison with frequency- domain design



Lead + lag



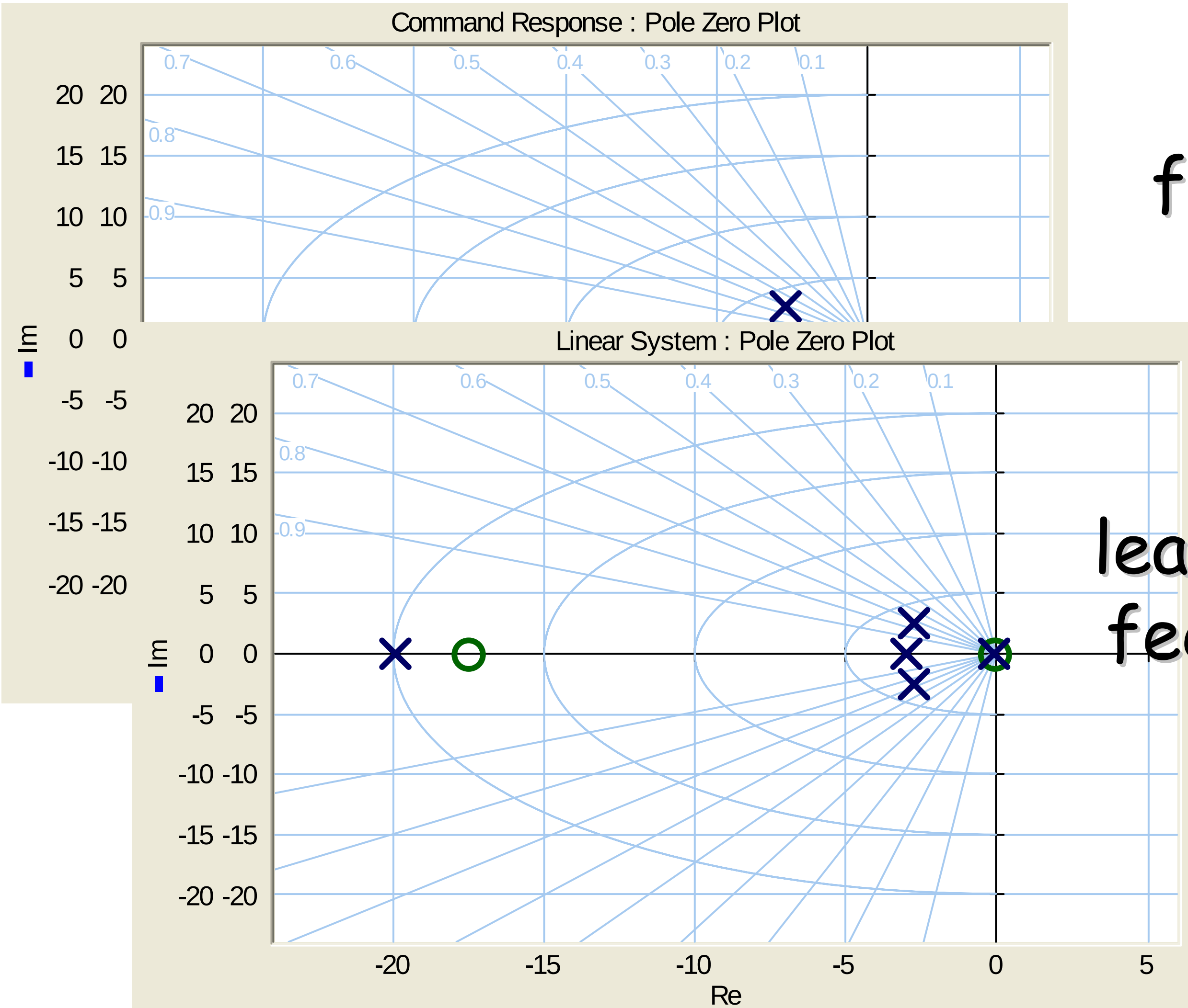
lead

lead +lag

System gain × 10

20 sim

Lead + lag



network
forward path

lead network in
feedback path

20 sim

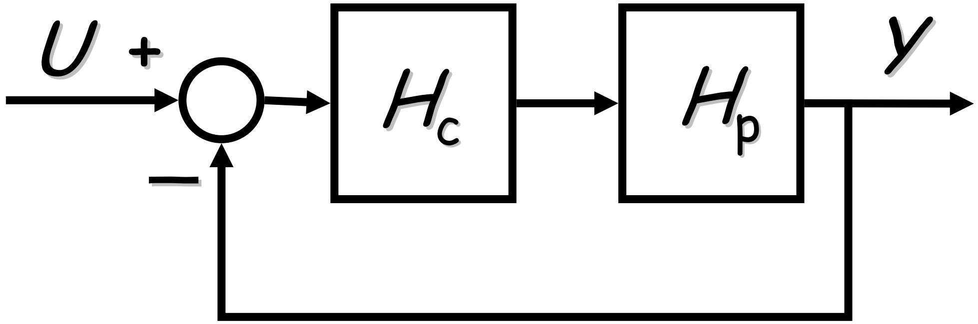
Conclusions

- Lag network:
  - dynamics approximately the same
  - (almost) no change in shape of root locus
  - root-locus gain the same, system gain a factor *a* higher
- Lead network
  - Faster dynamics (poles move away from origin)
  - accuracy improved

Conclusions

- Compensation networks can improve the dynamic performance (transients) and/or the accuracy
- Lead networks: add zero a little bit at the left of the dominant pole
- Lag networks: add zero a factor ten at the right of the dominant pole

