Frequency Domain Design

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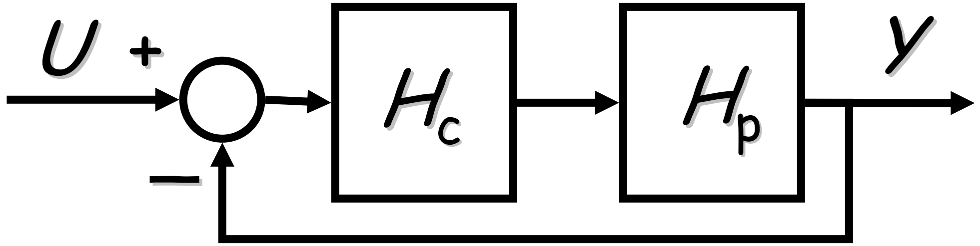
- Frequency diagrams of open and closed systems
- Sensitivity
- Design of lead and lag networks

Open and closed systems



$$H_p = \frac{1}{s+1}$$

$$H_c = K$$

	open loop	closed loop
	$H_L = \frac{K}{s+1}$	$H = \frac{K}{s+(1+K)}$
$\omega = 0$	K	$K/1+K$
$\omega \to \infty$	$\frac{K}{j\omega}$	$\frac{K}{j\omega}$

Open and closed systems

Feedback only effects the low-frequencies

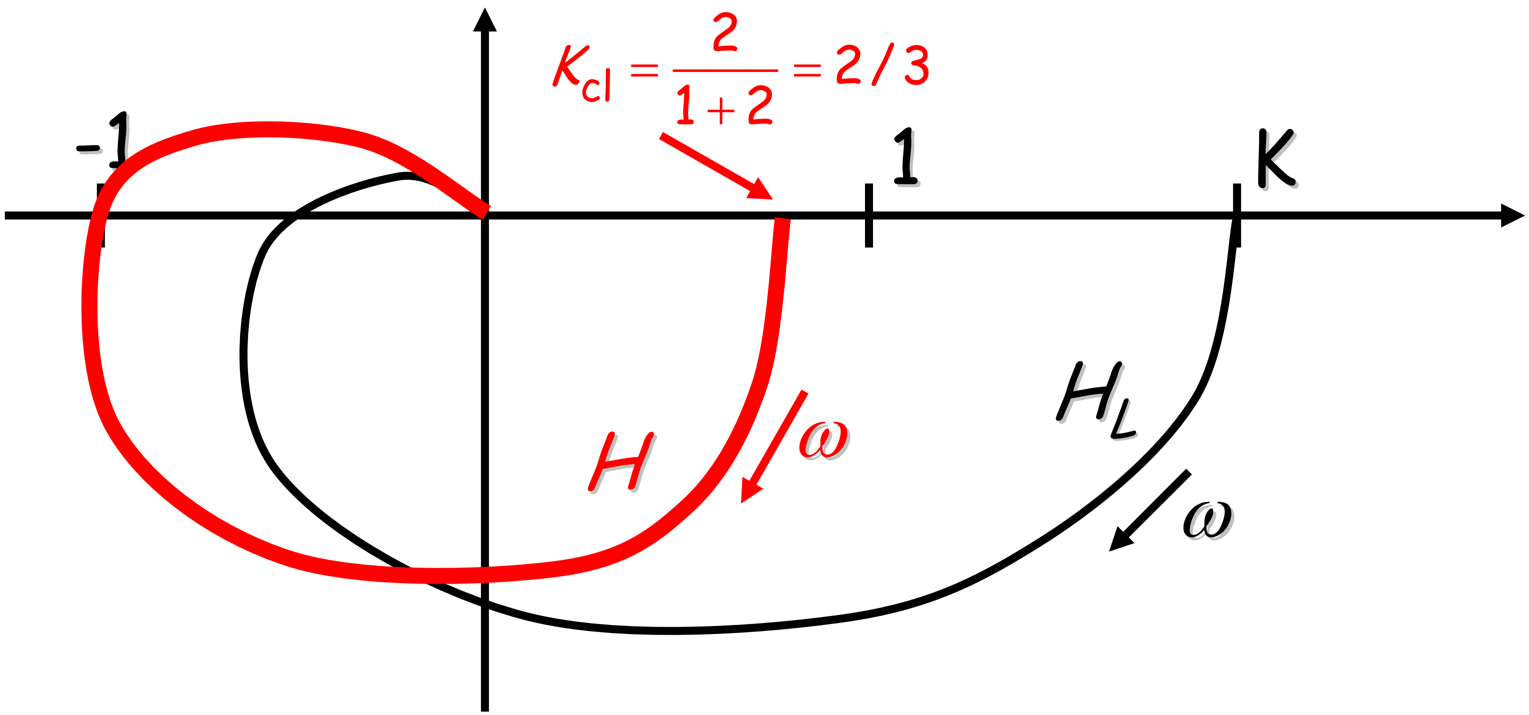
If K large, for $\omega \to 0$, $H_{\text{closed loop}} \to 1$

	open loop	closed loop
	$H_L = \dfrac{K}{s+1}$	$H = \dfrac{K}{s+(1+K)}$
$\omega = 0$	K	$K/(1+K)$
$\omega \to \infty$	$\dfrac{K}{j\omega}$	$\dfrac{K}{j\omega}$

Open and closed systems

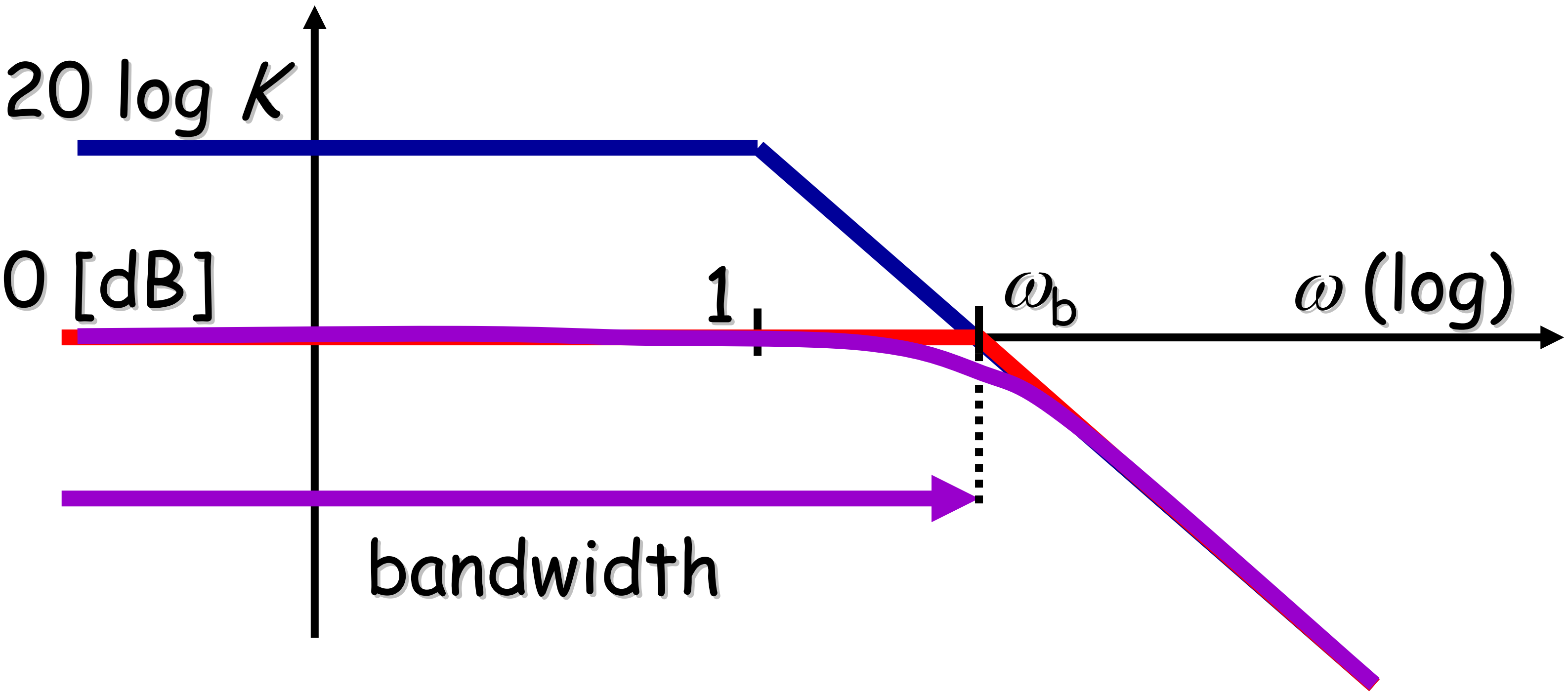
Feedback only effects the low-frequencies

If K large, for $\omega \to 0$, $H_{\text{closed loop}} \to 1$

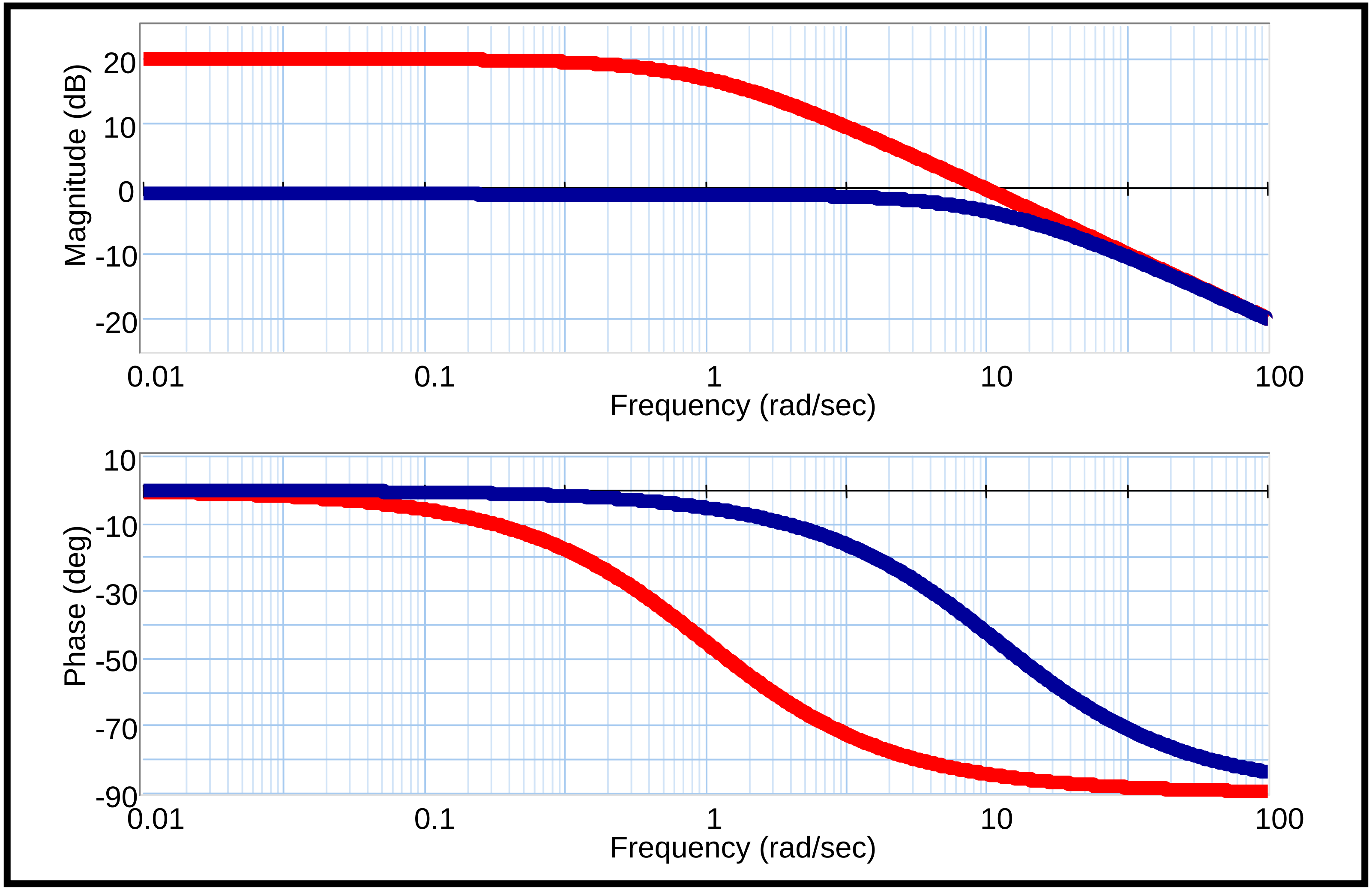


Feedback only effects the low-frequencies

If K large, for $\omega \to 0$, $H_L \to 1$



20-sim: 10/(s+1)


Magnitude (dB)
20
10
0
-10
-20
0.01
0.1
1
10
100
Frequency (rad/sec)
Phase (deg)
10
-10
-30
-50
-70
-90
0.01
0.1
1
10
100
Frequency (rad/sec)

