《自动控制原理》部分习题解答

2-7 $\Delta F = 12.11 \Delta y$

2-8 $\Delta e_d = -E_{do}(\sin \alpha_o)(\alpha - \alpha_o)$

2-9 $\Phi(s) = \dfrac{s^2 + 4s + 2}{(s+1)(s+2)}$ $\qquad k(t) = \dfrac{dc(t)}{dt} = \delta(t) + 2e^{-2t} - e^{-t}$

2-10 零初态响应 $c_1(t) = 1 - 2e^{-t} + e - 2t$

零输入响应 $c_2(t) = e^{-2t} - 2e^{-t}$

总输出 $c(t) = c_1(t) + c_2(t) = 1 - 4e^{-t} + 2e^{-2t}$

2-11 $\dfrac{C(s)}{R(s)} = \dfrac{100(4s+1)}{12s^2 + 23s + 25}$ $\qquad \dfrac{E(s)}{R(s)} = \dfrac{10(12s^2 + 23s + 5)}{12s^2 + 23s + 25}$

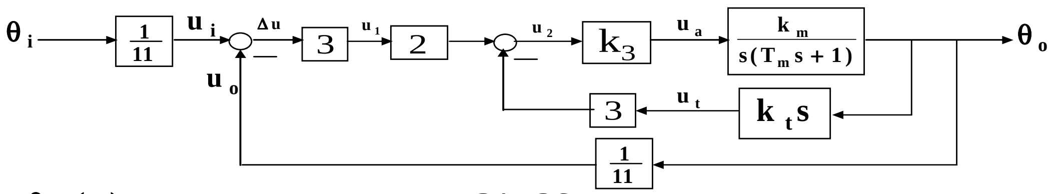
2-12(a) $\dfrac{U_o(s)}{U_i(s)} = -\dfrac{R_1}{R_o}(R_oC_os + 1)$ $\qquad$ **(b)** $\dfrac{U_o(s)}{U_i(s)} = -\dfrac{(R_1C_1s+1)(R_oC_os+10)}{R_oC_1s}$

(b) $\dfrac{U_o(s)}{U_i(s)} = -\dfrac{R_1}{R_o}\dfrac{(R_2C_2s+1)}{(R_1+R_2)C_2s+1}$

2-13 $\dfrac{U_o(s)}{U_i(s)} = -\dfrac{R_1R_2}{R_o^3R_1C_1C_2s^2 + R_o^3C_2s + R_1R_2}$

2-14 $\dfrac{\Omega_m(s)}{U_a(s)} = \dfrac{K_1}{T_ms + 1}$ $\qquad \dfrac{\Omega_m(s)}{M_a(s)} = -\dfrac{K_2}{T_ms + 1}$

2-15



$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{31.26}{T_ms^2 + (1 + 3k_3k_tk_m)s + 31.26k_3k_m}$$

2-22(a) $\dfrac{C(s)}{R(s)} = G_6 + \dfrac{G_1G_2G_3G_4G_5}{1 + G_3H_1 + G_3G_4H_3 + G_2G_3H_2}$

(b) 9个单独回路：

$L_1 = -G_2H_1, L_2 = -G_4H_2, L_3 = -G_6H_3, L_4 = -G_3G_4G_5H_4, L_5 = -G_1G_2G_3G_4G_5G_6H_5$

$L_6 = -G_7G_3G_4G_5G_6H_5, L_7 = -G_1G_8G_6H_5, L_8 = G_7H_1G_8G_6H_5, L_9 = G_8H_4H_1$

6对两两互不接触回路： L_1L_2 L_1L_3 L_2L_3 L_7L_2 L_8L_2 L_9L_2

三个互不接触回路1组： $L_1L_2L_3$

4条前向通路及其余子式： $P_1=G_1G_2G_3G_4G_5G_6$，$\Delta_1=1$； $P_2=G_7G_3G_4G_5G_6$，$\Delta_2=1$；
$P_3=-G_7H_1G_8G_6$，$\Delta_3=1+G_4H_2$； $P_4=G_1G_8G_6$，$\Delta_4=1+G_4H_2$；

$$\frac{C(s)}{R(s)} = \frac{\sum_{k=1}^{4} P_k\Delta_k}{1 - \sum_{a=1}^{9} L_a + \sum_{1}^{6} L_bL_c - L_1L_2L_3}$$