System with time delay

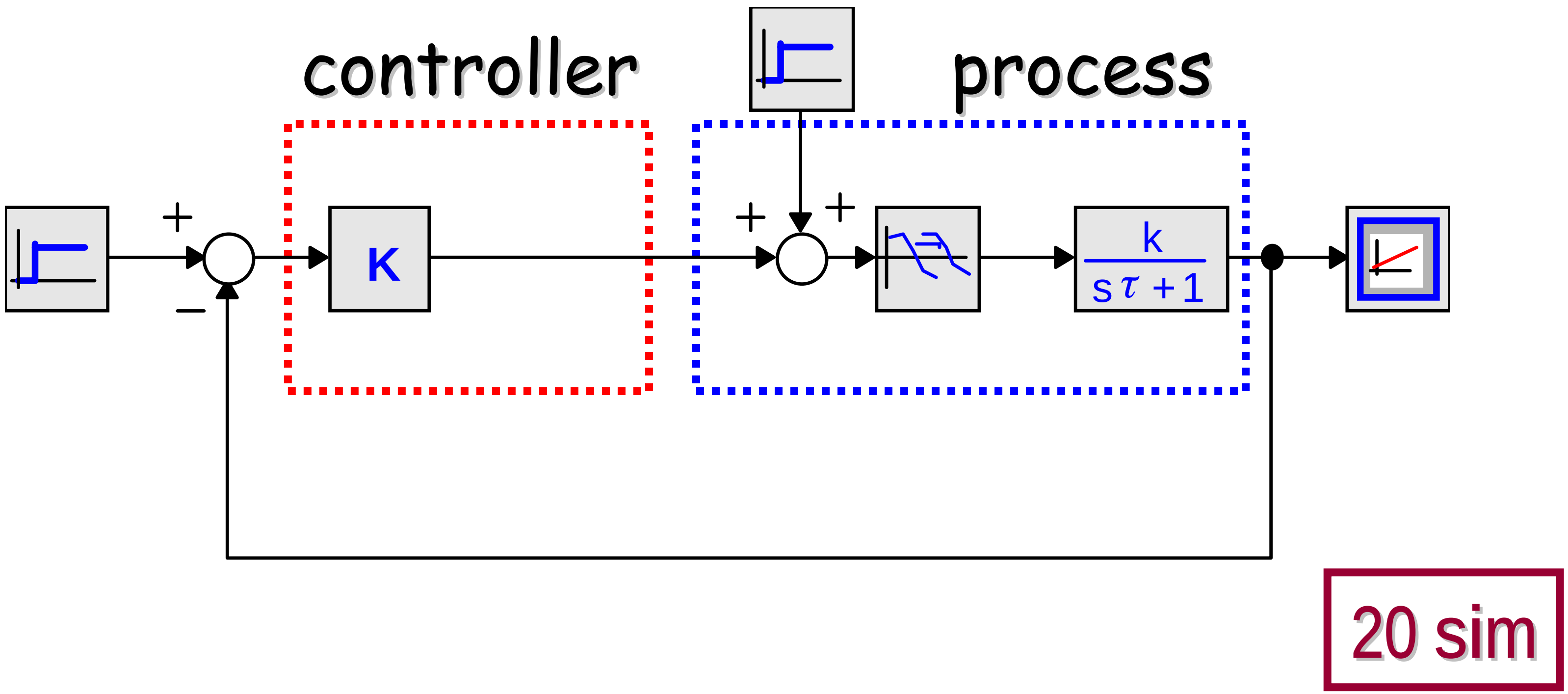


$$H_p(s) = \frac{e^{-sT_d}}{(s+1)}, \quad T_d = 1$$

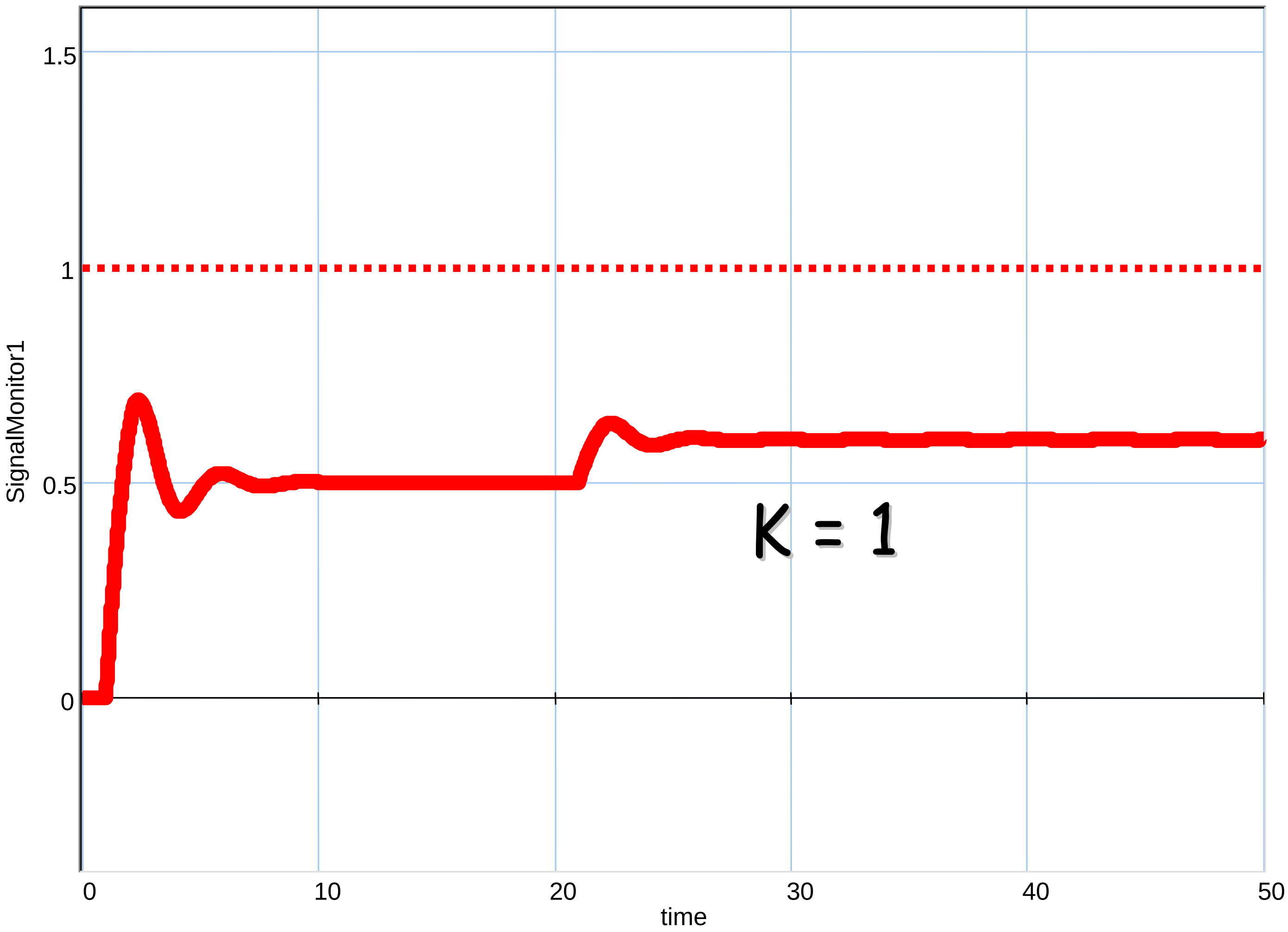
delay is present in systems

- with transportation lag (shower, refinery)
- digital control systems

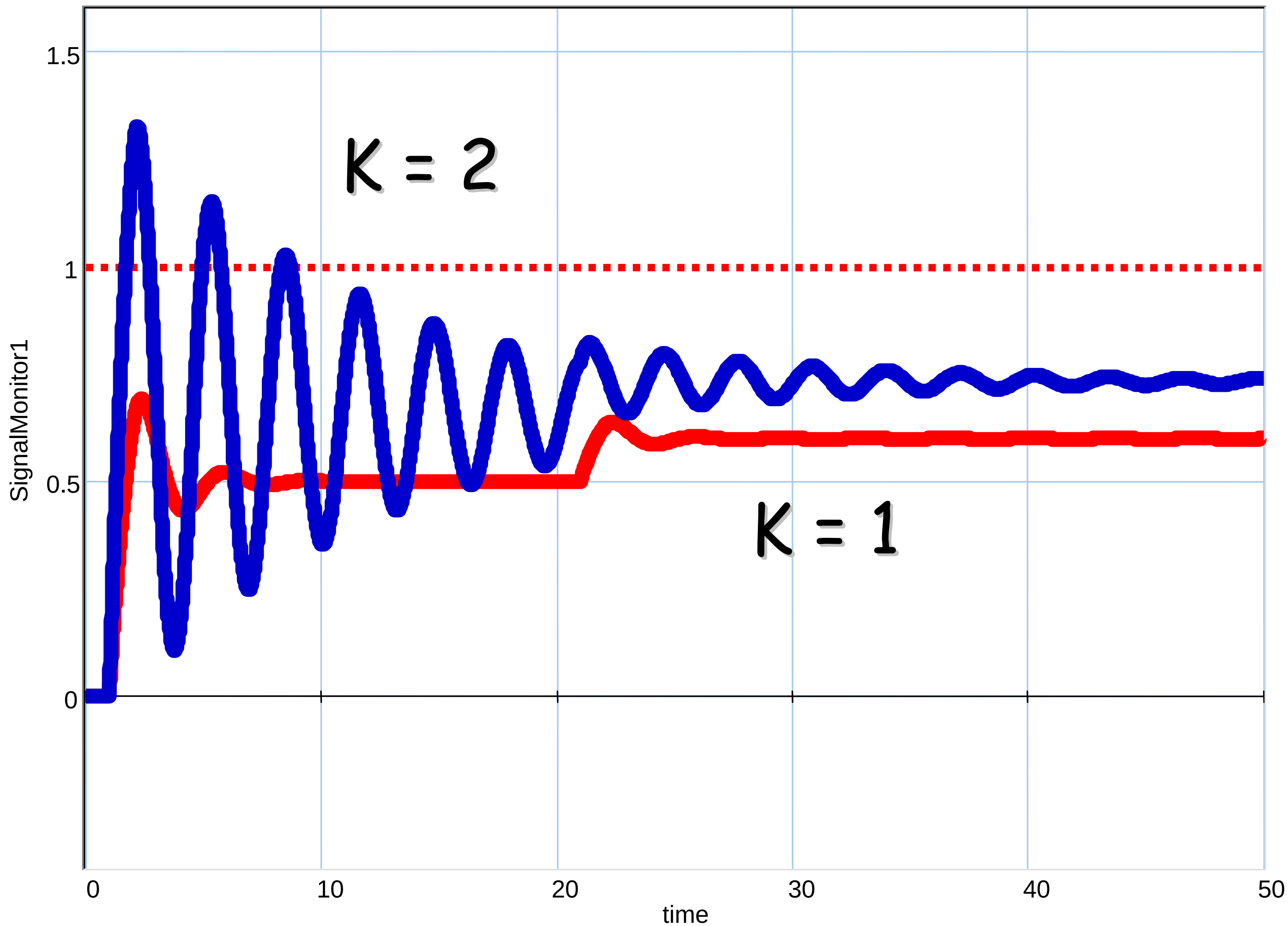