Open and closed systems

$$H_{cl} = \frac{H_L}{1 + H_L}$$

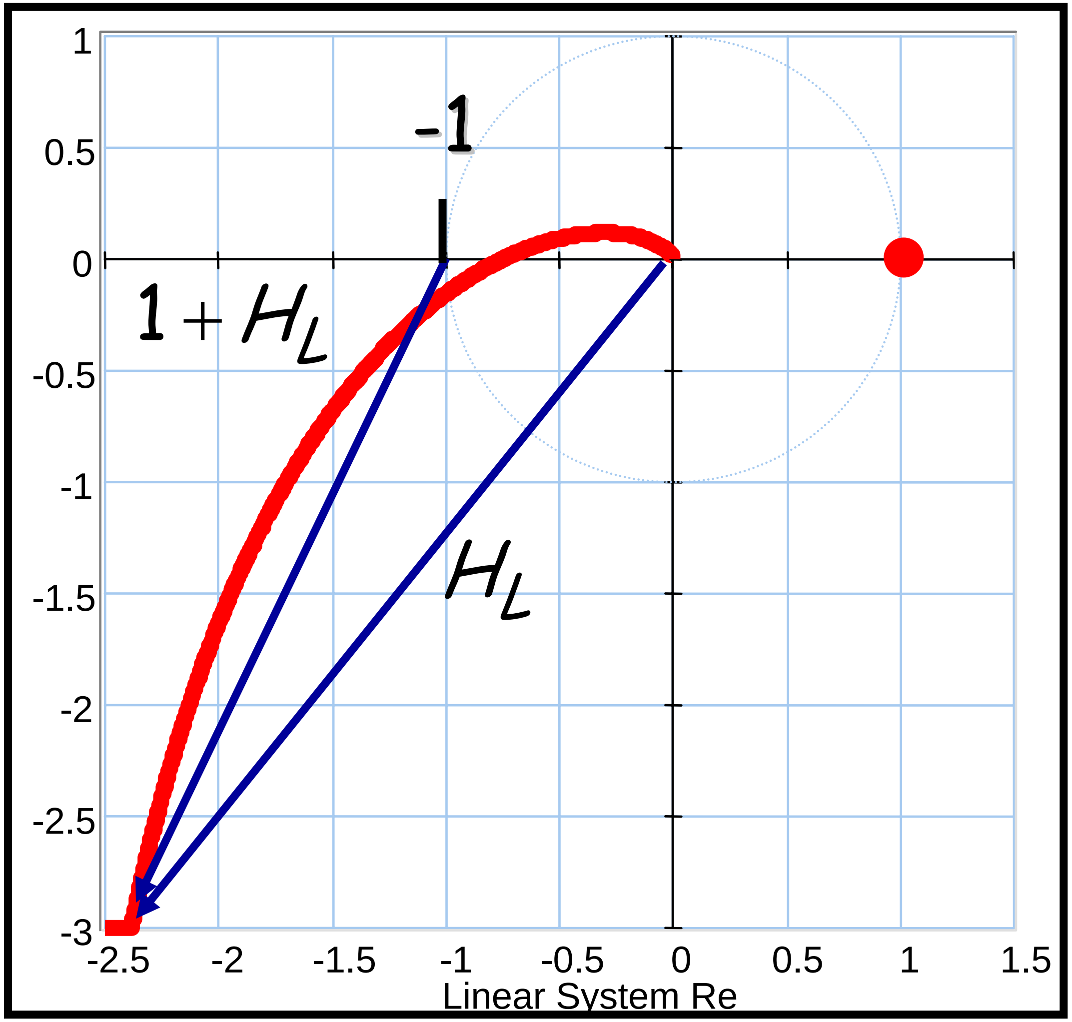
$$\omega = 0$$

$$\left|H_L\right| \approx \left|1 + H_L\right|$$

$$\arg\left(H_L\right) \approx \arg\left(1 + H_L\right)$$

$$\approx -\frac{\pi}{2}$$

$$H_{cl} \to 1$$



Open and closed systems

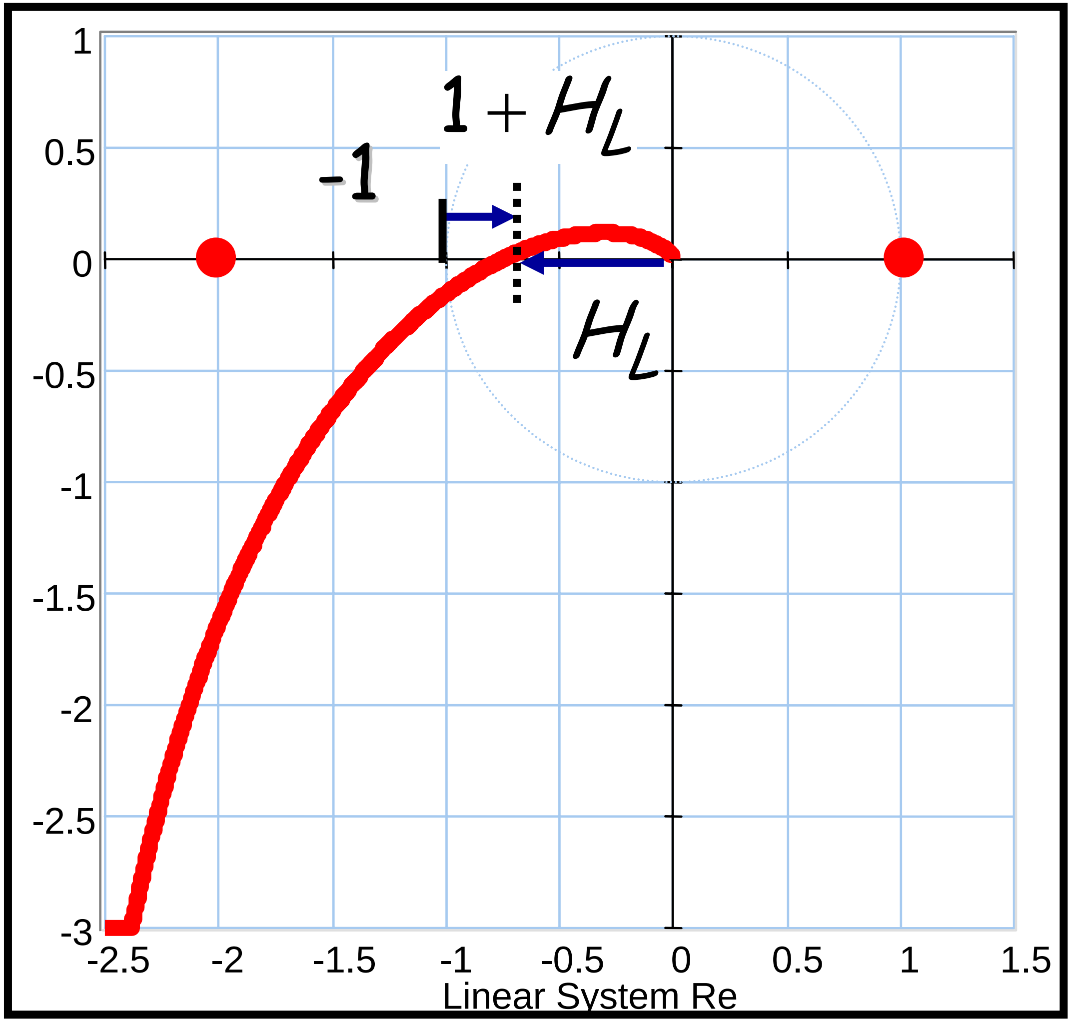
$$H_{cl} = \frac{H_L}{1 + H_L}$$

$$|H_L| \approx \frac{2}{3} \qquad |1 + H_L| \approx \frac{1}{3}$$

$$\arg(H_L) = -\pi$$

$$\arg(1 + H_L) = 0$$

$$H_{cl} \rightarrow 2e^{-j\pi}$$



Open and closed systems

$$H_{cl} = \frac{H_L}{1+H_L}$$

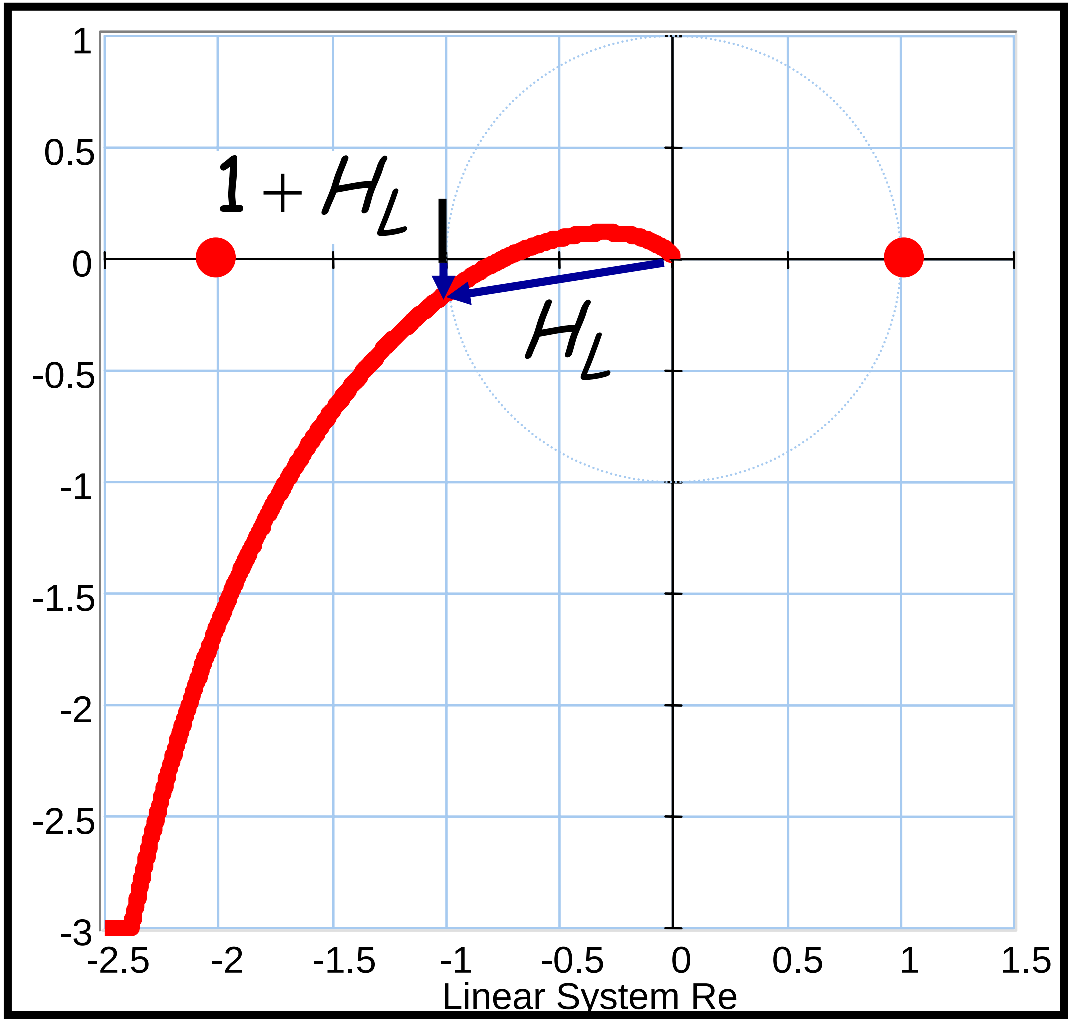
$$|H_L| \approx 1.25$$

$$|1+H_L| \approx 0.25$$

$$\arg(H_L) \approx -\pi$$

$$\arg(1+H_L) \approx -\frac{\pi}{2}$$

$$H_{cl} \to 5e^{-j\frac{\pi}{2}}$$

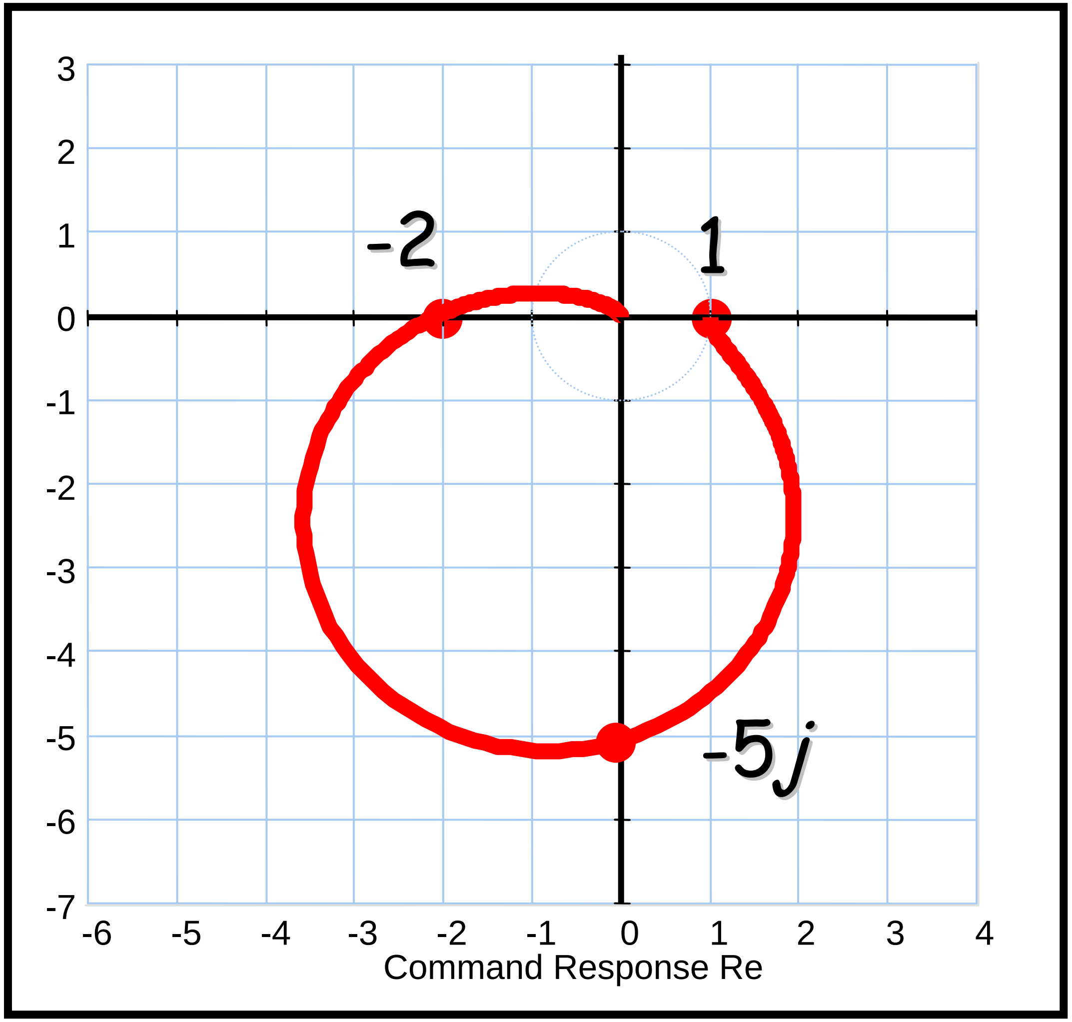


Open and closed systems

$$H_{cl} = \frac{H_L}{1 + H_L}$$

$$H_{cl} \rightarrow 5e^{-j\frac{\pi}{2}}$$

Stable because
-1 outside plot of H_L
(open system)

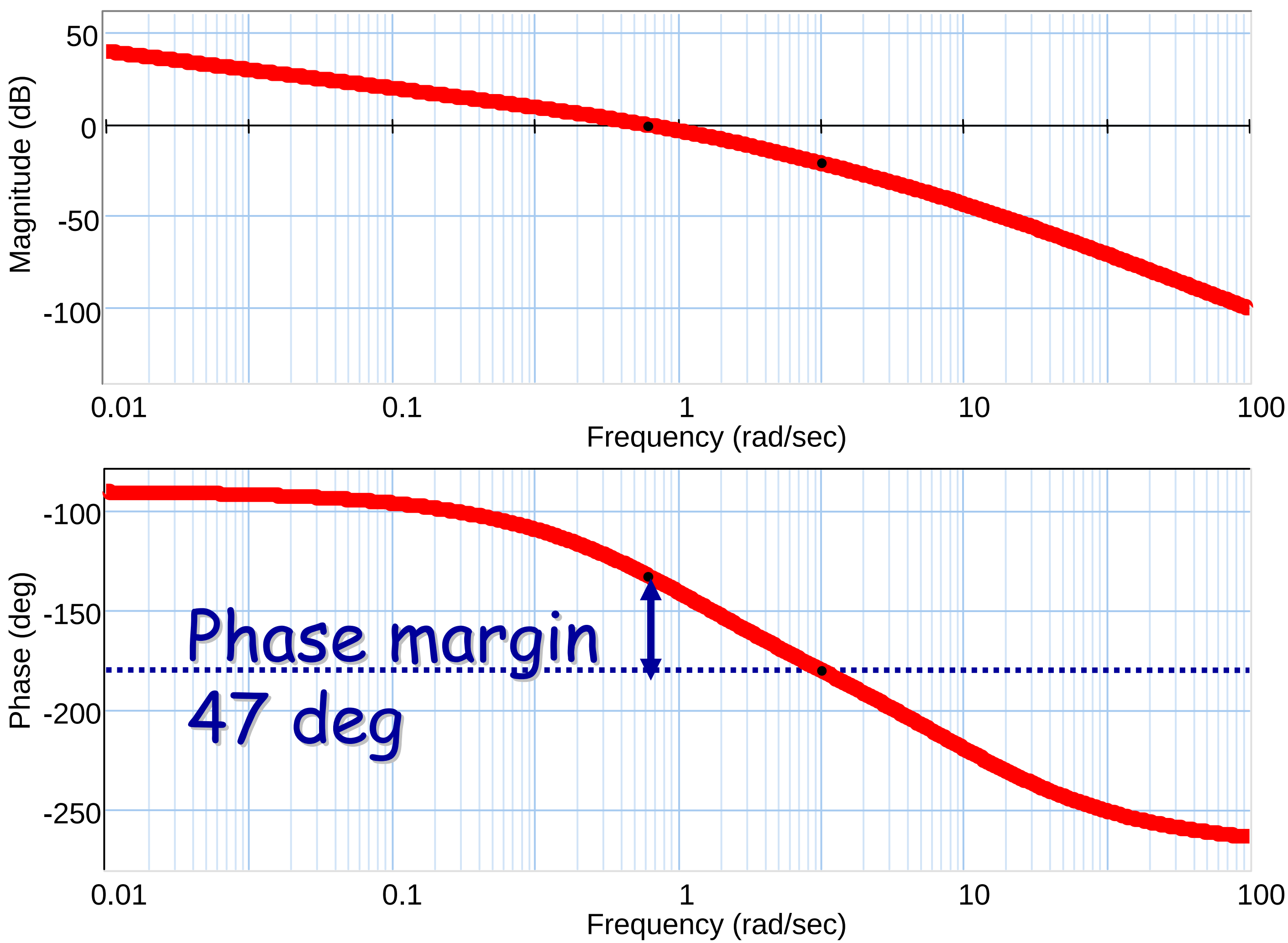


Design

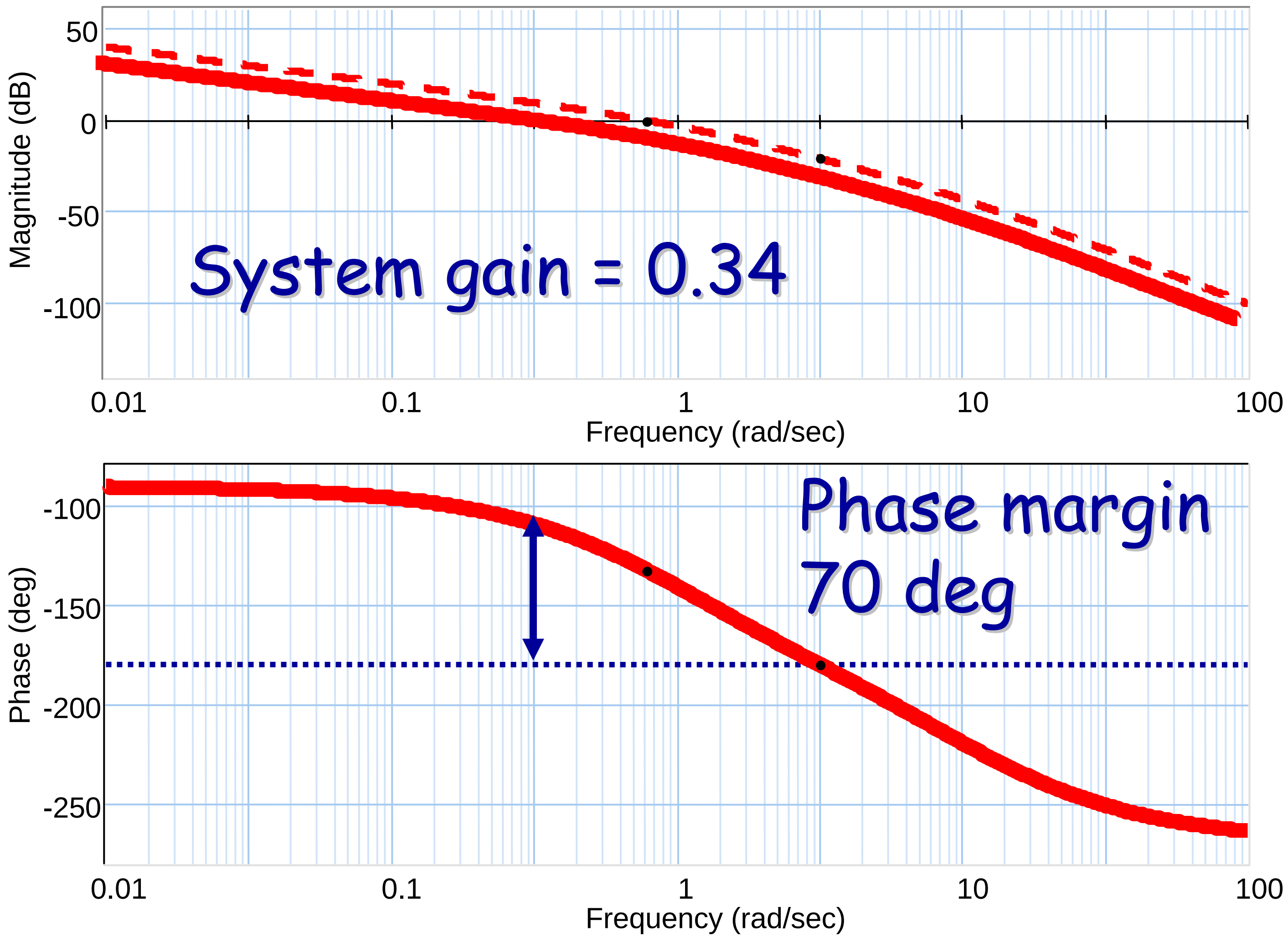
- Design a proportional controller such that the system has a phase margin of 70 degrees ($z \approx 0.7$) for the process:

$$H(j\omega) = \frac{10}{j\omega(j\omega+1)(j\omega+10)}$$

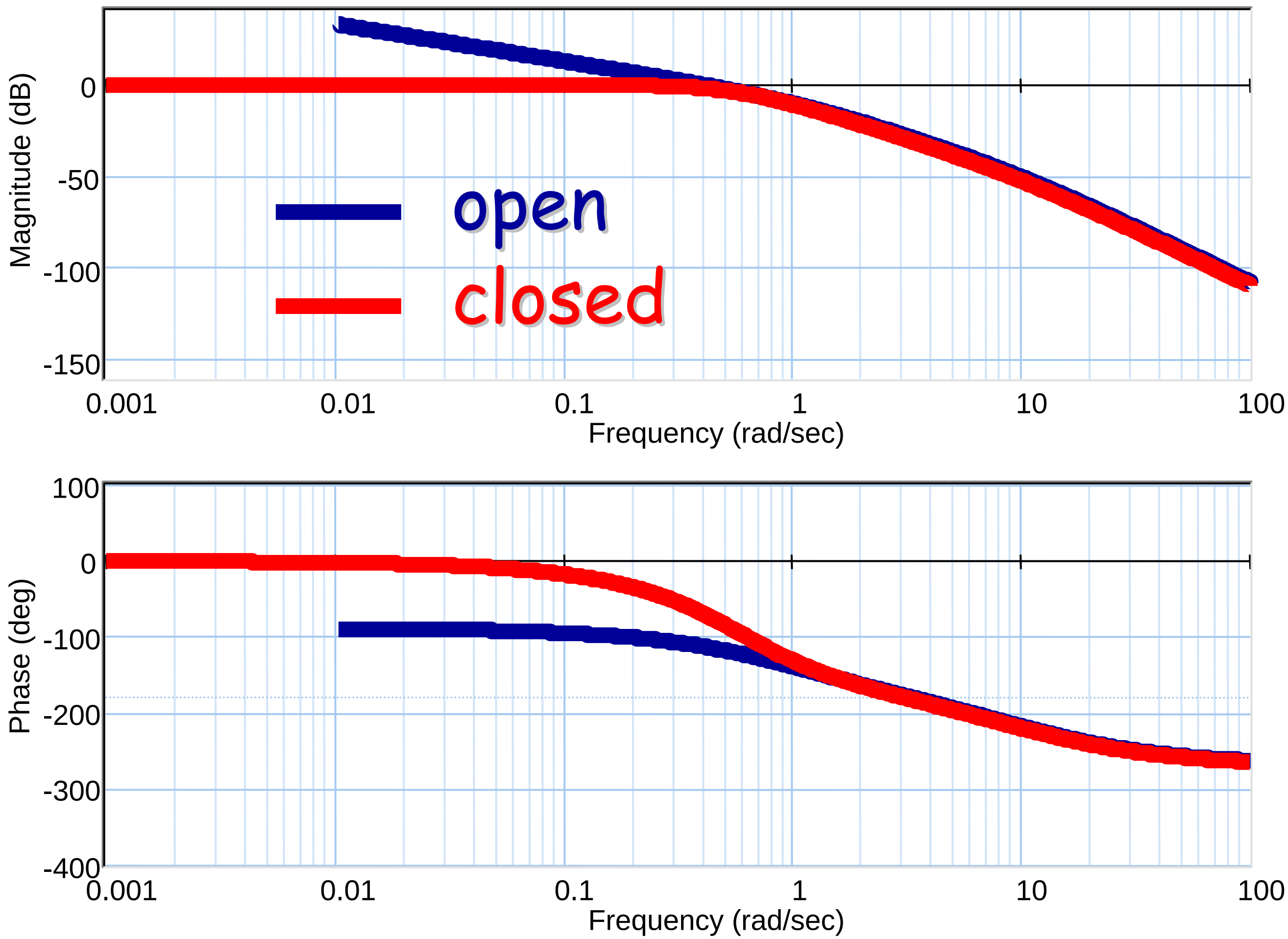
Design


50
0
-50
-100
Magnitude (dB)
0.01
0.1
1
10
100
Frequency (rad/sec)
-100
-150
-200
-250
Phase (deg)
Phase margin
47 deg
0.01
0.1
1
10
100
Frequency (rad/sec)