(c) $\dfrac{C(s)}{R(s)} = \dfrac{590}{39} = 15.128$ **(d)** $\dfrac{C(s)}{R(s)} = \dfrac{abcd + ed(1-bg)}{1 - af - bg - ch - ehgf + afch}$

(e) $\dfrac{C(s)}{R_1(s)} = \dfrac{bcde + ade + (a+bc)(1+eg)}{1 + cf + eg + bcdeh + cefg + adeh}$ $\dfrac{C(s)}{R_2(s)} = \dfrac{le(1+cf) - lehbc - leha}{1 + cf + eg + bcdeh + cefg + adeh}$

(f) $\dfrac{C(s)}{R_1(s)} = \dfrac{[ah(1-fg) + aej + aegi] + (bdh + bdej + b\deg i) + (cfdh + cfdej + ci)}{1 - f\deg - fg}$

$\dfrac{C(s)}{R_2(s)} = \dfrac{fdh + fdej + i + fj}{1 - f\deg - fg}$ $\dfrac{C(s)}{R_3(s)} = \dfrac{h(1-fg) + ej + egi}{1 - f\deg - fg}$

3-1 $h(t) = 1 - \dfrac{T-\tau}{T}e^{-\frac{t}{T}}$ **3-2 (1)** $k(t) = 10$ $h(t) = 10t$

3-2 (2) $k(t) = \dfrac{25}{4}e^{-3t}\sin 4t$ $h(t) = 1 - \dfrac{5}{4}e^{-3t}\sin(4t + 53.13^o)$

3-3 (1) $\Phi(s) = \dfrac{0.0125}{s + 1.25}$ **(2)** $\Phi(s) = \dfrac{5}{s^2} + \dfrac{\sqrt{50}(s+4)}{s^2 + 16}$ **(3)** $\Phi(s) = \dfrac{0.1}{s(3s+1)}$

3-4 $\xi = 0.6$ $\omega_n = 2$ $\sigma\% = 9.478\%$ $t_p = 1.96\ s$ $t_s = 2.917\ s$

3-5 $r = 1.0066$ $\omega_n = 1$ $\xi_d = 0.5$ $z = 2.5$ $\beta = \frac{\pi}{2}$ $\psi = -1.686$
$t_r = 1.45\ s$ $t_p = 3.156\ s$ $t_s = 6.0133\ s$ $\sigma\% = 17.99\%$

3-6 $\xi = 1.43$ $\omega_n = 24.5$ **3-7** $k_1 = 1.44$ $k_2 = 0.311$

3-8 (a) $\xi = 0$ $\omega_n = 1$ 系统临界稳定

(b) $\Phi(s) = \dfrac{s+1}{s^2 + s + 1}$ $\xi = 0.5$ $\omega_n = 1$ $\sigma\% = 29.8\%$ $t_s = 7.51\ s$

(c) $\Phi(s) = \dfrac{1}{s^2 + s + 1}$ $\xi = 0.5$ $\omega_n = 1$ $\sigma\% = 16.3\%$ $t_s = 8.08\ s$

(b)比(c)多一个零点,附加零点有削弱阻尼的作用

3-9 (1) $G(s) = \dfrac{5}{s(0.5s+1)}$ $k = 5$ $\xi = \dfrac{1}{\sqrt{10}}$ $\omega_n = \sqrt{10}$ $\sigma\% = 35.09\%$ $t_s = 3.5s$ $e_{ss} = 0.2$

(2) $G(s) = \dfrac{10(0.1s+1)}{s(s+1)}$ $k = 10$ $\xi = \dfrac{1}{\sqrt{10}}$ $\omega_n = \sqrt{10}$ $z = 10$ $r = 1$ $\beta_d = 1.249$

$\psi = -\dfrac{\pi}{2}$ $\sigma\% = 37.06\%$ $t_s = 3s$ $e_{ss} = 0.1$

3-11 劳斯表变号两次，有两个特征根在s右半平面,系统不稳定。

3-12 (1) 有一对纯虚根： $s_{1,2}=\pm j2$ 系统不稳定。

(2) $s_{1,2}=\pm j\sqrt{2}$ $s_{3,4}=\pm 1$ $s_5=1$ $s_6=-5$ 系统不稳定。

(3) 有一对纯虚根： $s_{1,2}=\pm j\sqrt{5}$ 系统不稳定。

3-13 $0<k<1.7$ **3-14** $\tau>0$ $\tau\neq 0$

3-15 (1) $k=20$ $e_{ss}=\infty$ $e_{ss}=\infty$ (2) $k=10$ $e_{ss}=0.2$ $e_{ss}=\infty$