Simulation



Responses



Responses



Root locus equation

$$1 + K_L' \frac{e^{-sT_d}}{(s+\pi)} = 0 \quad \longrightarrow \quad \frac{e^{-sT_d}}{(s+\pi)} = -\frac{1}{K_L'}$$

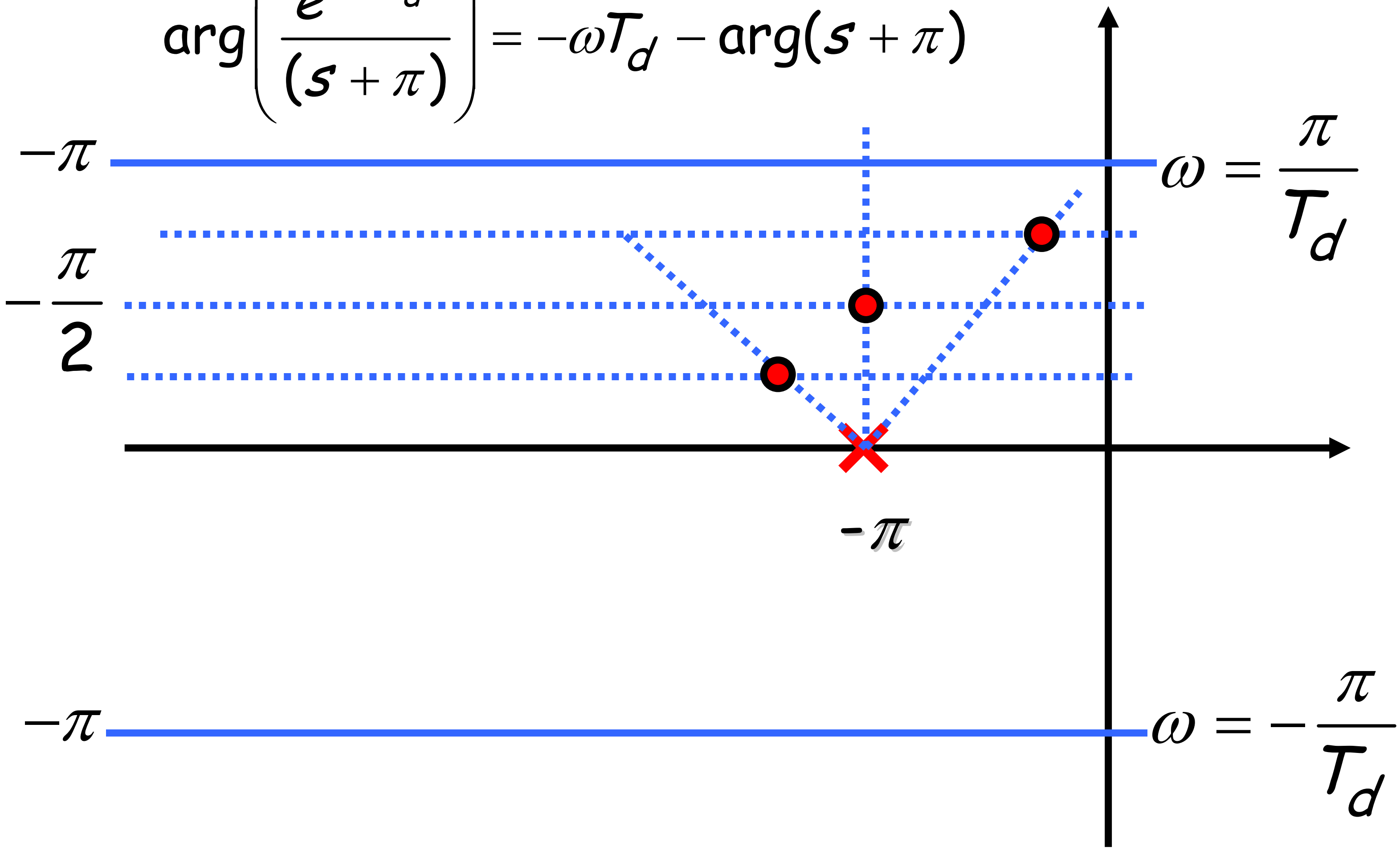
$$\left| \frac{e^{-sT_d}}{(s+\pi)} \right| = \frac{e^{-aT_d}}{|s+\pi|}$$