Design


50
0
-50
-100
Magnitude (dB)
System gain = 0.34
0.01
0.1
1
10
100
Frequency (rad/sec)
-100
-150
-200
-250
Phase (deg)
Phase margin
70 deg
0.01
0.1
1
10
100
Frequency (rad/sec)

Closed system (Bode)



Frequency (rad/sec)

Step response

Closed system (Bode)

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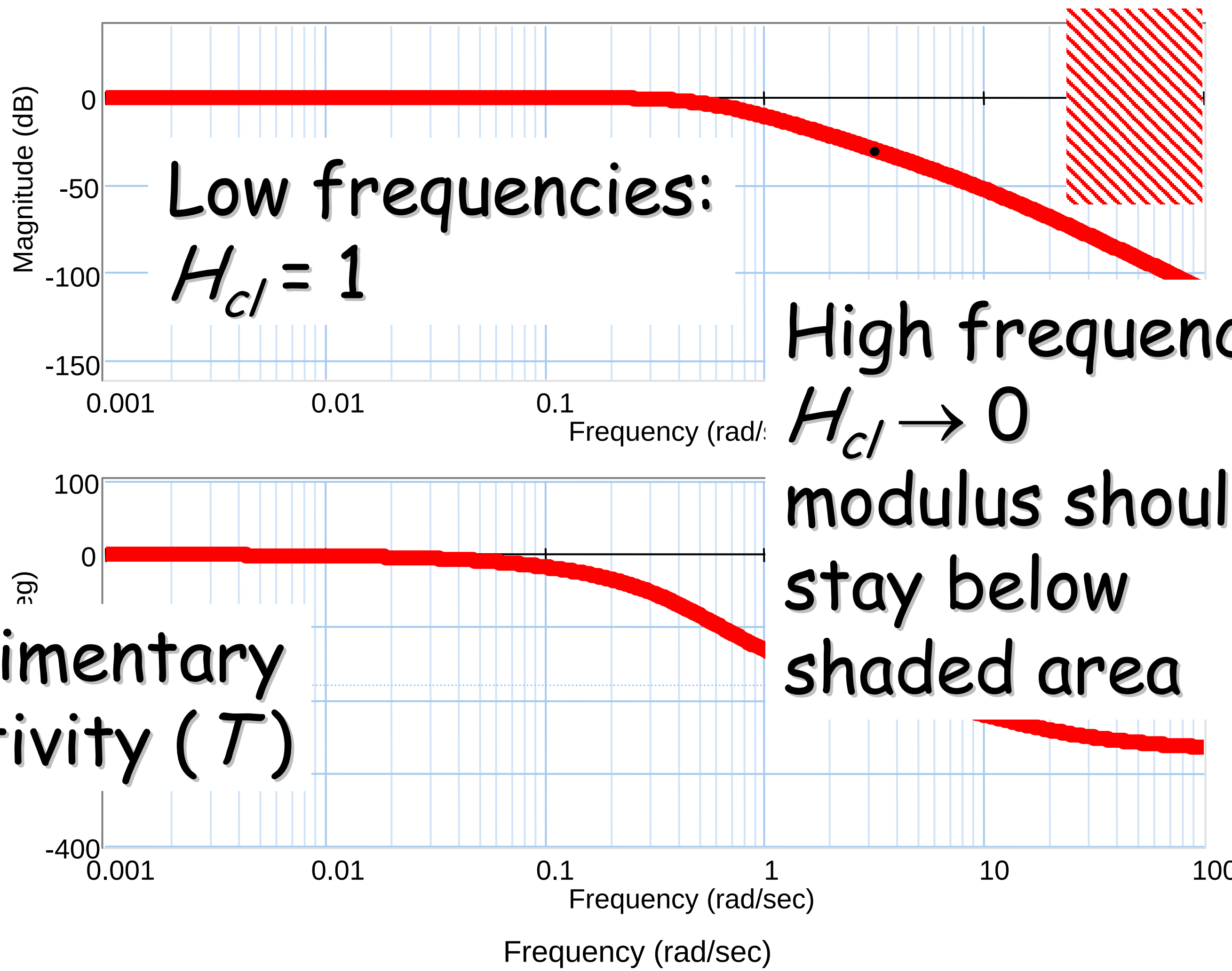

Low frequencies:
$H_{cl} = 1$

High frequencies:
$H_{cl} \rightarrow 0$
modulus should stay below shaded area

C/R

Complimentary Sensitivity (T)

Sensitivity (S)

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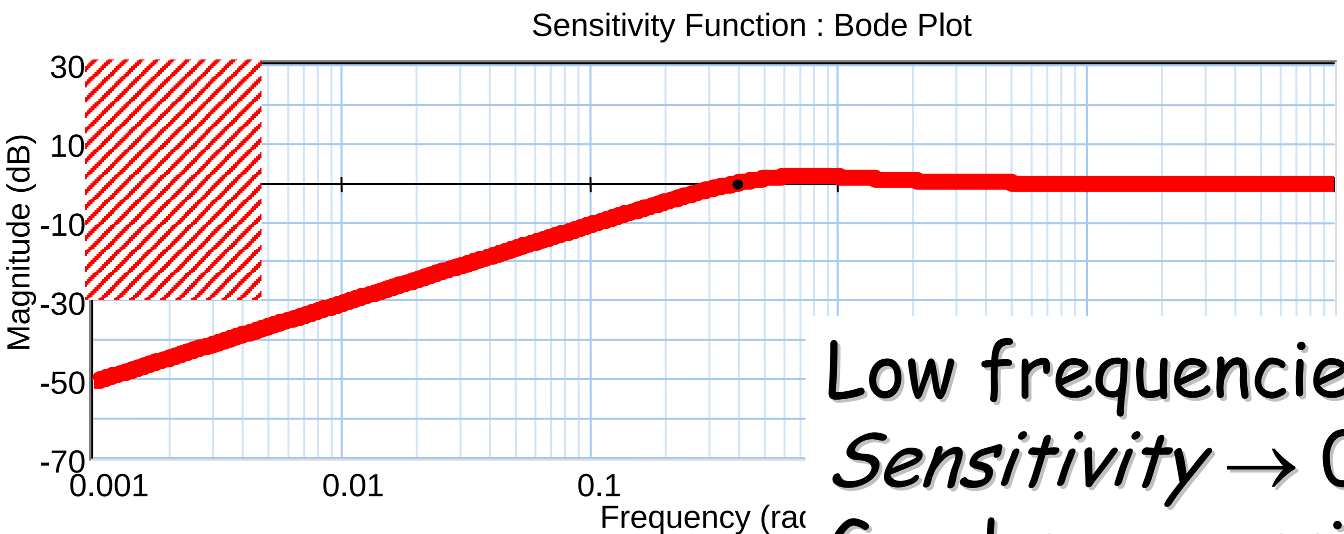


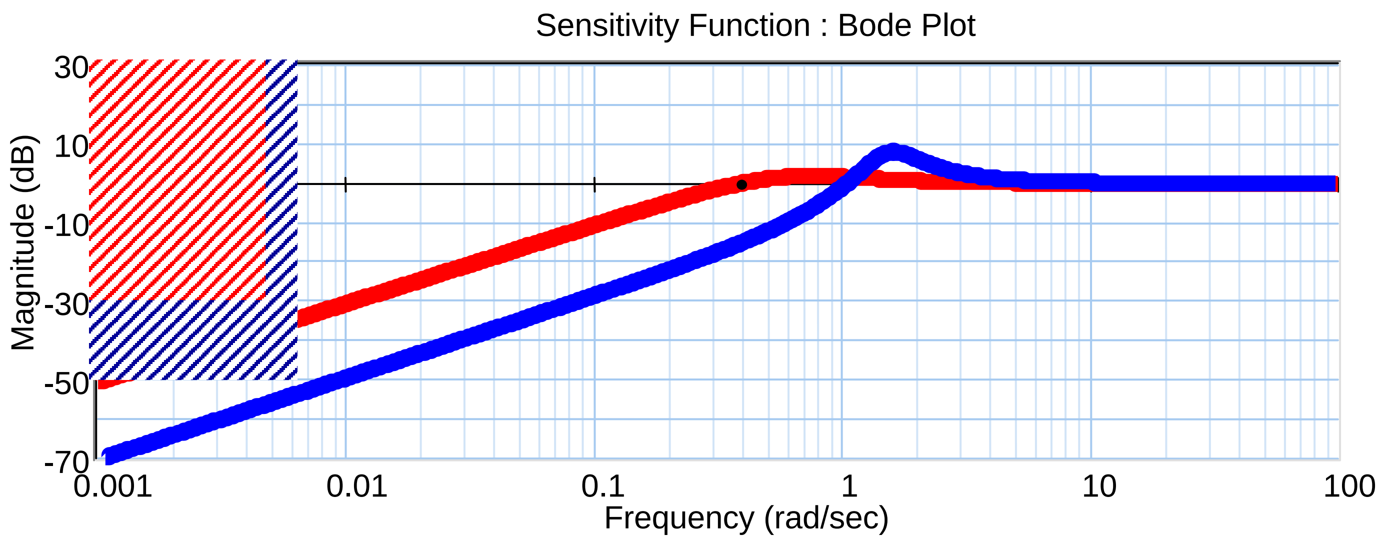
Low frequencies:
Sensitivity → 0
Good suppression of low-frequency disturbances

20-sim

demo

Frequency (rad/sec)

Sensitivity (S)



Higher gain: Better suppression of low-frequency disturbances, but amplification of higher frequencies

Frequency (rad/sec)

Bode Sensitivity integral (theorem of Westcott)

$S(j\omega)$ = Sensitivity (Afwijkingsverhouding)

$$S = \frac{1}{1 + H_L}$$

If H_L has at least two more poles than zero's:

$$\int_0^\infty \log\left|S(j\omega)\right| d\omega = 0$$

All improvements in one area, have to be paid for by a deterioration in another area (compare waterbed)

Sensitivity (S)

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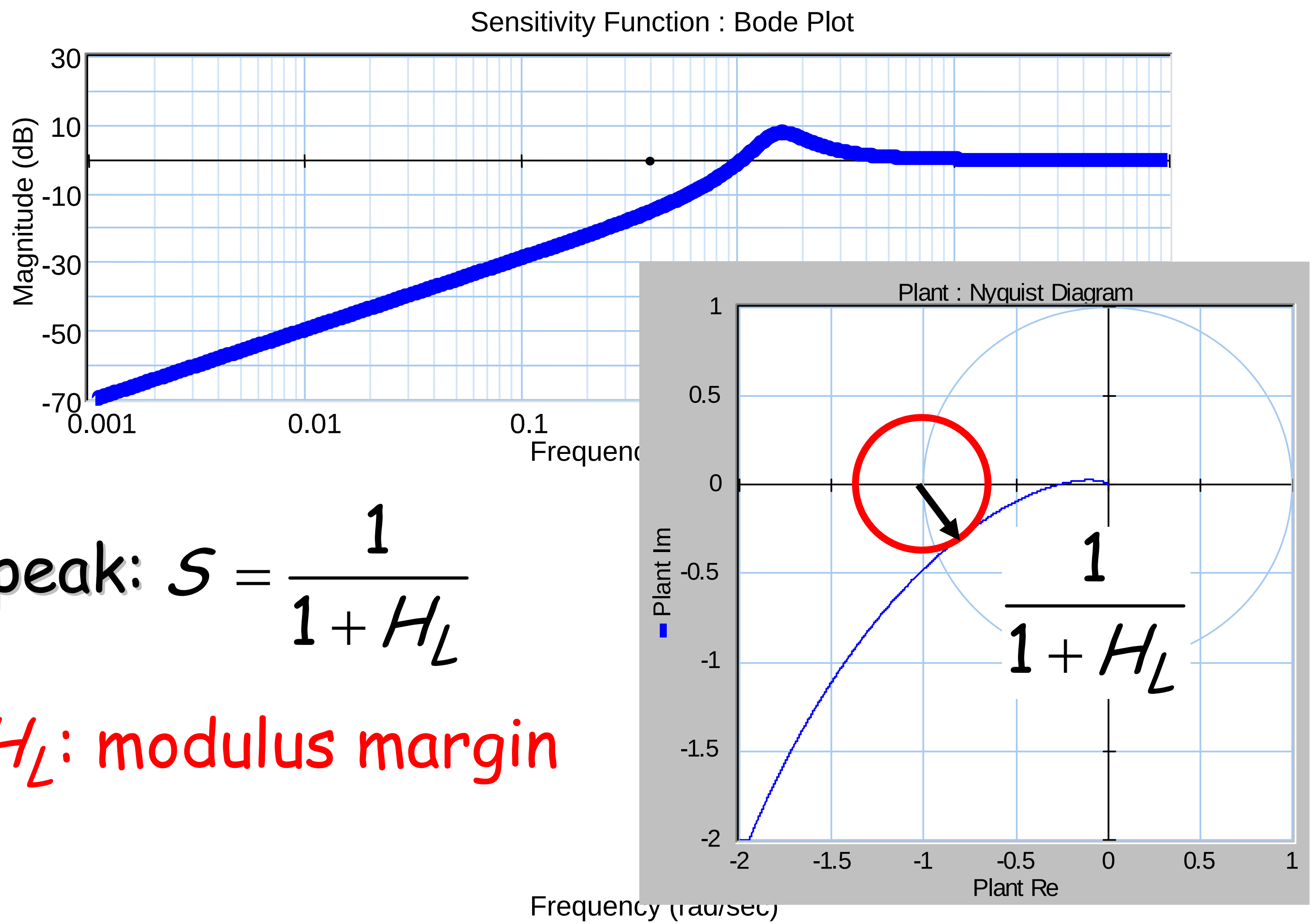