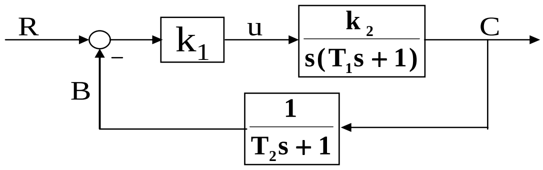
(3) $k=0.1$ $e_{ss}=0$ $e_{ss}=20$

3-16 (1) $k_p=50$ $k_v=0$ $k_a=0$ (2) $k_p=\infty$ $k_v=\frac{k}{200}$ $k_a=0$

(3) $k_p=\infty$ $k_v=\infty$ $k_a=1$

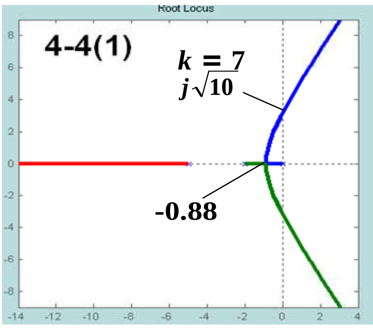
3-18 (1) $e_{ssr}=0$ (2) $e_{ssn1}=0$ (3) $e_{ssn2}=0$

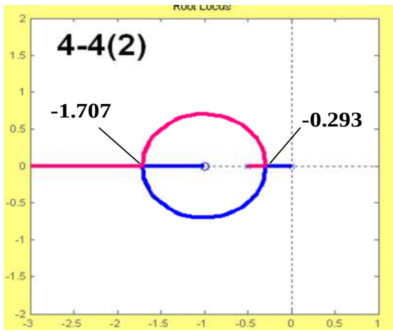
3-20

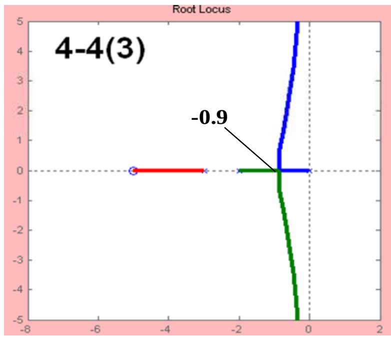


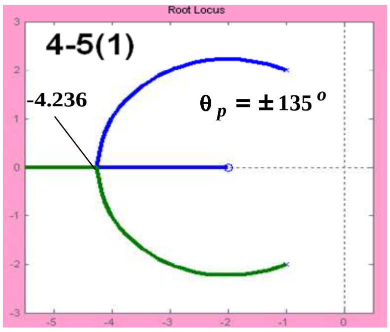
由题意得： E(s)=R(s)-C(s)

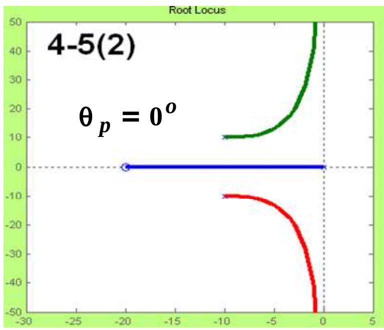
$$\frac{1}{k_2(\varepsilon_o+T_2)}\le k_1<\frac{T_1+T_2}{k_2T_1T_2}$$