$$s = a + j\omega$$

$$\arg\left(\frac{e^{-sT_d}}{(s+\pi)} \right) = -\omega T_d - \arg(s+\pi)$$

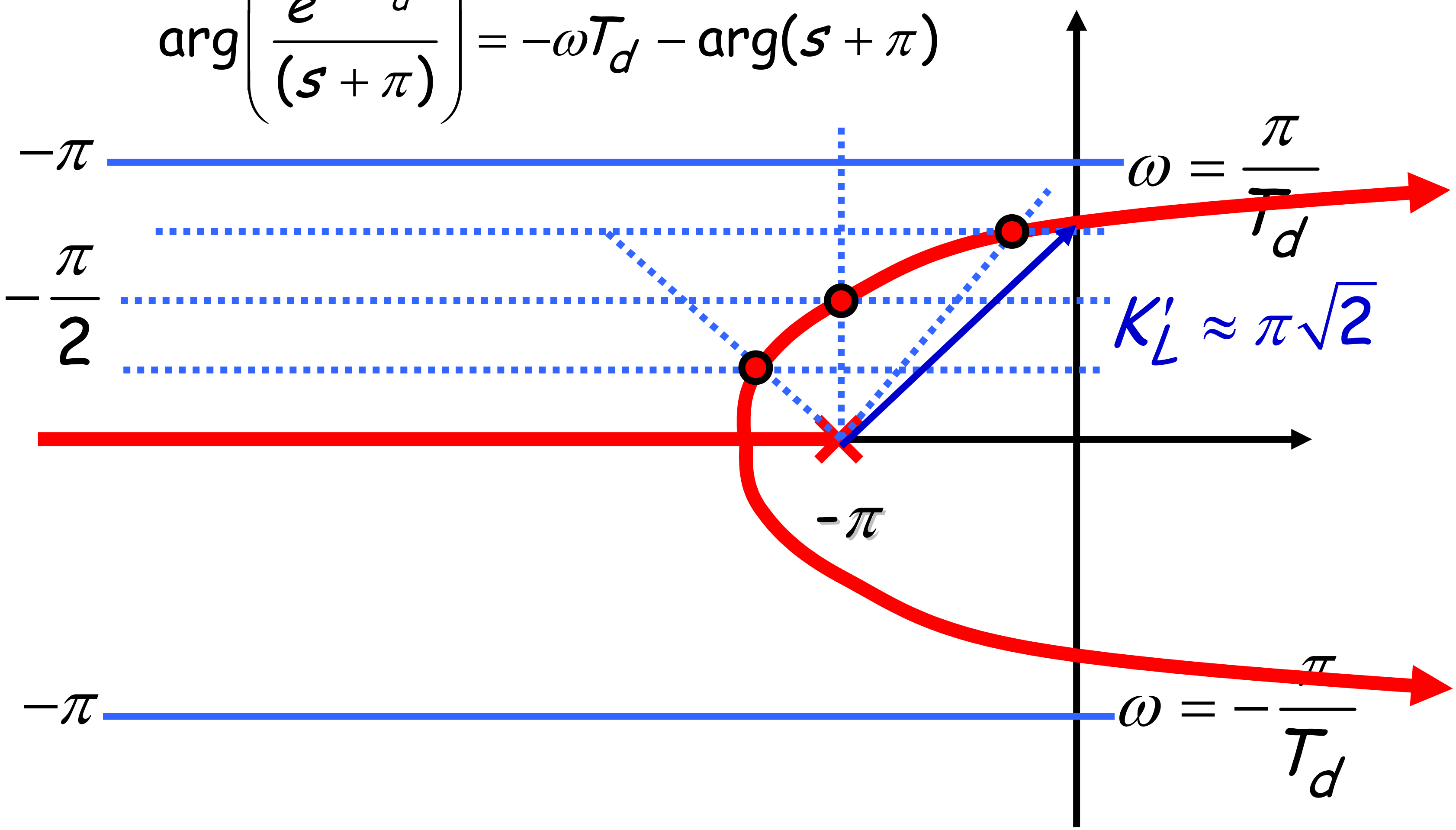
Root locus ($T_d = 1$)

$$\arg\left(\frac{e^{-sT_d}}{(s+\pi)}\right) = -\omega T_d - \arg(s+\pi)$$

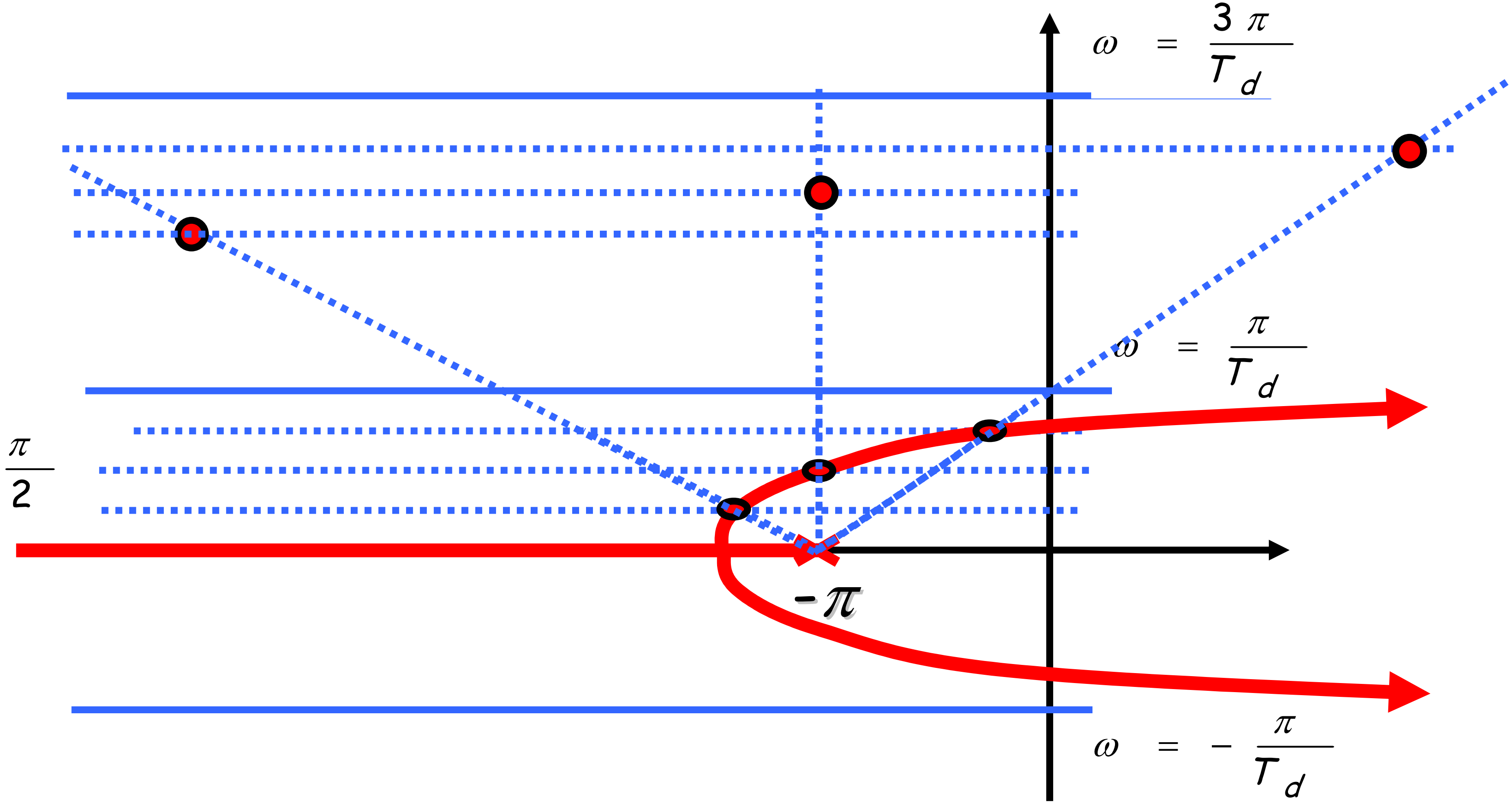


Root locus ($T_d = 1$)