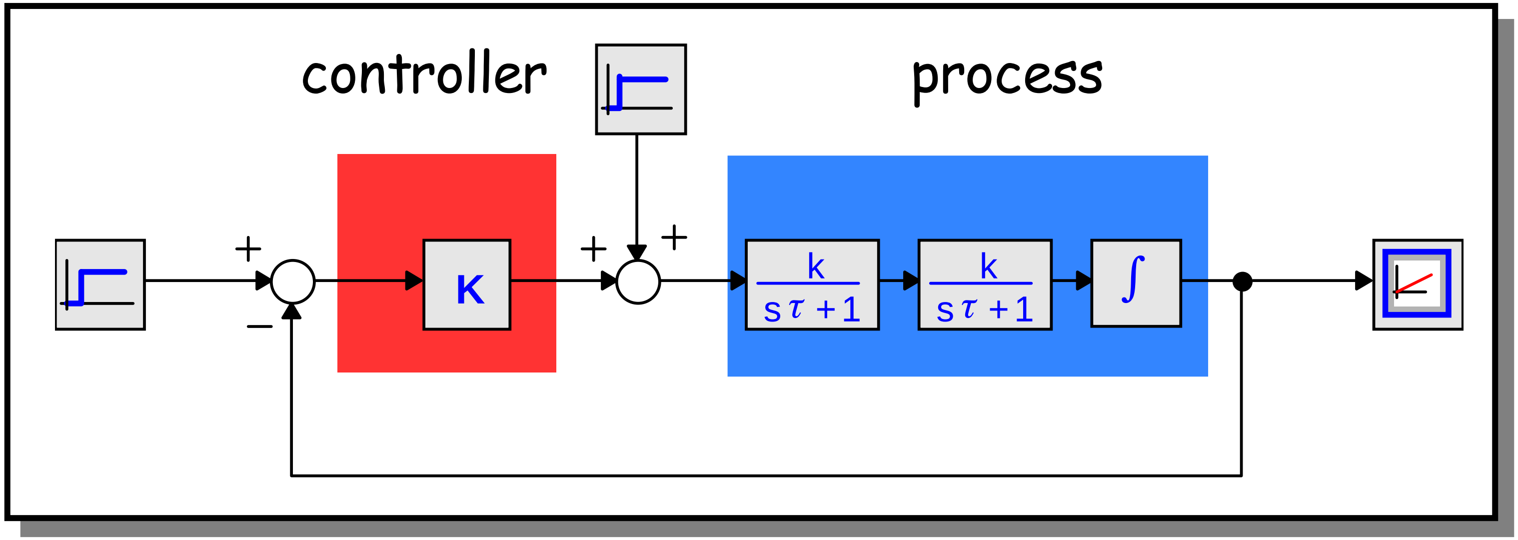
peak: $S = \dfrac{1}{1+H_L}$

$\dfrac{1}{1+H_L}$

$1+H_L$: modulus margin

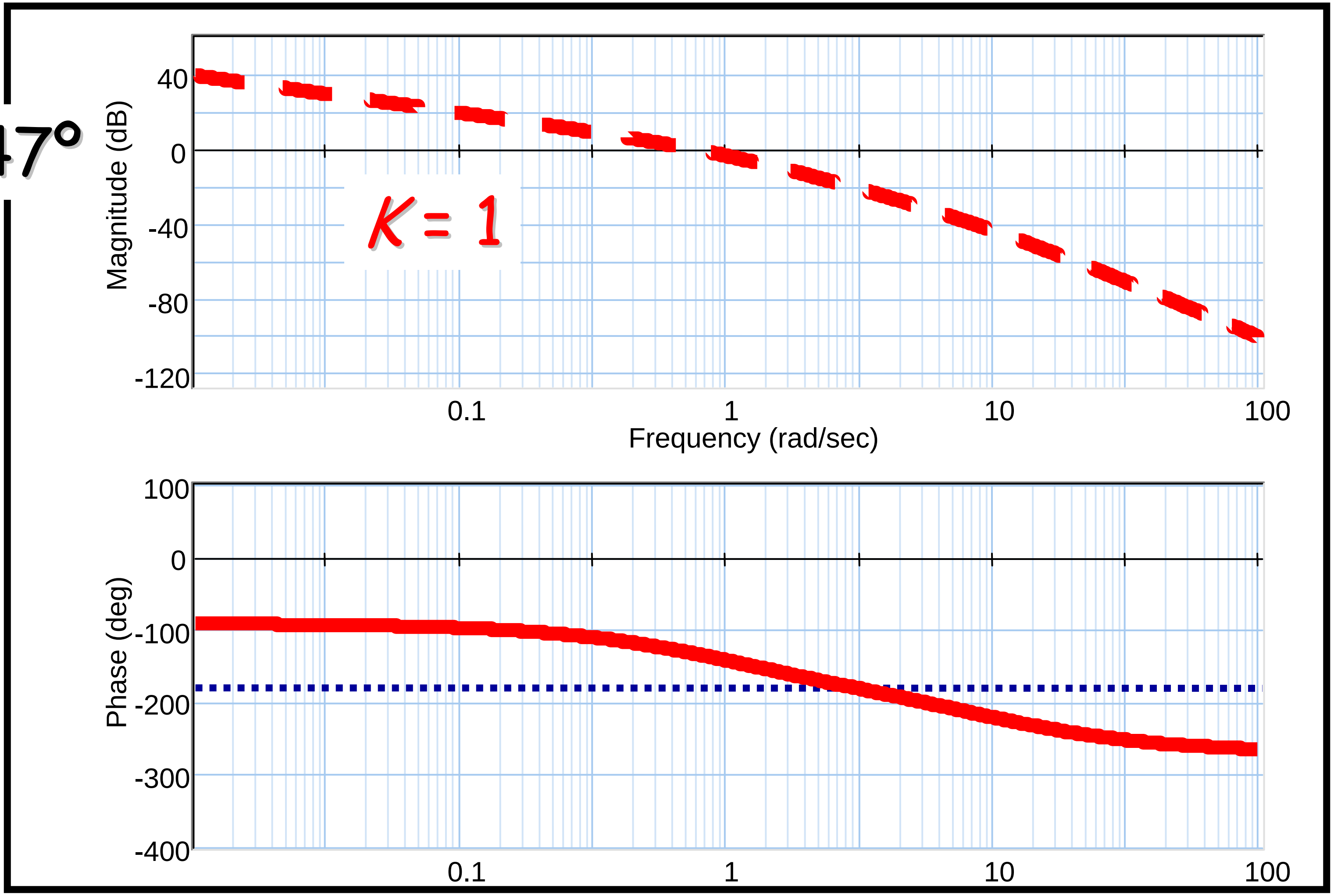
Disturbance suppression

Consider the following system

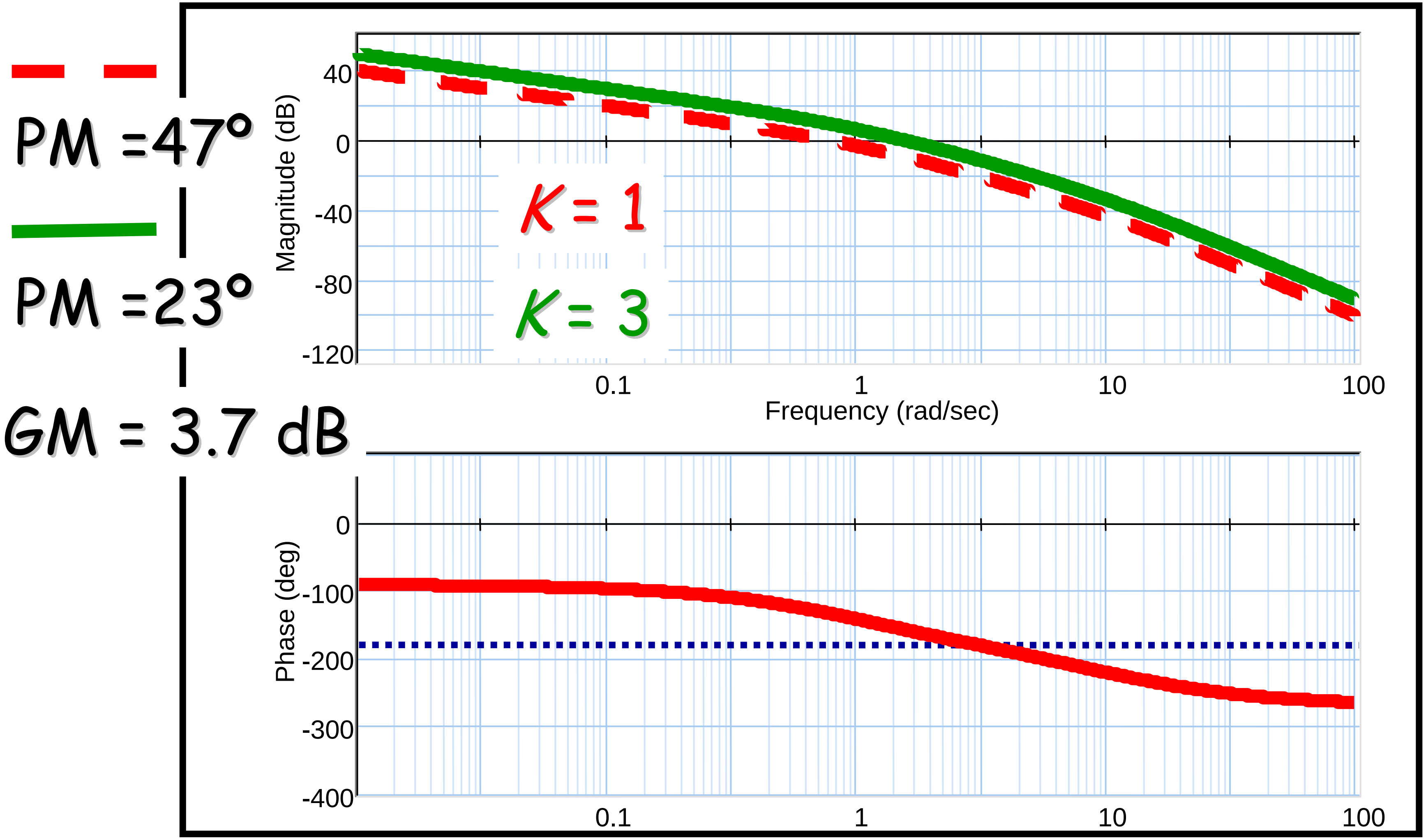


Bode

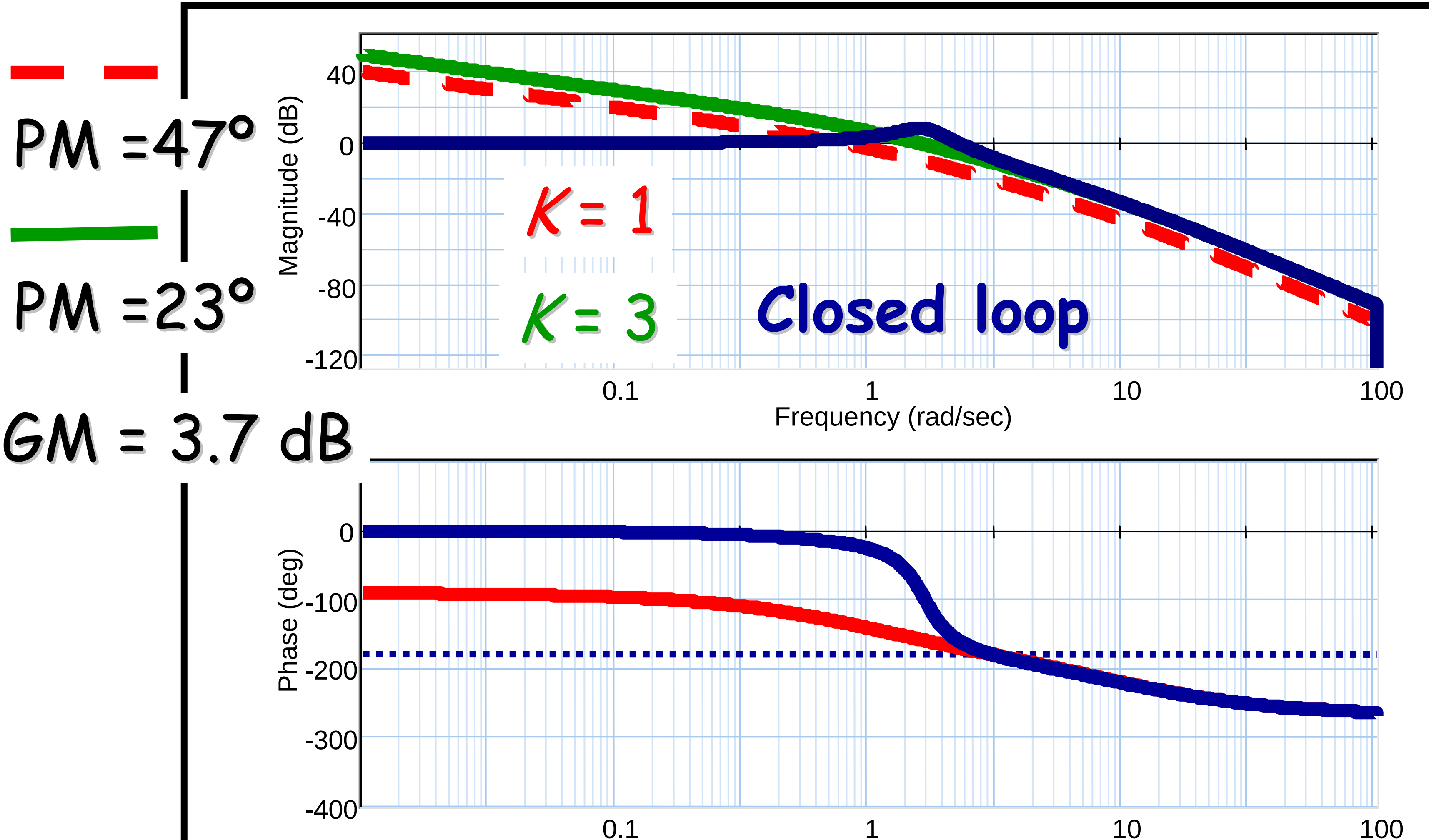
PM =47°


K = 1
40
0
-40
-80
-120
Magnitude (dB)
0.1
1
10
100
Frequency (rad/sec)
100
0
-100
-200
-300
-400
Phase (deg)
0.1
1
10
100

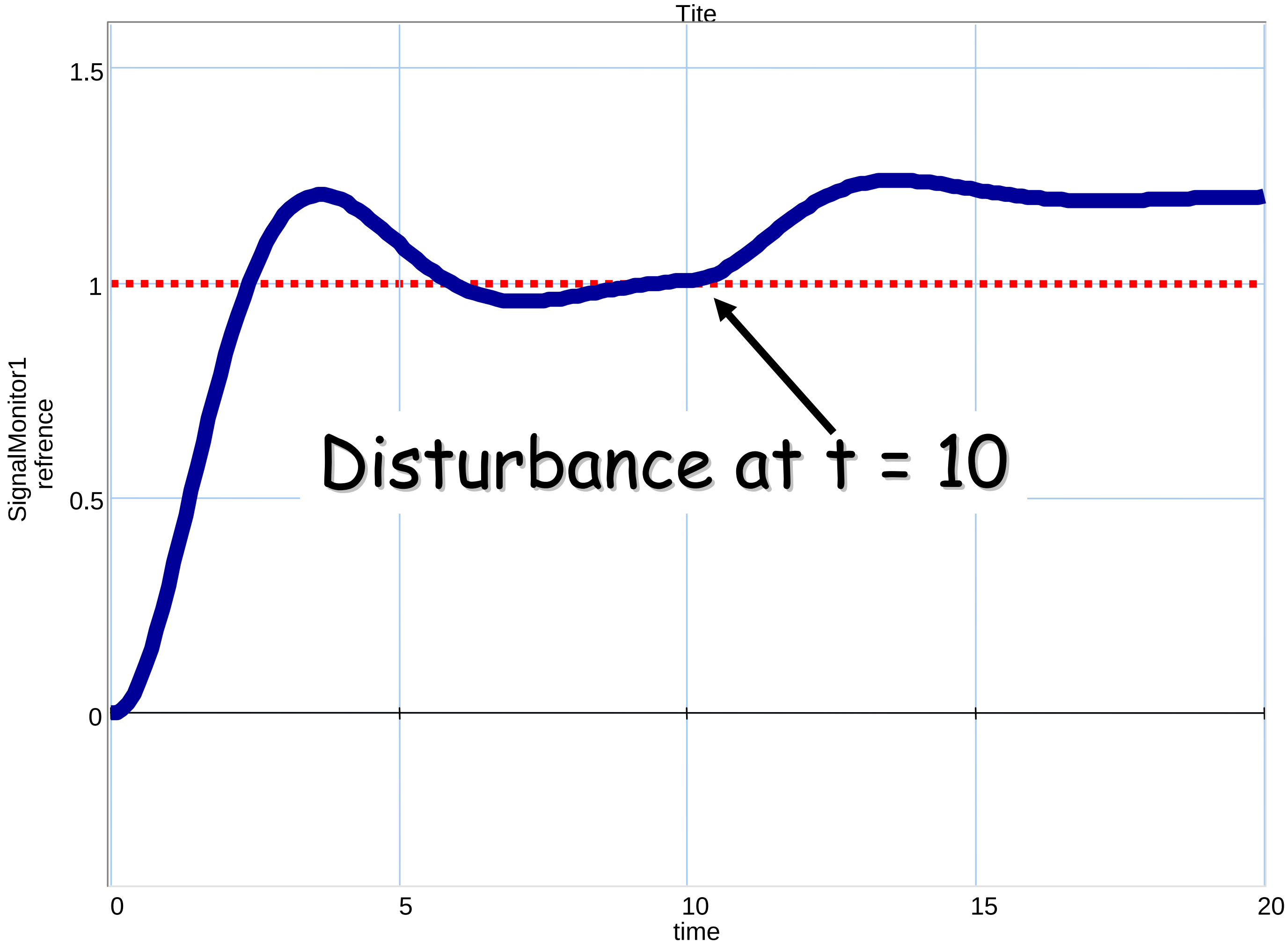
Bode


PM =47°
PM =23°
GM = 3.7 dB
K= 1
K= 3
40
0
-40
-80
-120
Magnitude (dB)
0.1
1
10
100
Frequency (rad/sec)
0
-100
-200
-300
-400
Phase (deg)
0.1
1
10
100

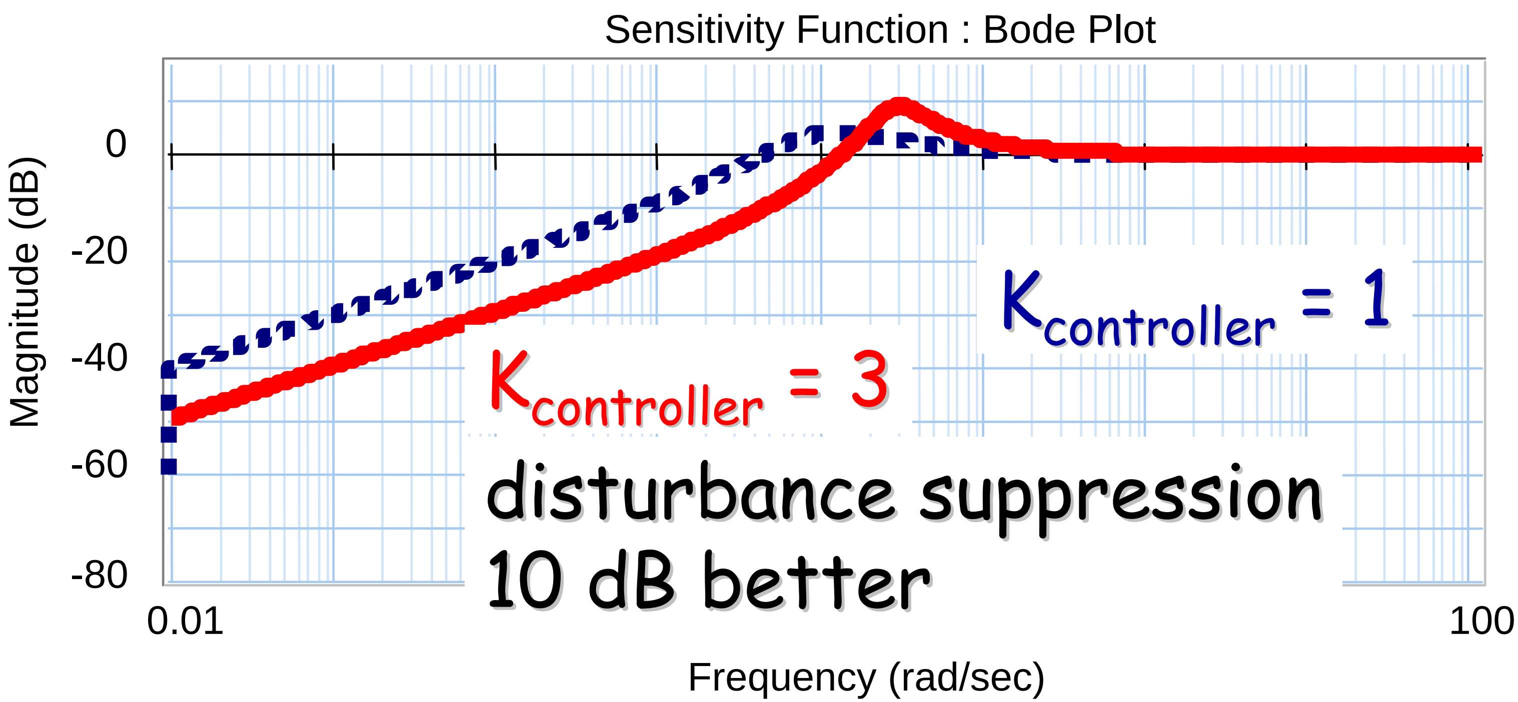
Bode


PM =47°
PM =23°
GM = 3.7 dB
K= 1
K= 3
Closed loop
Magnitude (dB)
40
0
-40
-80
-120
0.1
1
10
100
Frequency (rad/sec)
Phase (deg)
0
-100
-200
-300
-400
0.1
1
10
100