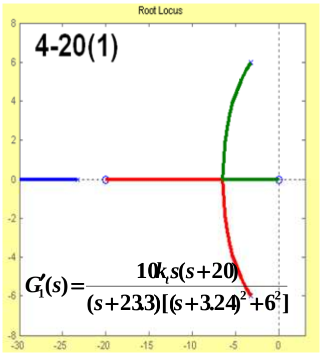


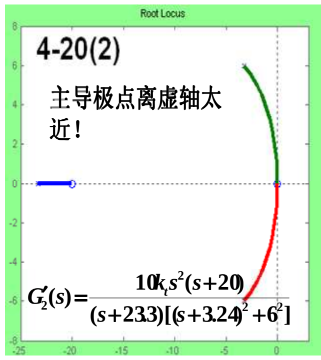


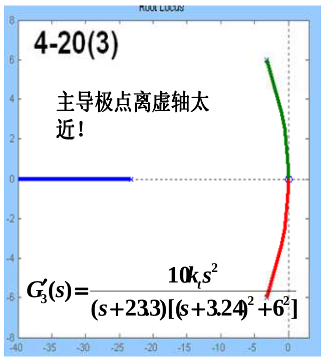


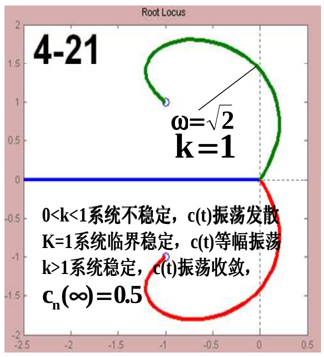












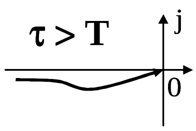
5-2 $\Phi(s)=\dfrac{36}{(s+4)(s+9)}$ $\Phi(j\omega)=\dfrac{36}{(36-\omega^2)+j13}$

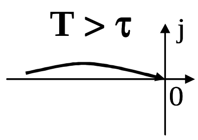
5-3 $e_{ss}(t)=0.632\sin(t+48.4^\circ)-0.79\cos(2t-26.57^\circ)$

或 $c_{ss}(t)=0.447\sin(t+3.4^\circ)-0.707\cos(2t-90^\circ)$ $e_{ss}(t)=r(t)-c_{ss}(t)$

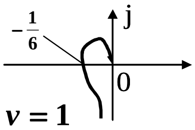
5-4 $\xi=0.653$ $\omega_n=1.848$

5-5

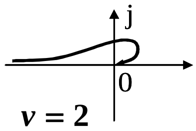




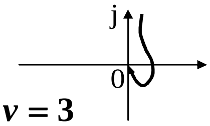
5-6



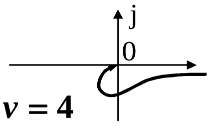
$v=1$



$v=2$



$v=3$



$v=4$

5-7 $G(s)=\dfrac{10(-0.05s+1)}{s(20s+1)}$

5-8 $G(j0.5)=17.9\angle-153.4^\circ$ $G(j2)=0.383\angle-327.53^\circ$

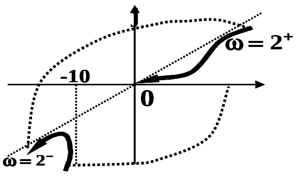
5-9

$G(j0^+)=\infty\angle-90^{\circ}$

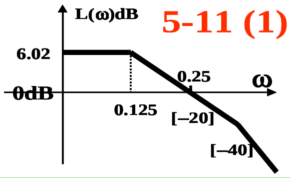
$G(j2^-)=\infty\angle-153.4^{\circ}$

$G(j2^+)=\infty\angle-333.4^{\circ}$

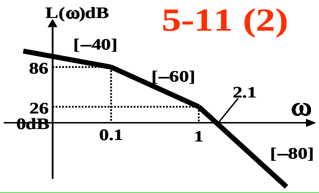
$G(j\infty)=0\angle-360^{\circ}$



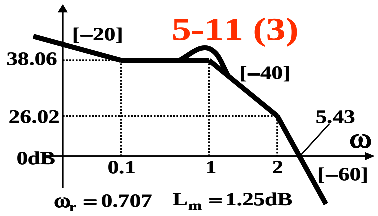
5-11 (1)



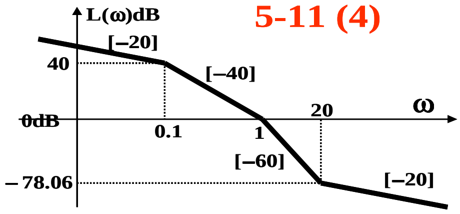
5-11 (2)



5-11 (3)



5-11 (4)