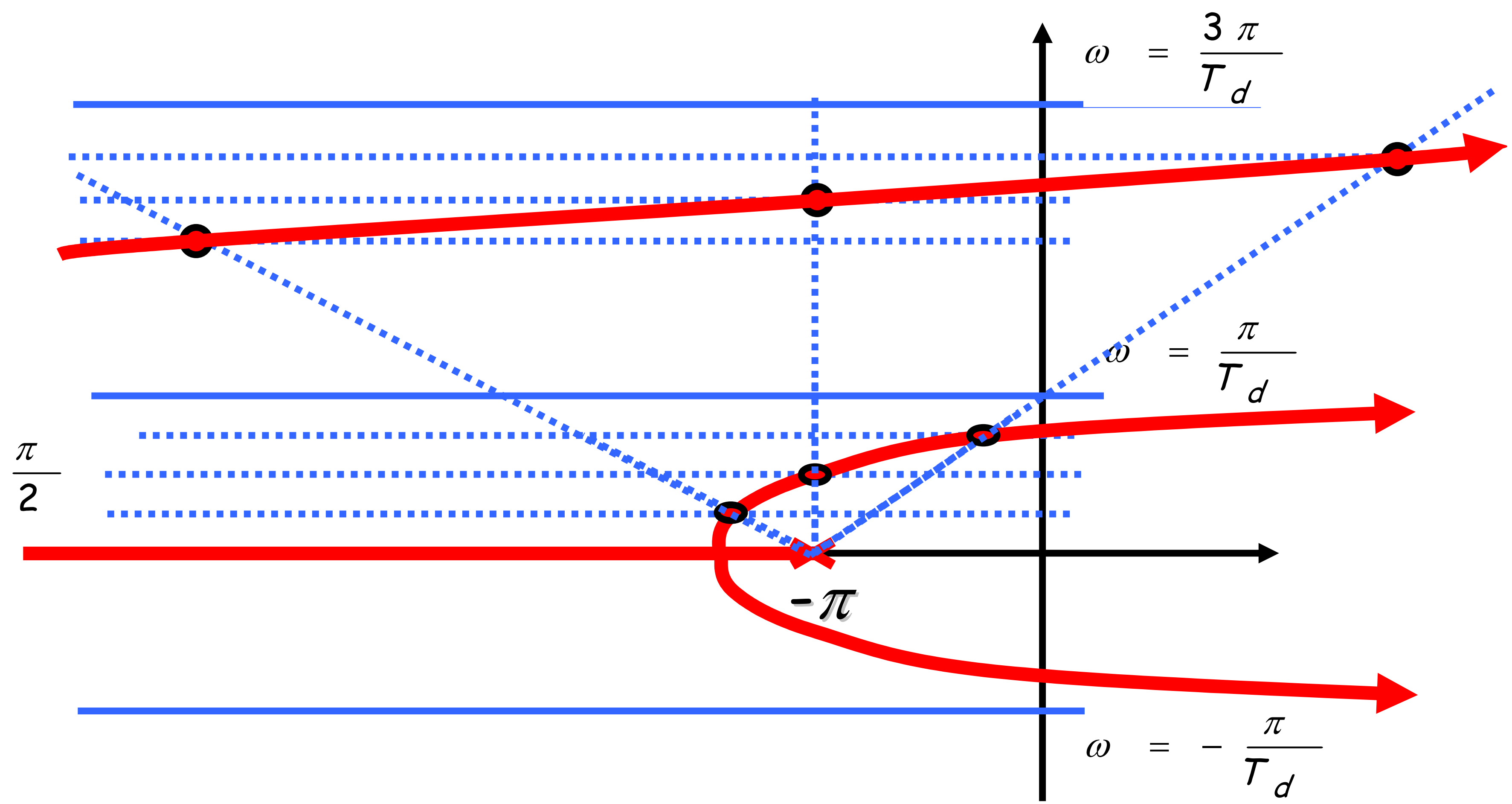
$$\arg\left(\frac{e^{-sT_d}}{(s+\pi)}\right) = -\omega T_d - \arg(s+\pi)$$



Root locus (2)



Root locus (2)



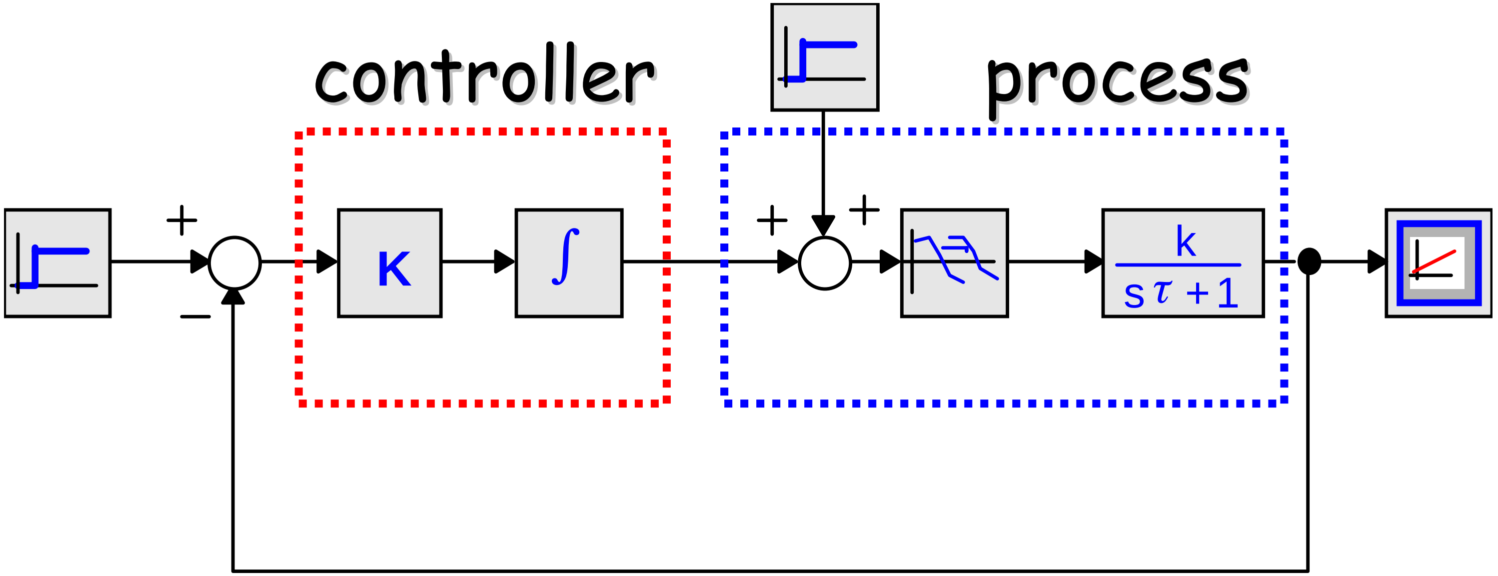
Root locus (3)

- This leads to an infinite number of branches
- delay can be modelled as an infinite number of poles in $-\infty$
- stability is completely determined by primary region
- plays a role in sampled data systems

Compensation(?)