Response K = 1

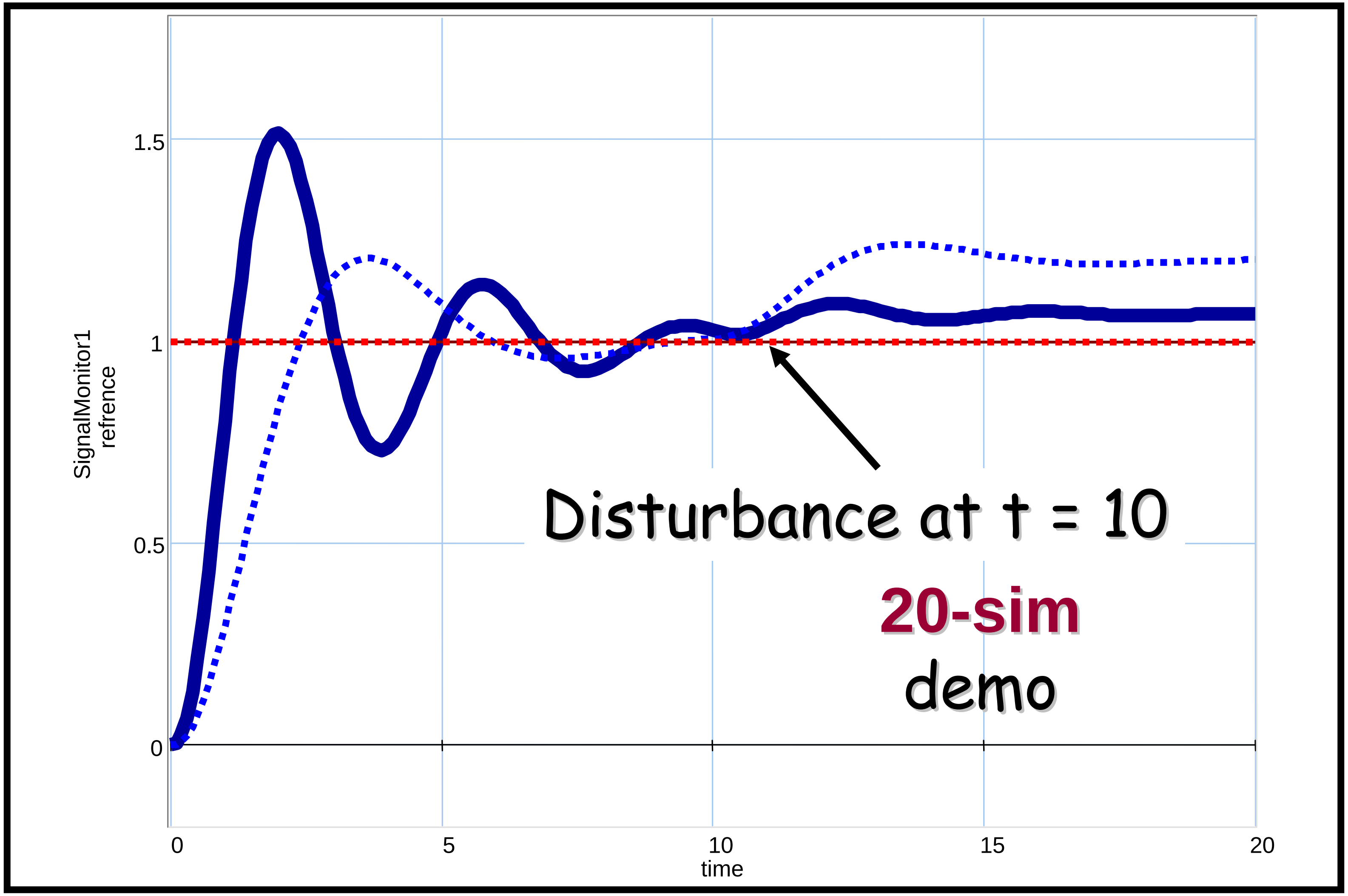

Tite
1.5
1
0.5
0
SignalMonitor1
refrence
0
5
10
15
20
time
Disturbance at t = 10

Sensitivity

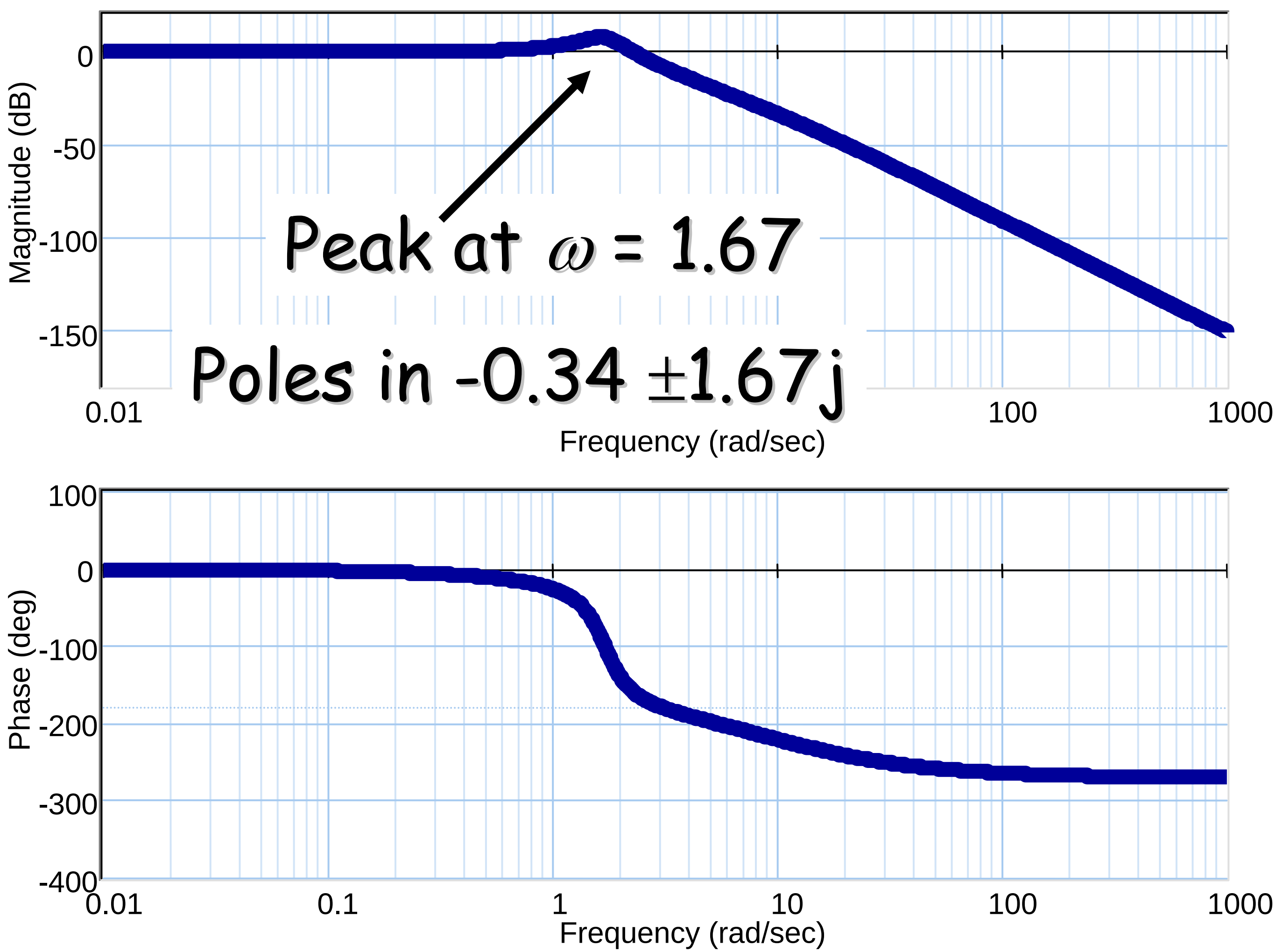


disturbance suppression
10 dB better

Response K = 3

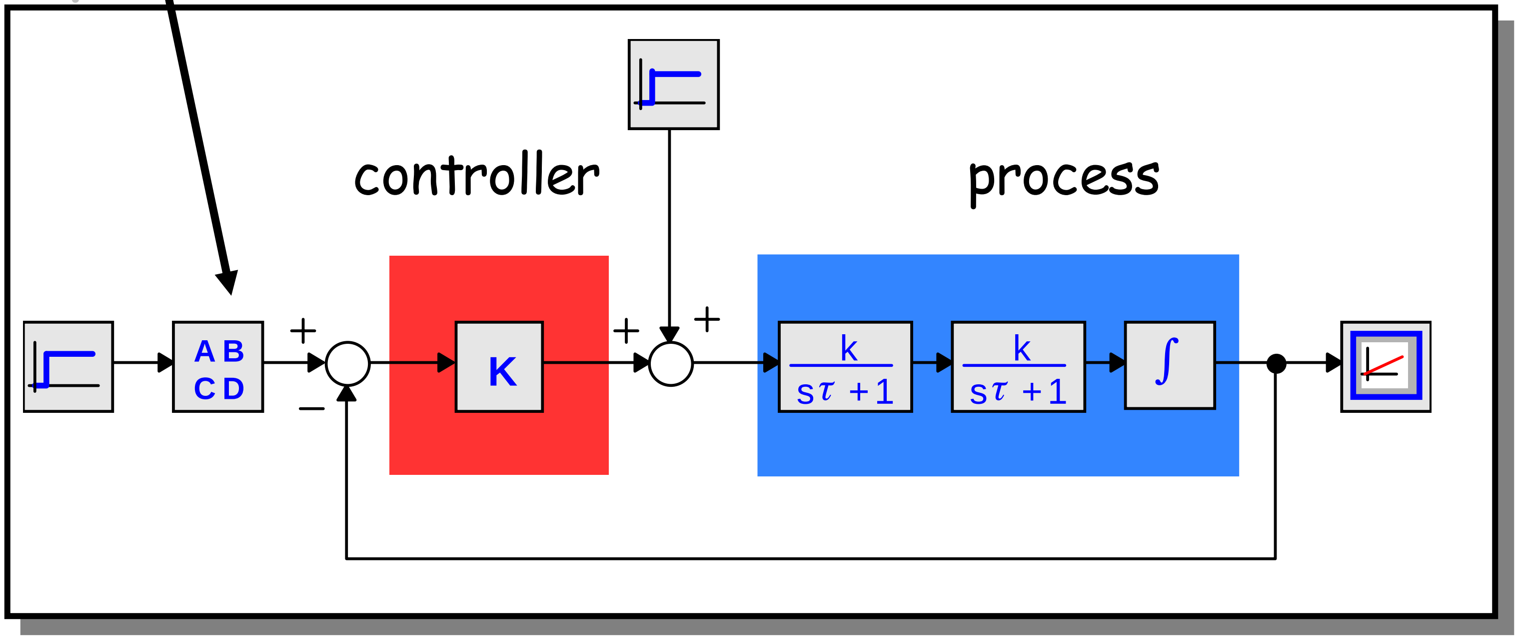

1.5
1
0.5
0
SignalMonitor1
refrence
0
5
10
15
20
time
Disturbance at t = 10
20-sim
demo

Bode

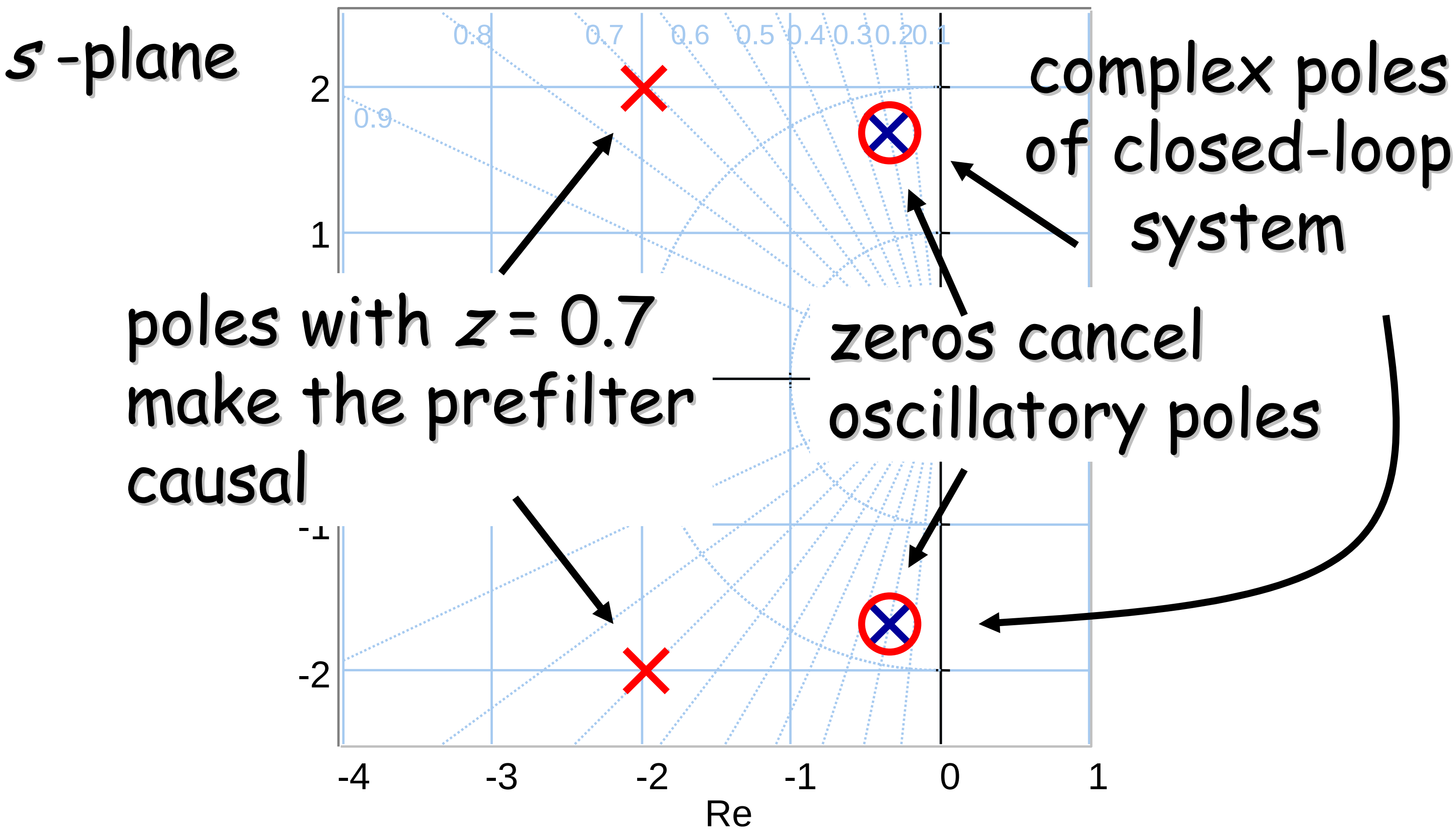

0
-50
-100
-150
Magnitude (dB)
0.01
100
1000
Frequency (rad/sec)
Peak at ω = 1.67
Poles in -0.34 ±1.67j
100
0
-100
-200
-300
-400
Phase (deg)
0.01
0.1
1
10
100
1000
Frequency (rad/sec)