5-13

5-5： τ >T时系统稳定，T> τ 时系统不稳定。

5-6： v=1时系统稳定，其余不稳定。

5-14 (1) Z=0-2(-1)=2 不稳定 (2) z=0 稳定 (3) z=0-2(-1)=2 不稳定

(4) z=0 稳定 (5) Z=0-2(-1)=2 不稳定 (6) z=0-2(1-1)=0 稳定 (7) z=0 稳定

(8) z=0 稳定 (9) Z=1 不稳定 (10) z=2 不稳定

5-15 z=0-2(-1)=2 不稳定

5-16 (1) k<1.5 (2) T<1/9 (3) k-1<1/T **5-17** z=0-0=0 稳定

5-18（左图）原系统稳定，改变k值。使 $\omega_c<\omega_1$ 或 $\omega_c>\omega_2$ 时系统稳定，

其中 $\varphi(\omega_1)=\varphi(\omega_2)=-180^{\circ}$

（右图）原系统z=0-2(-1)=2不稳定。改变k值，使 $\omega_c<\omega_1$ 时系统稳定

其中 $\varphi(\omega_1)=180^{\circ}$

5-19 K>0时应有 0<k<2.64 ； k<0时应有 -1<k<0

5-20 $0\le\tau<1.3686$ **5-21** $a=\dfrac{1}{\sqrt[4]{2}}=0.84$

5-22 $\omega_c=1.94455$ $\gamma=65.156^{\circ}$

5-23 $\xi=0.517$ $\omega_n=2.24$ $\gamma=53.17^{\circ}$

5-24 $\gamma_1=90^{\circ}$ $\gamma_2=-151.8^{\circ}$ $\gamma_3=-60^{\circ}$ $\gamma_4=-33.56^{\circ}$

6-1 (1) $k=6$ $\omega_c=2.924$ $\gamma=4.0521^o$ $\omega_x=3.1623$ $h=1.1667$

(2) $\omega_c=3.8473$ $\gamma=29.7673^o$ $\omega_x=7.83$ $h=3.1249$

6-2 (1) $k=57$ (2) $t_s=2.2673\,s$ $k_v=\frac{57}{27}=2.11$

(3) $G_c(s)=\frac{(s+3)(s+0.091)}{(s+0.01)}$ $k_v=28.2226$ $t_s=0.7s$ $\sigma\%=3.2\%$

6-3 取 $k=20$ $\omega_c''=8$ $G_c(s)=\frac{1+0.4s}{1+0.04s}$ 验算得： $\omega_c''=7.93,\gamma''=62.1^o$

6-5 (1) $G_c(s)=\frac{1+0.19s}{1+0.004s}$ 验算得： $\gamma''=30.1^o$ $h''=16.7dB$

(2) $G_c(s)=\frac{2.6s+1}{180s+1}$ 验算得： $\gamma''=50.4^o$ $h''=30.3dB$

6-6 $\omega_c'=1.969$ $\gamma'=14.25^o$ $\omega_c''=0.7962$ $\gamma''=74.5131^o$

6-7 (1) 取 $k_v=5$ $\omega_c''=0.5$ $G_{c1}(s)=\frac{20s+1}{200s+1}$ 验算得： $\omega_c''=0.45,\gamma''=53.4^o$

(2) $G_c(s)=\frac{20s+1}{200s+1}\bullet\frac{6s+1}{0.3s+1}$ 验算得： $\omega_c''=2.05,\gamma''=51.3^o$

（7-3题~7-10题）

7-3 (1) $e(nT)=10(2^n-1)$

(2) $e^*(t)=-\sum_{n=0}^{\infty}(2n+3)\delta(t-nT)=-3\delta(t)-5\delta(t-T)-7\delta(t-2T)-\Lambda$

7-4 (1) $E(z)=\frac{1}{3}z^{-1}(1-z^{-1})[\sum_{n=0}^{\infty}\frac{a_n}{3^n}z^{-2n}],a_n=3^n-a_{n-1}$ (2) $e(n)=1.5^{-n}+\frac{(3n-1)(-0.5)^n}{2.25}$

7-5 (1) $e(\infty)=\infty$ (2) $e(\infty)=0$

7-7 $c(2)=4$ $c(3)=15$ $c(4)=56$ Λ

7-8 (1) $c(1)=0$ $c^*(t)=\sum_{n=0}^{\infty}(\frac{1}{3}-\frac{1}{2}2^{\frac{t}{T}}+\frac{1}{6}4^{\frac{t}{T}})\delta(t-nT)$

(2) $c(nT)=\frac{T}{4}(n-1)[1-(-1)^n]$ (3) $c(n)=11(-1)^n+7(-2)^n+5(-3)^n$

7-9 (a) $G(z)=\frac{10z^2}{z^2-(e^{-2T}+e^{-5T})z+e^{-7T}}$ (b) $G(z)=\frac{10}{3}(\frac{z}{z-e^{-2T}}-\frac{z}{z-e^{-5T}})$