- Because the gain is constant and the phase lag increases linearly with the frequency
- (and thus exponentially in the Bode plot)
  - lead network can do no good
  - consider pure integral control

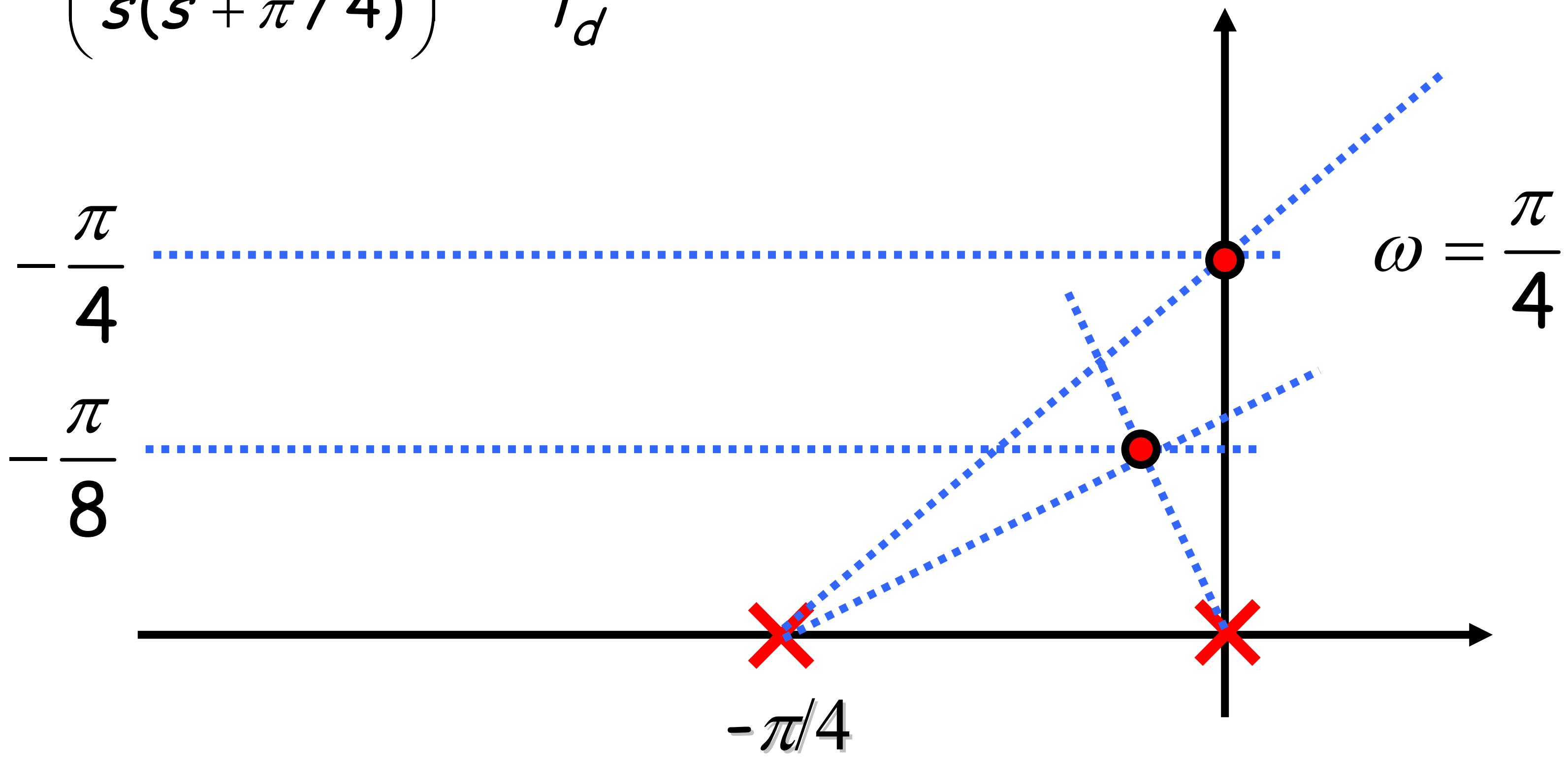
Pure integral control



20 sim

Root locus ($T_d = 1$)

$$\arg\left(\frac{e^{-sT_d}}{s(s+\pi/4)}\right) = -\frac{\omega}{T_d} - \arg(s+\pi/4) - \arg(s)$$

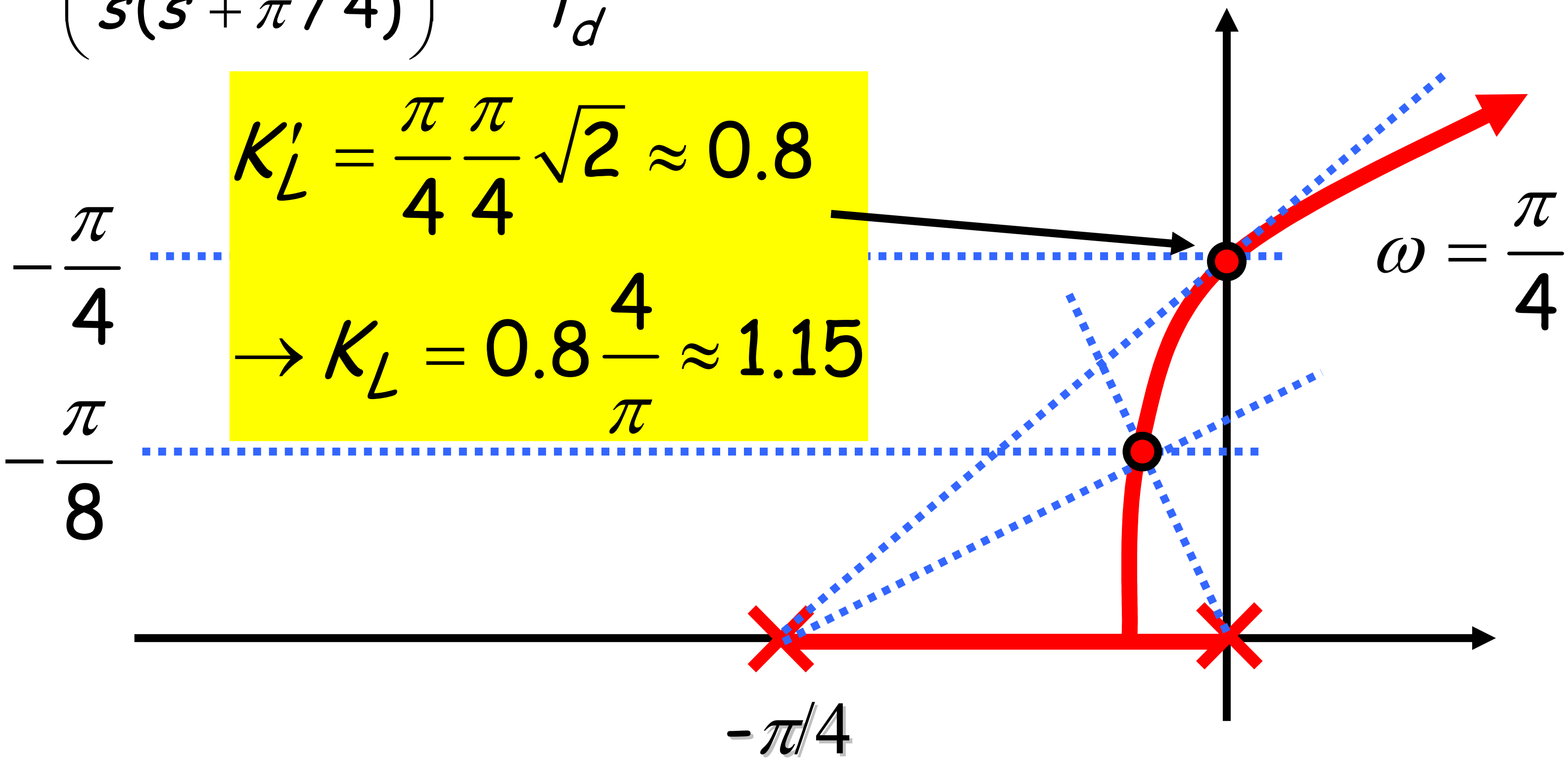


Root locus ($T_d = 1$)

$$\arg\left(\frac{e^{-sT_d}}{s(s+\pi/4)}\right) = -\frac{\omega}{T_d} - \arg(s+\pi/4) - \arg(s)$$