Prefilter

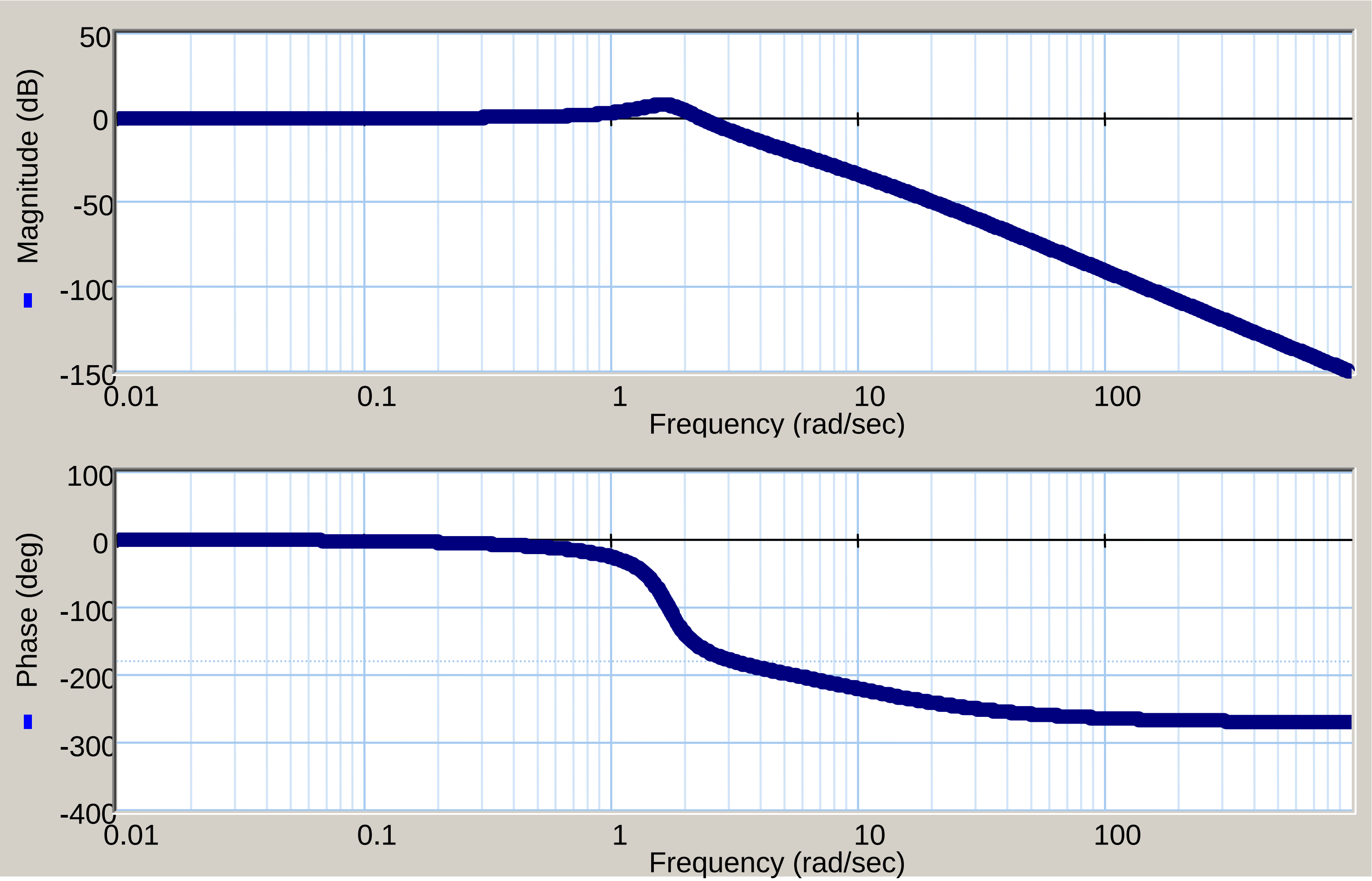
attenuate the resonance frequencies



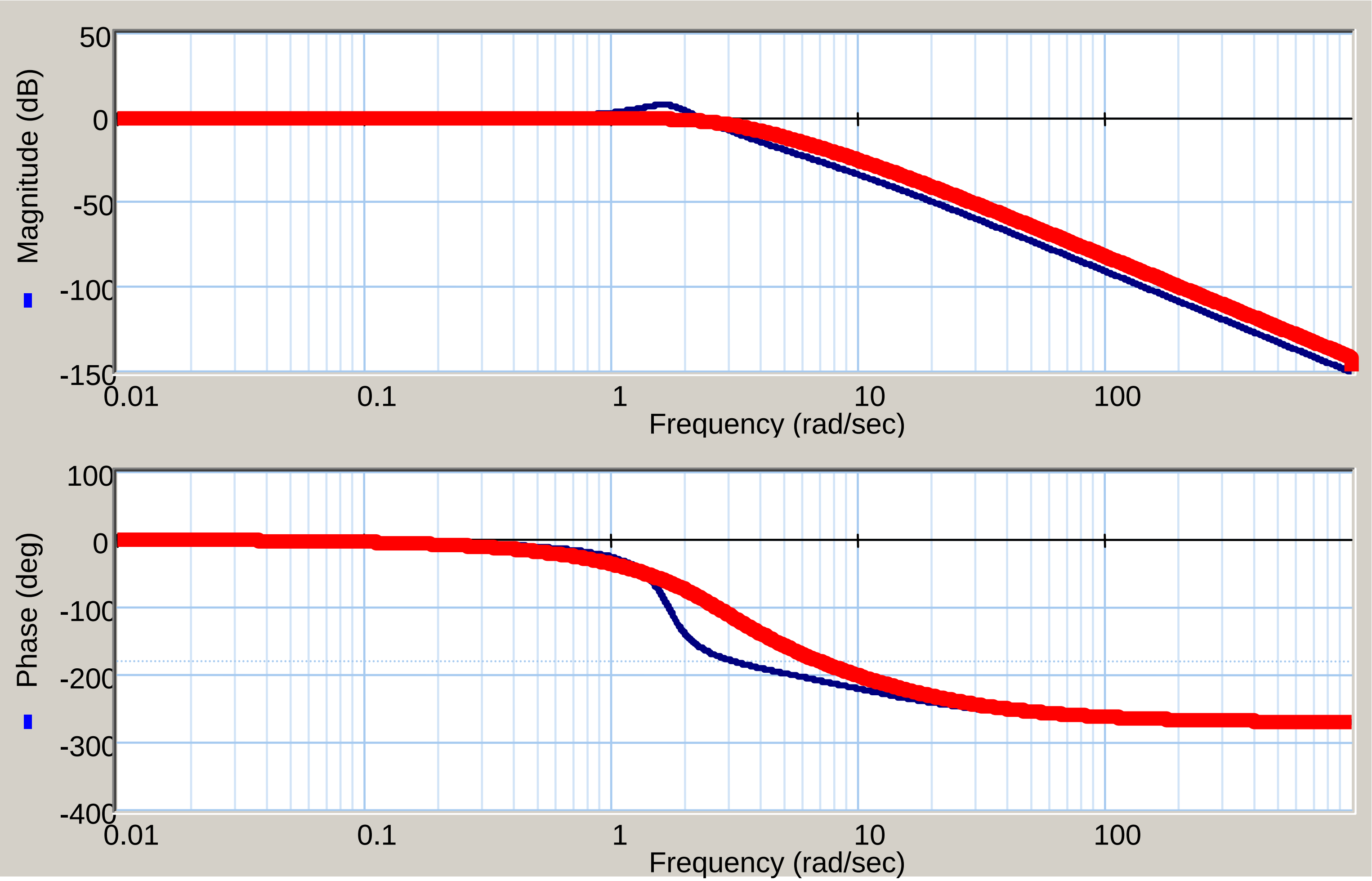
Prefilter


s -plane
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.9
2
1
-1
-2
-4
-3
-2
-1
0
1
Re
complex poles of closed-loop system
poles with z = 0.7 make the prefilter causal
zeros cancel oscillatory poles

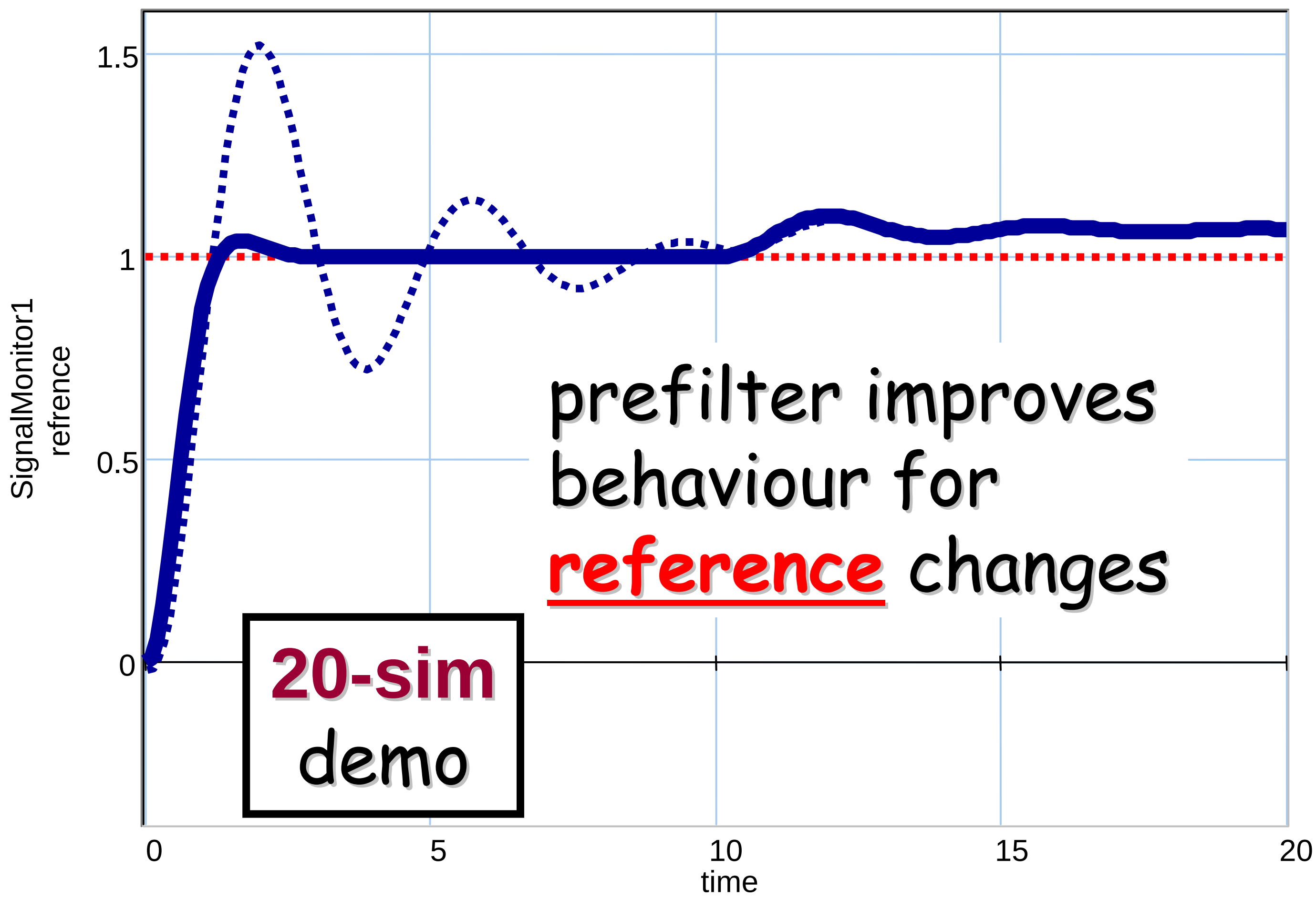
Closed-loop Bode



Closed-loop Bode + prefilter



Response



prefilter improves behaviour for **reference** changes

20-sim demo

Review

- Disturbance suppression does not require high damping ratios
- Response on reference changes can be improved by means of a prefilter
- But…
  - gain and phase margins were small
  - robustness for parameter variations is small

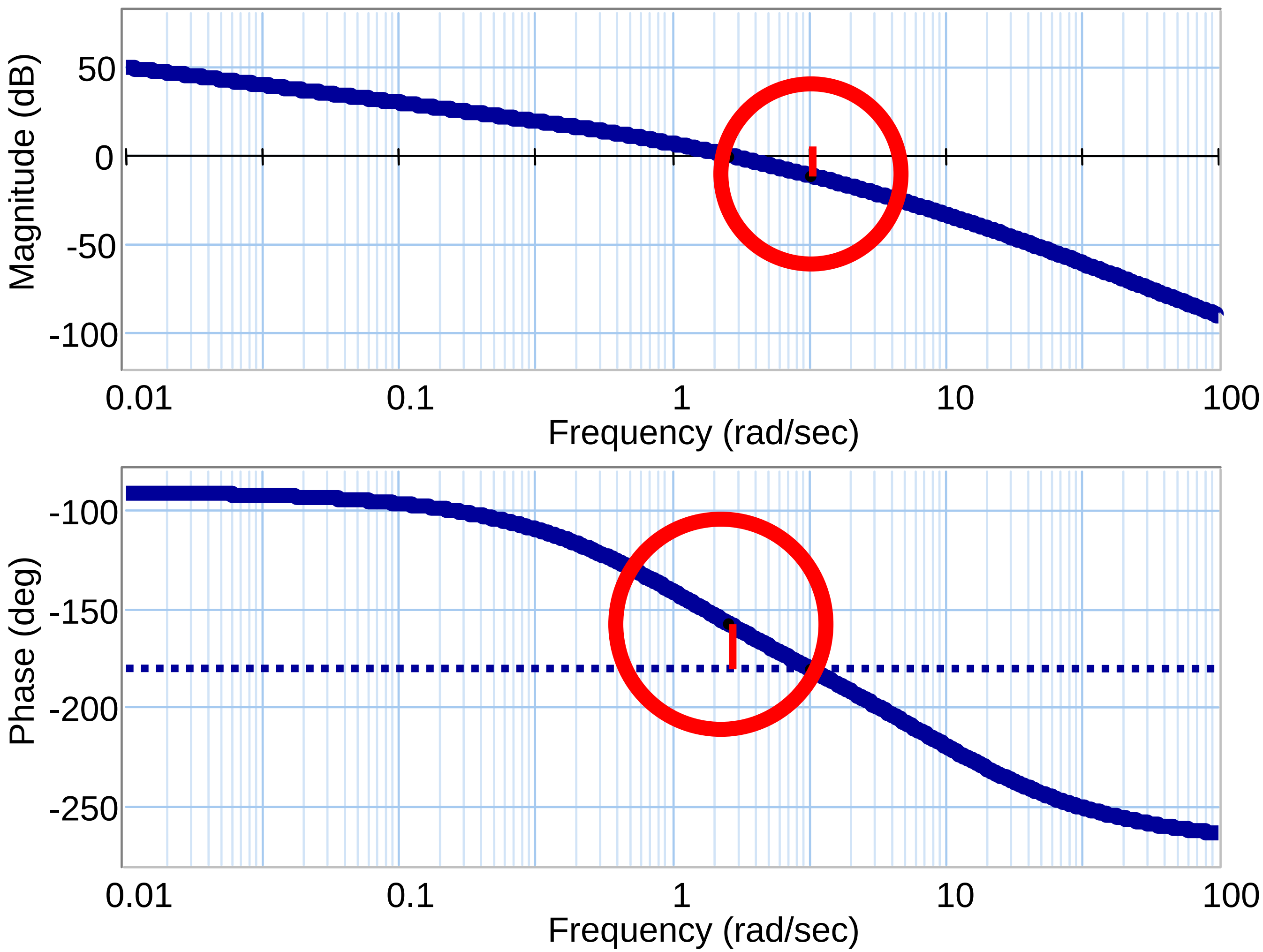
Solution

- Try to improve the robustness by designing more advanced compensators that simultaneously guarantee
  - good transients
  - high disturbance suppression