7-10 (a) $\Phi(z)=\frac{G_1(z)}{1+G_1G_2(z)+G_1(z)G_3(z)}$ (b) $C(z)=\frac{RG_1(z)G_hG_3G_4(z)+RG_2G_4(z)}{1+G_hG_3G_4(z)}$

(c) $C(z)=\frac{R(z)[D_1(z)+D_2(z)]G_hG_1G_2(z)+NG_2(z)}{1+D_1(z)G_hG_1G_2(z)}$

（7-11题~7-21题）

7-11 $c(nT)=(1-0.47\times 0.37^{n})\delta(t-nT)$

7-13 (1) $c(z)=\dfrac{0.16z^{3}+0.0384z^{2}}{z^{4}-2.8467z^{3}+2.8987z^{2}-1.0587z+0.0067}$

(2) $c^{*}(t)=0.16\delta(t-T)+0.4938\delta(t-2T)+0.94\delta(t-3T)+1.415\delta(t-4T)+\Lambda$

(3) 因为系统不稳定，所以 $c(\infty)$ 不存在。

7-15 (1) 不稳定 (2) 不稳定 (3) 不稳定

7-16 (1) 不稳定 (2) 0<k<3.31时稳定

7-17 $G(z)=\dfrac{1.2z-0.8}{(z-1)^{2}}$ $k_a=0.4$ $e(\infty)=0.1$

7-18 $k_p=\infty$ $k_v=0.1$ $k_a=0$ $e(\infty)=1$

7-19 0<k<2.272时系统稳定；$e_{ss}<0.1$时k应大于10，此时系统不稳定，所以k无值可取。

7-21 $D(z)=\dfrac{z-0.368}{0.632z}=1.58-0.58z^{-1}$

7-22 $D(z)=\dfrac{2z-1}{k(z-1)}$

（8-12题~8-19题）

8-12 (2)的准确性高

8-14 (a) $G(s)=[1+H_1(s)]G_1(s)$ (b) $G(s)=\dfrac{G_1(s)H_1(s)}{1+G_1(s)}$

8-15 (a) $N(A)=k-\dfrac{2k}{\pi}\arcsin\dfrac{a}{A}+\dfrac{2ka}{\pi A}\sqrt{1-(\dfrac{a}{A})^{2}}$ $A\ge a$

(b) $N(A)=\dfrac{4M}{\pi A}[\sqrt{1-(\dfrac{a}{A})^{2}}+\sqrt{1-(\dfrac{b}{A})^{2}}]$ $A\ge b$

(c) $N(A)=\dfrac{4M}{\pi A}\sqrt{1-\dfrac{(\frac{h}{k}+\Delta)^{2}}{A^{2}}}$ $A>\dfrac{h}{k}+\Delta$

8-16 $|G(j2)|=\dfrac{15}{2\sqrt{2}}=5.3$ 有自振 $x(t)=5.3\sin t$

8-17 $0<k<\dfrac{2}{3}$ 时系统稳定 $2<k<\infty$ 时系统不稳定

$\dfrac{2}{3}<k<2$ 时有自振 $x(t)=\dfrac{6k-4}{2-k}\sin t$

8-18 $x(t)=0.8\sin 3.91t$ $c(t)=-0.16\sin 3.91t$

8-19 $x(t)=0.796\sin 2t$ $y(t)=\dfrac{4}{\pi}\sin 2t$ $c(t)=-0.398\sin 2t$

9-2:(1) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$$x = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(2) $T = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