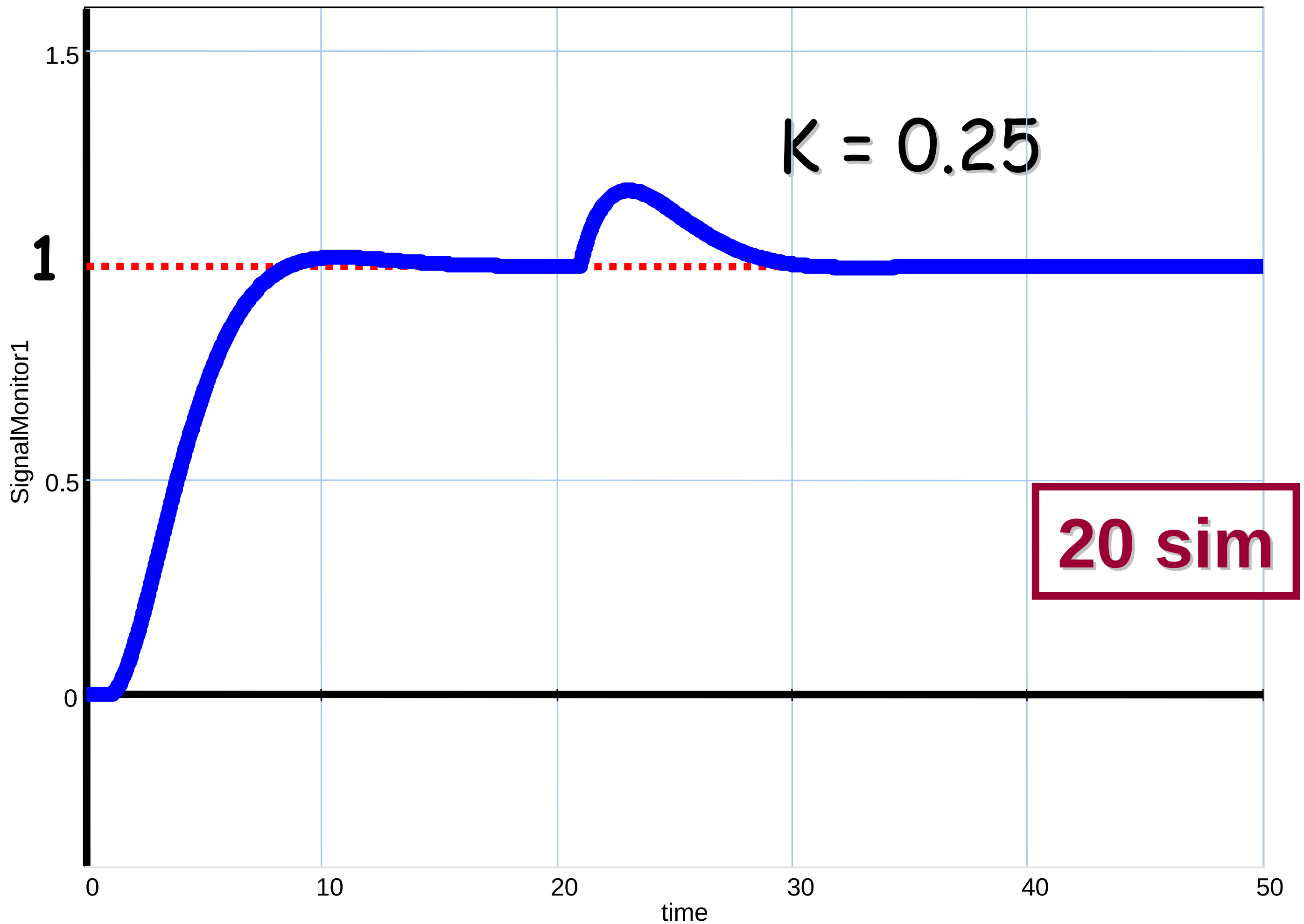
$$K_L' = \frac{\pi}{4}\frac{\pi}{4}\sqrt{2} \approx 0.8$$

$$\rightarrow K_L = 0.8\frac{4}{\pi} \approx 1.15$$



Response

Control Engineering
University of Twente



Conclusions

- Bandwidth of systems with delay is limited
- slow
- integral control improves the accuracy