Uncompensated system

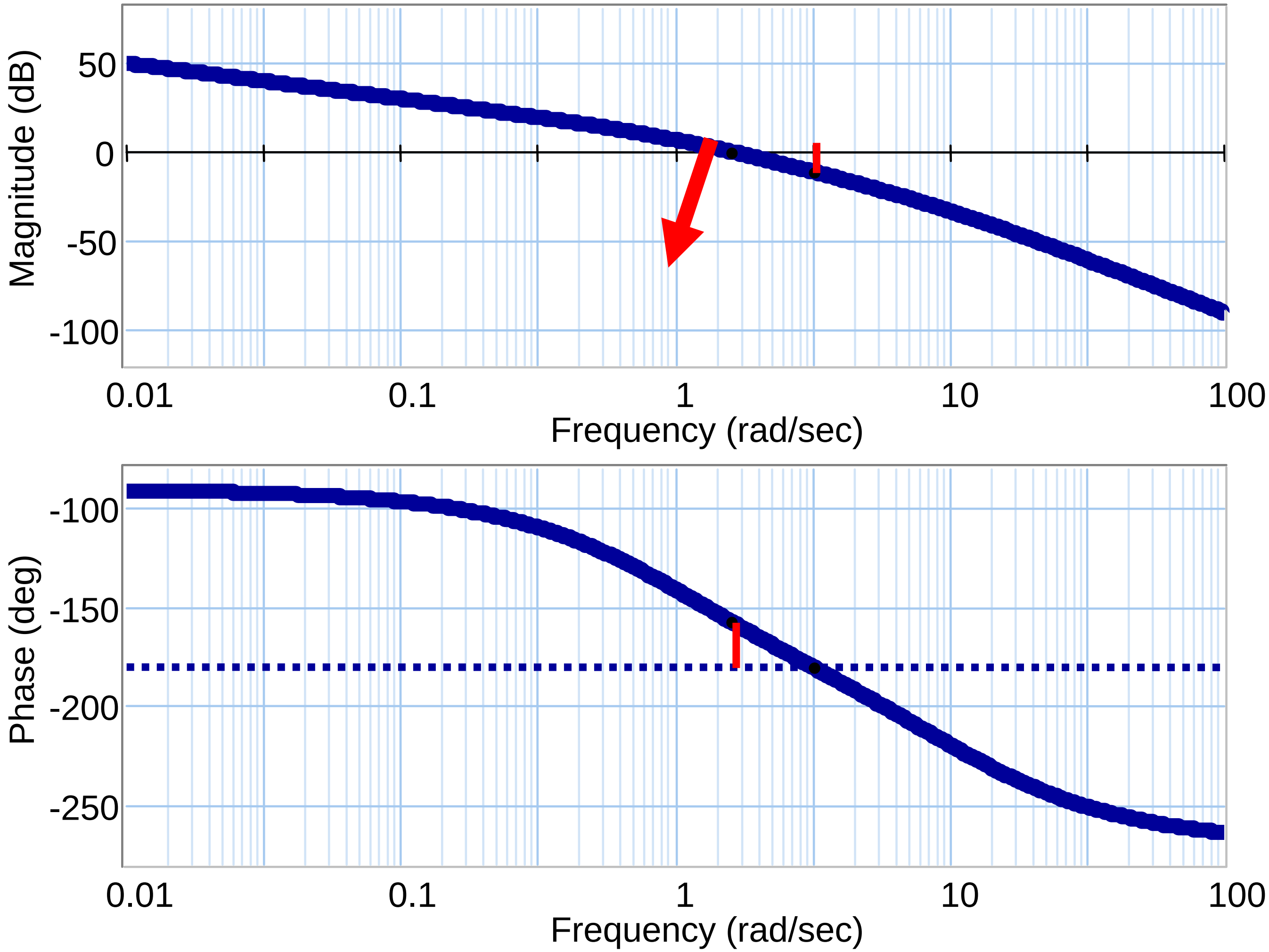
1: decrease HF-gain

2: decrease HF-phase shift

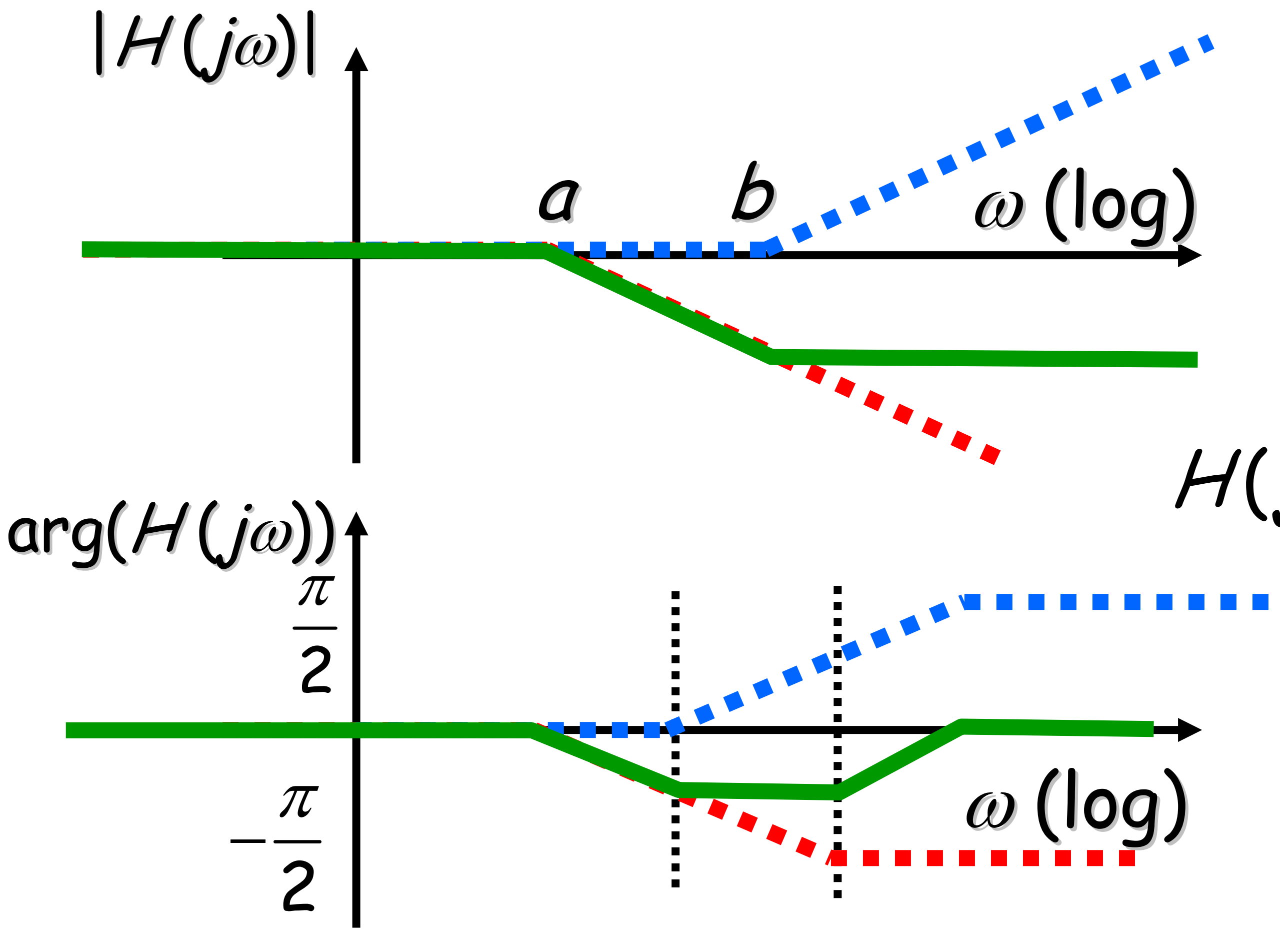


Uncompensated system

1:
decrease HF-gain



Decrease HF gain



Lag network

$$H(j\omega) = \frac{a}{b}\frac{j\omega + b}{j\omega + a}$$

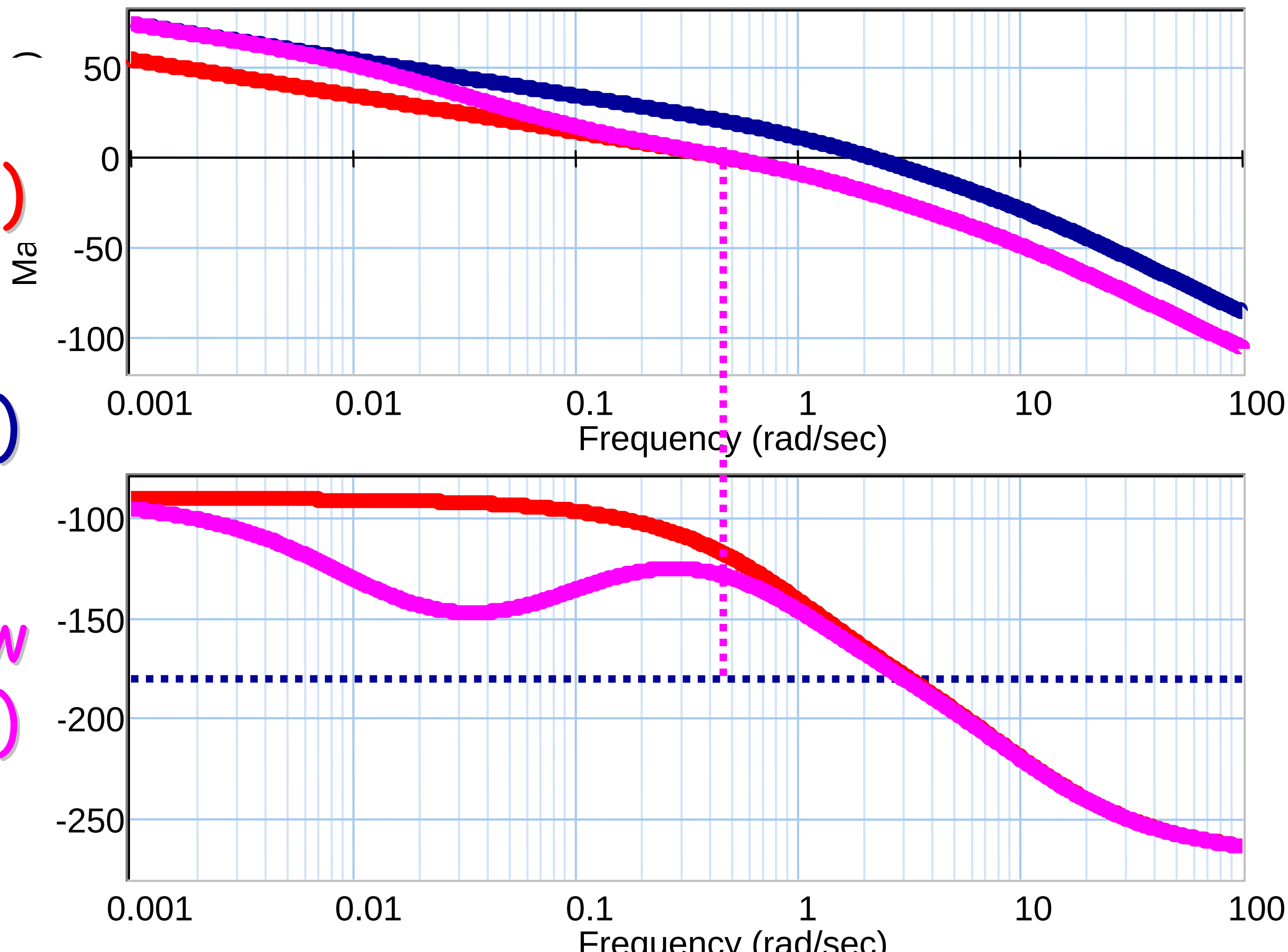
$$a < b$$

Lag network

K = 0.5
(PM = 63°)

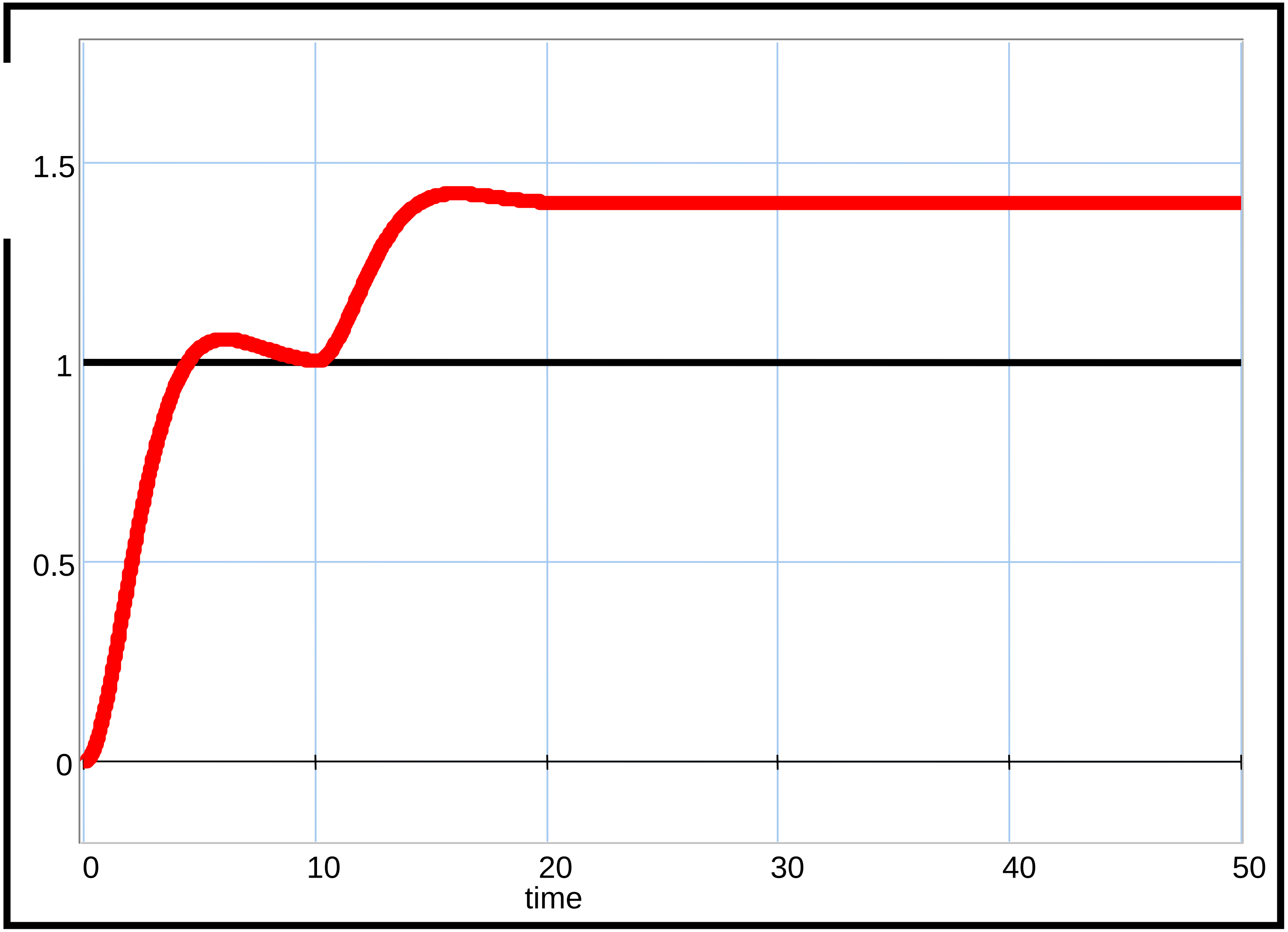
K = 5
(PM = 13°)

K = 5
+ Lag netw
(PM = 52°)



Responses

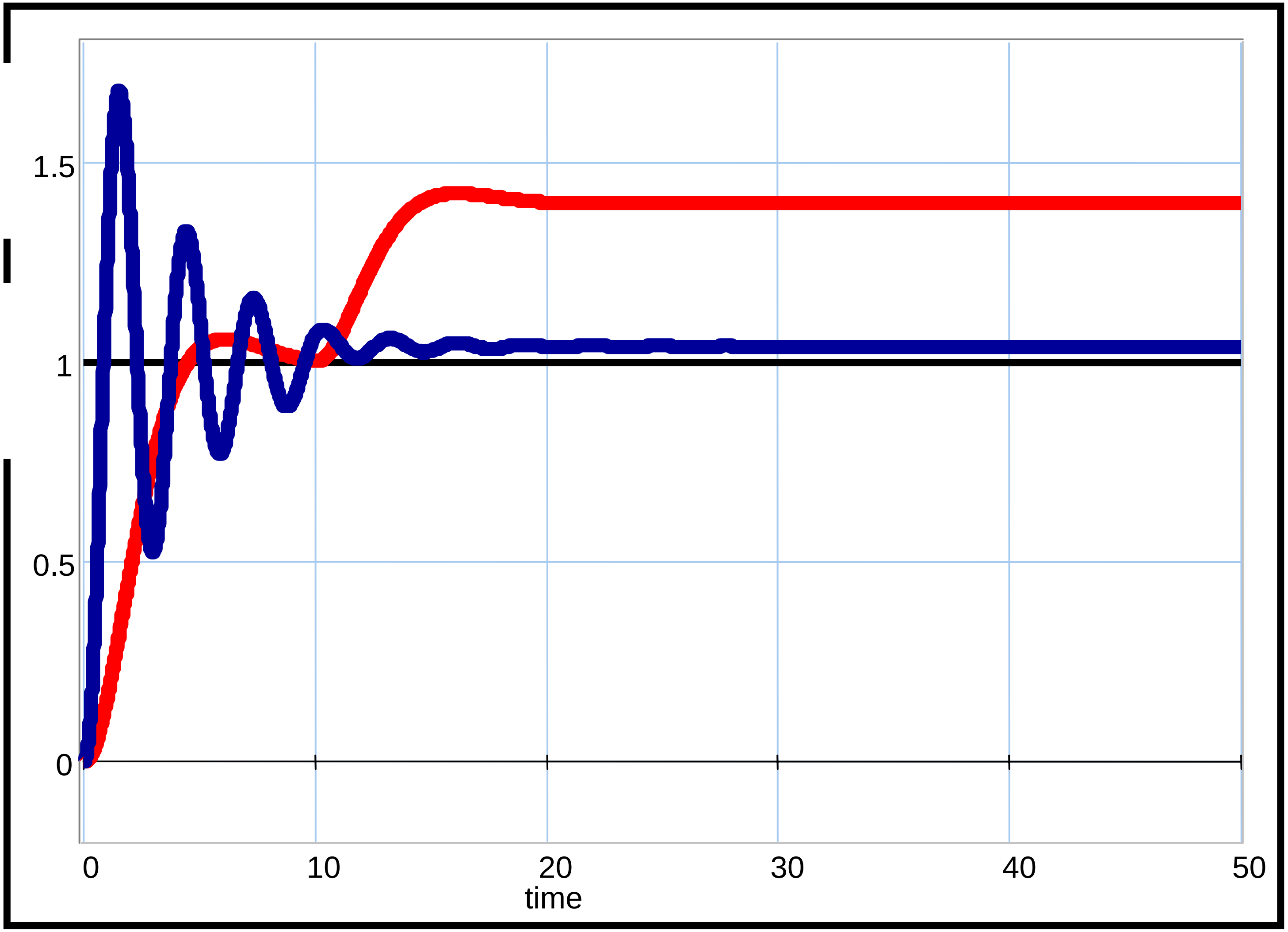
K = 0.5
(PM = 63°)



Responses

K = 0.5
(PM = 63°)

K = 5
(PM = 13°)

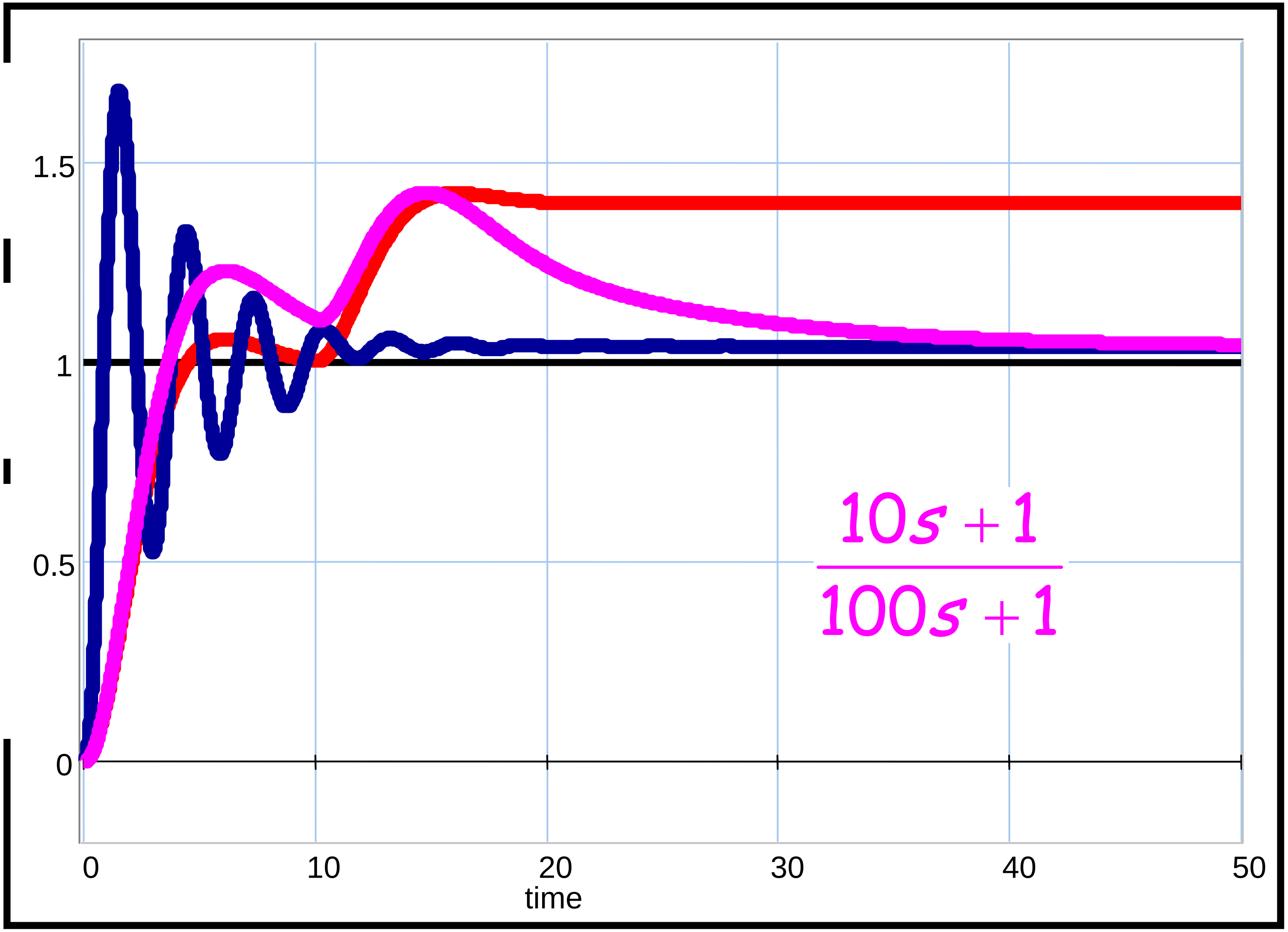


Responses

K = 0.5
(PM = 63°)

K = 5
(PM = 13°)

K = 5
+ Lag netw
(PM = 52°)