$$\dot{\bar{x}} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$

$$x = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}$$

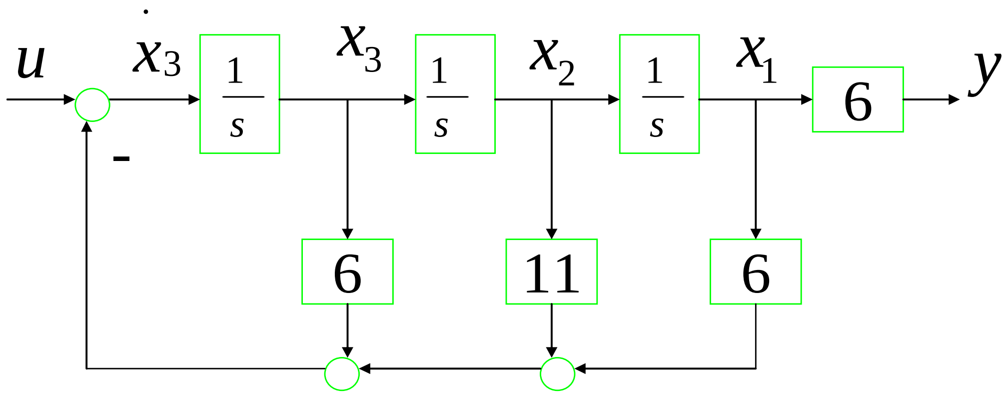
9-3:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 6 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$ 能控型

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$ 能观型

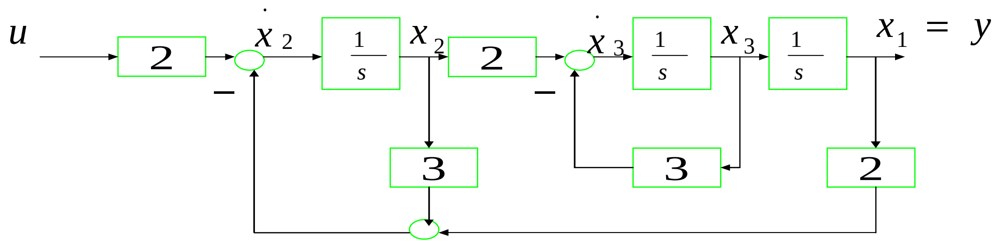


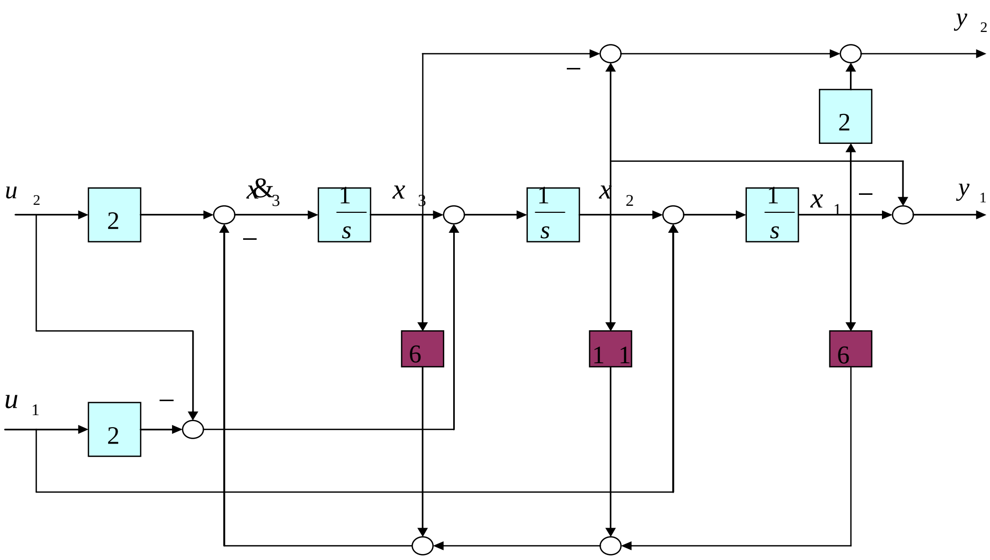
能控型

9-4:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 0 & 1\\ -2 & -3 & 0\\ 0 & 2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 2\\ 0\end{bmatrix}u$$

$$y=\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$





9-6:能控型：

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 5 & 2 \end{bmatrix} x + u$$

能观型：

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 5 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x + u$$

9-13: $G(s) = \dfrac{2s^2 + 7s + 3}{(s+1)(s+2)(s-3)}$

9-15:

$$x(k) = \begin{bmatrix} -\frac{1}{2}(-1)^k + \frac{1}{6} + \frac{1}{3}(-2)^k \\ \frac{1}{2}(-1)^k + \frac{1}{6} - \frac{2}{3}(-2)^k \end{bmatrix}$$

$$y(k) = \begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$= -\frac{1}{2}(-1)^k + \frac{5}{6} - \frac{1}{3}(-2)^k$$