Non-minimum phase system

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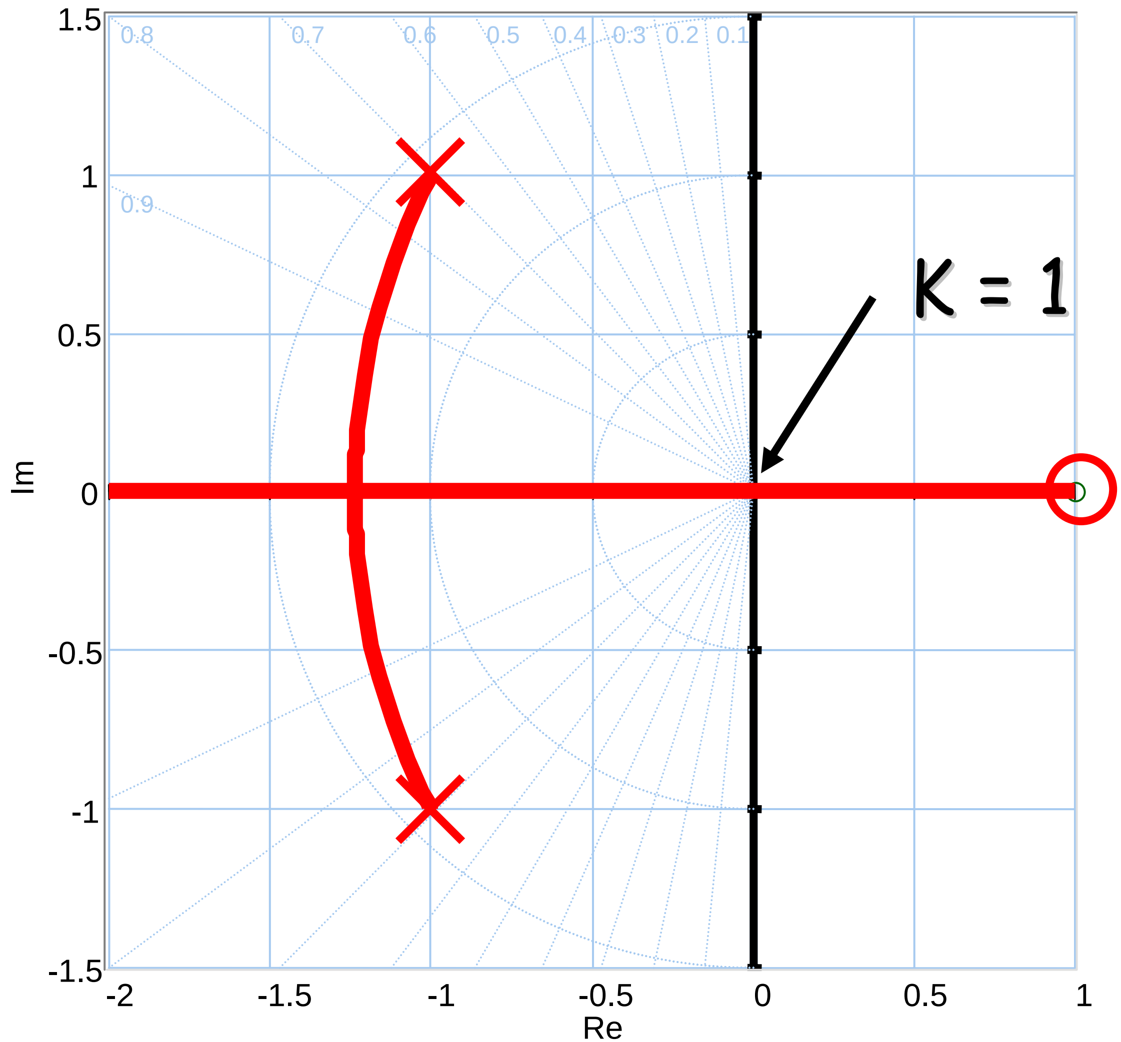
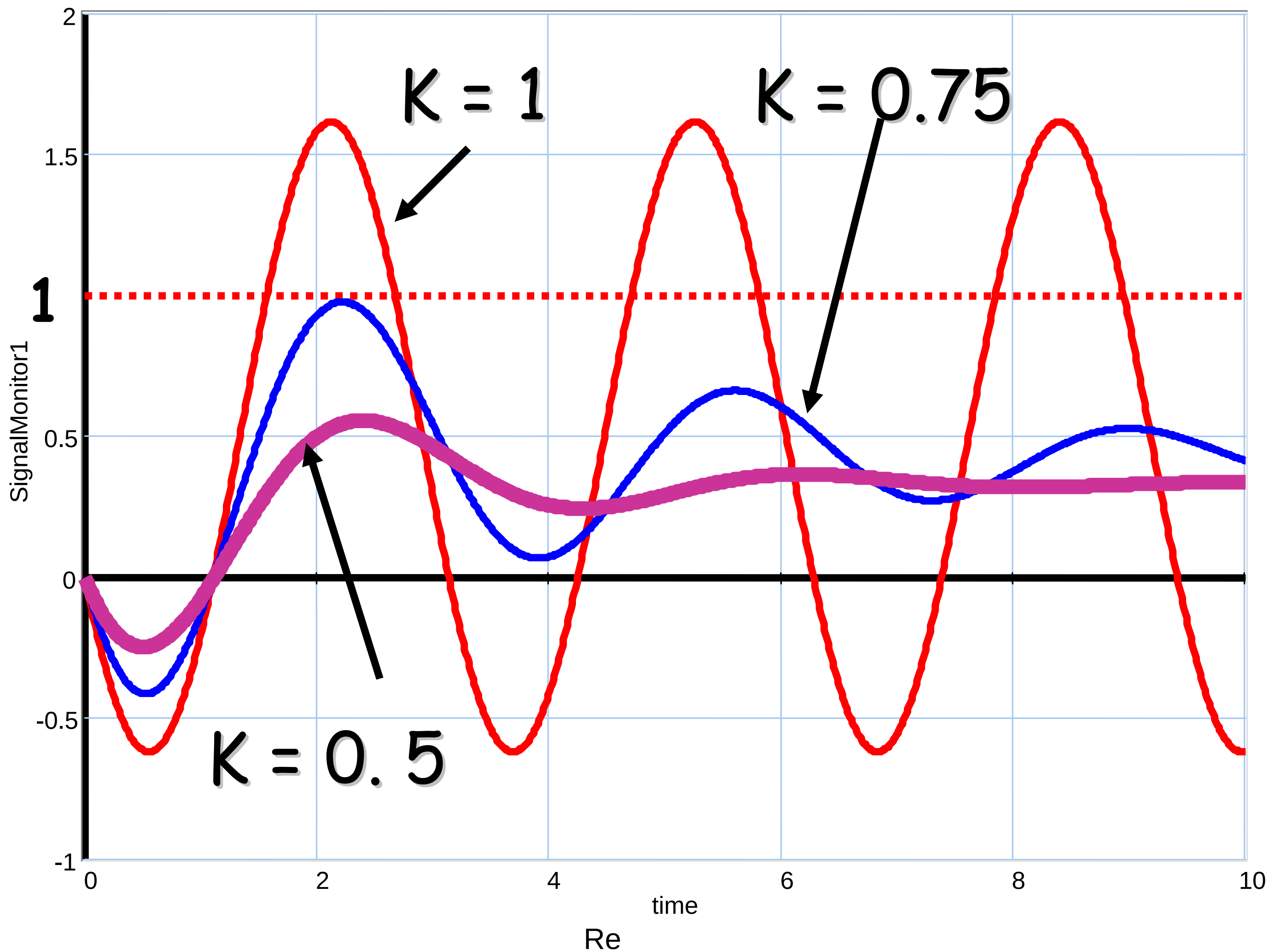


Responses

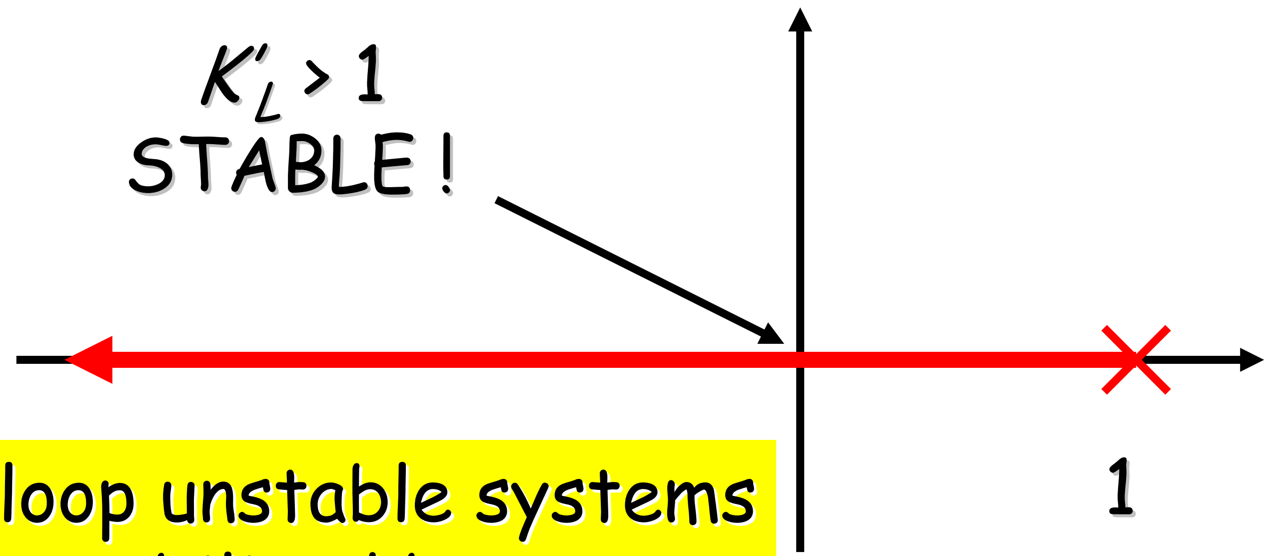
20 sim



Conclusions

- Performance of non-minimum phase systems is limited
- for high gains, always unstable

Open loop unstable



Open loop unstable systems can be stabilised by means of feedback