Responses

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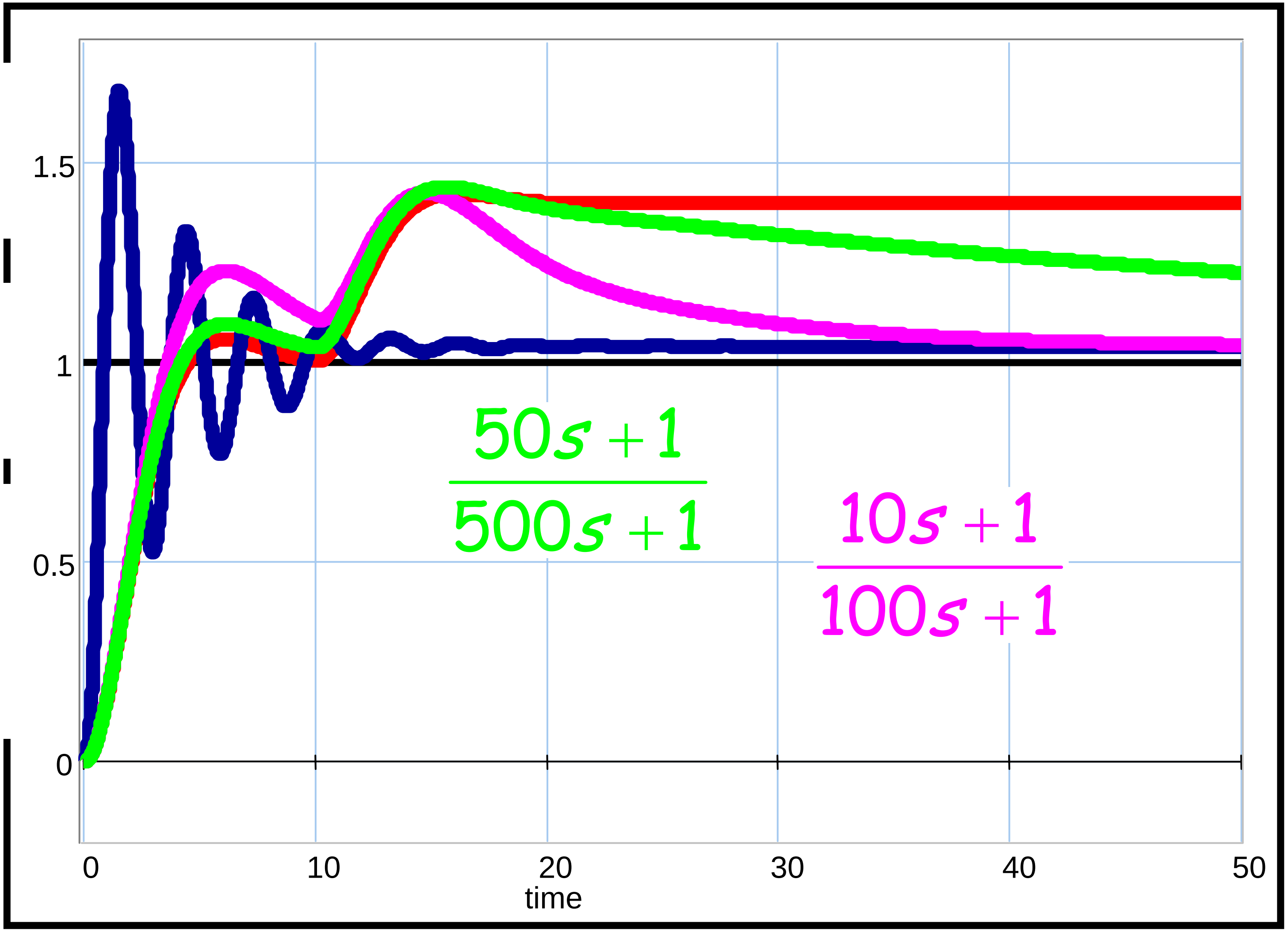
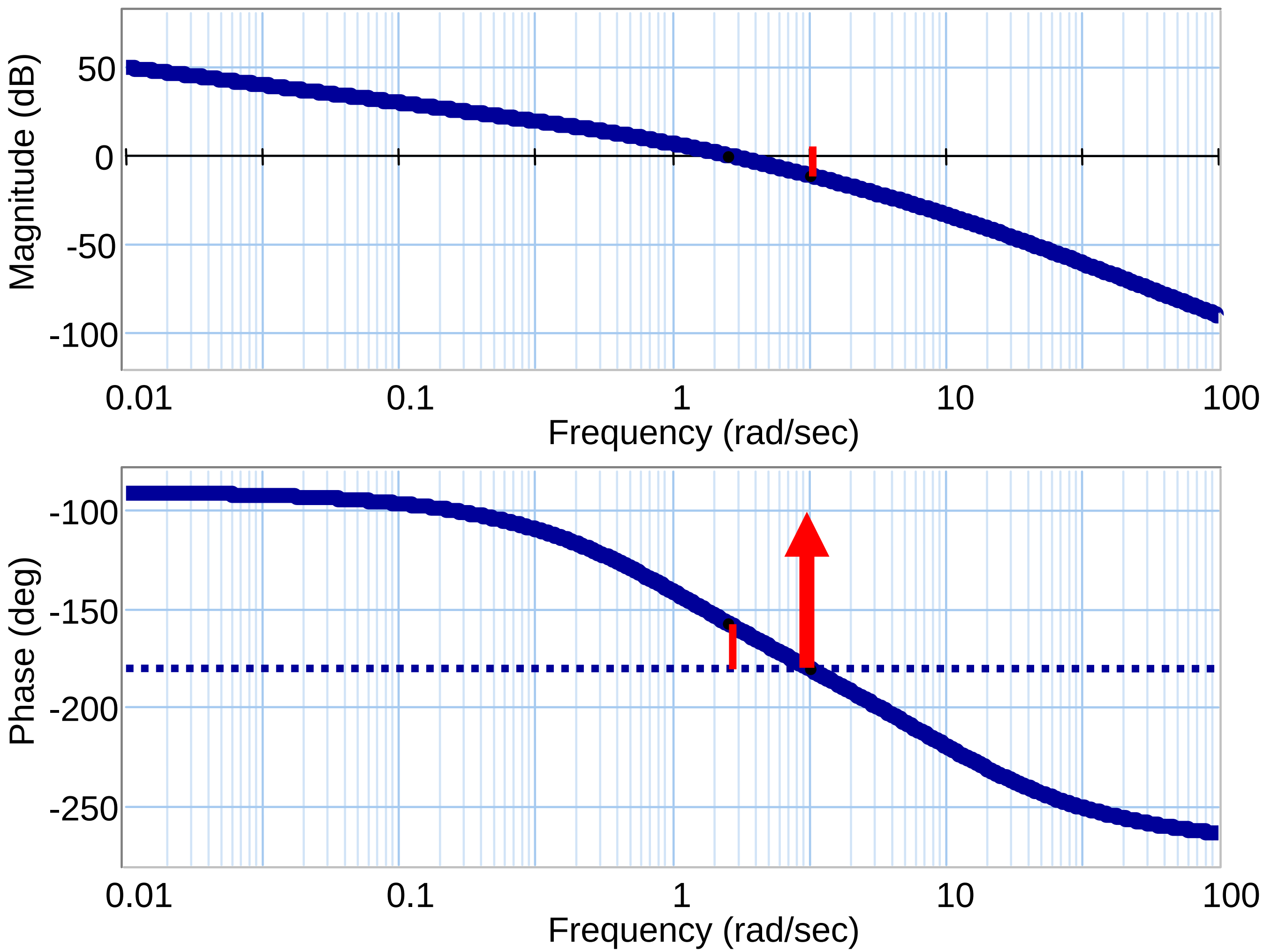
K = 0.5
(PM = 63°)

K = 5
(PM = 13°)

K = 5
+ Lag netw
(PM = 52°)

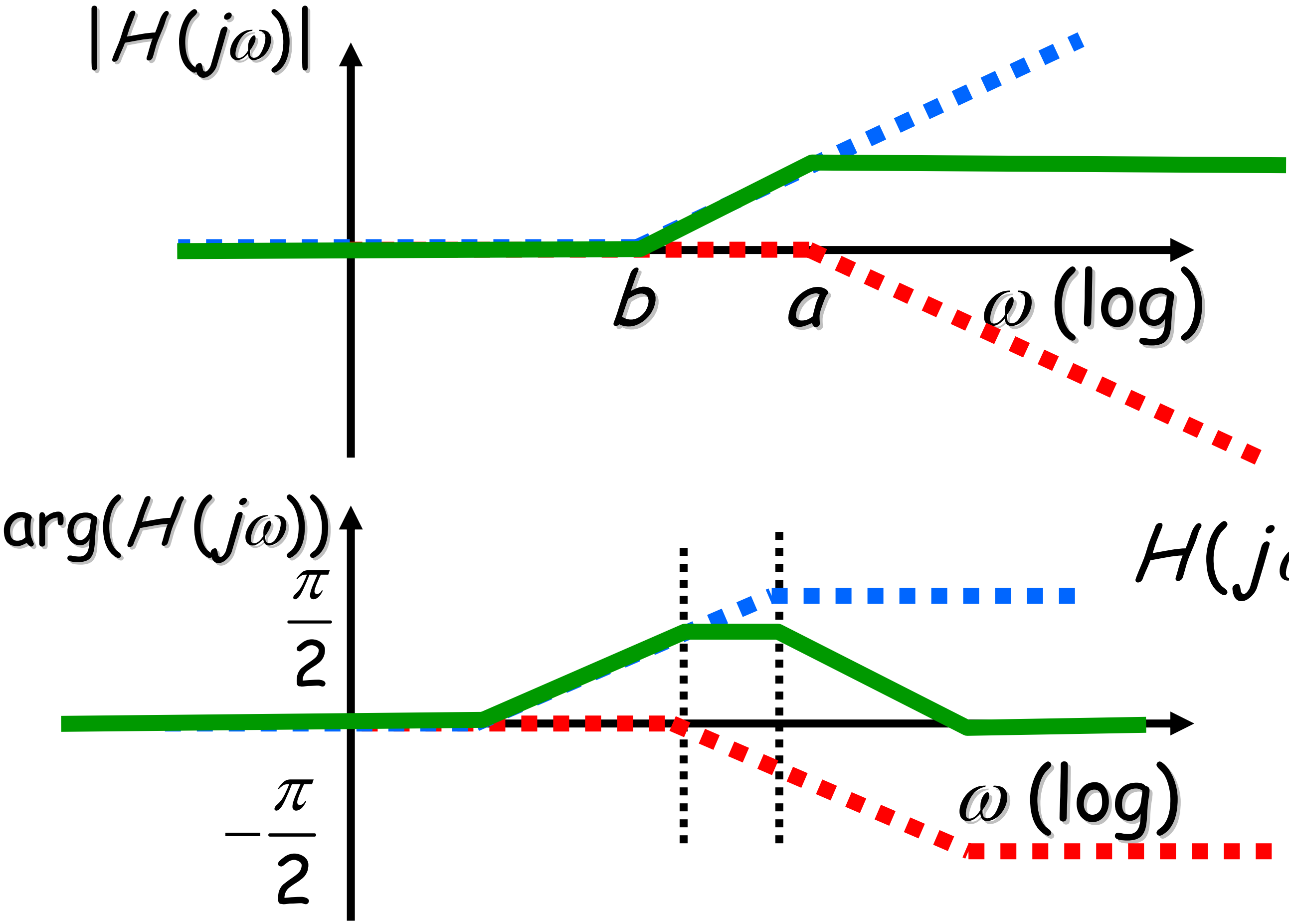


Decrease HF phase shift



2: decrease HF-phase shift

Combinations



Lead network

$$H(j\omega) = \frac{a}{b}\,\frac{j\omega + b}{j\omega + a}$$

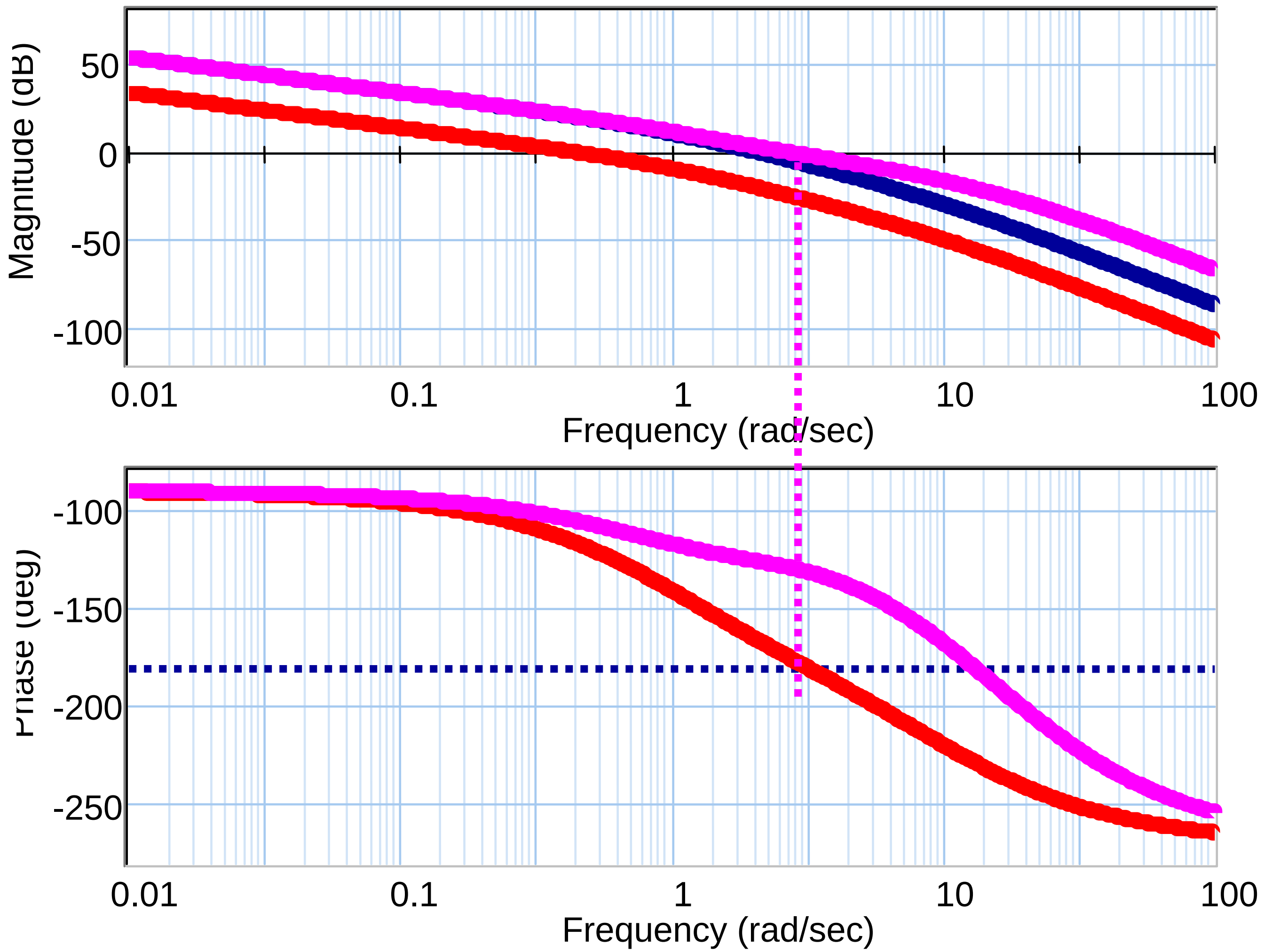
$$b < a$$

Lead network (phase lead)

K = 0.5
(PM = 63°)

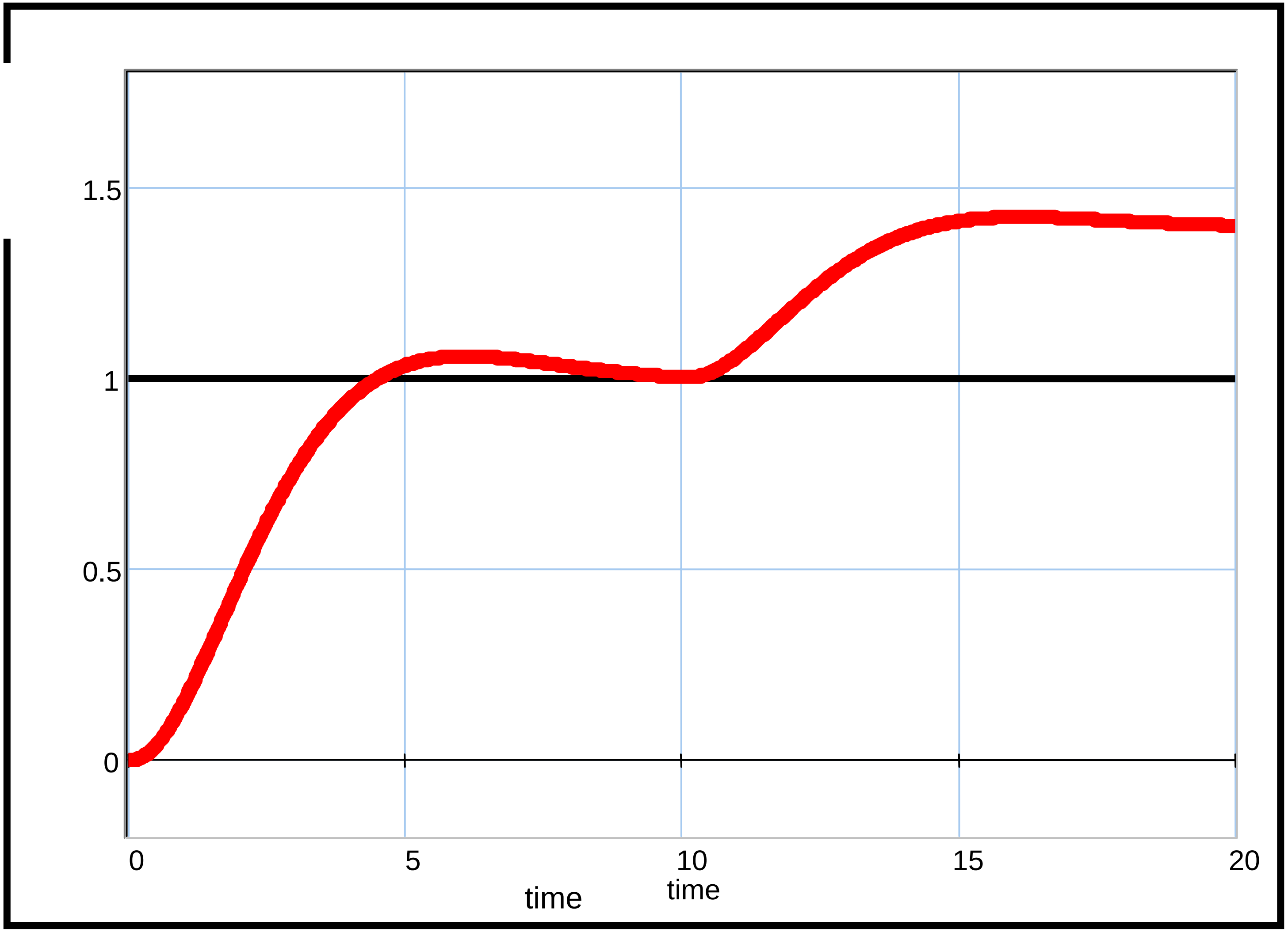
K = 5
(PM = 13°)

K = 5
+ Lead netw
(PM = 50°)



Responses

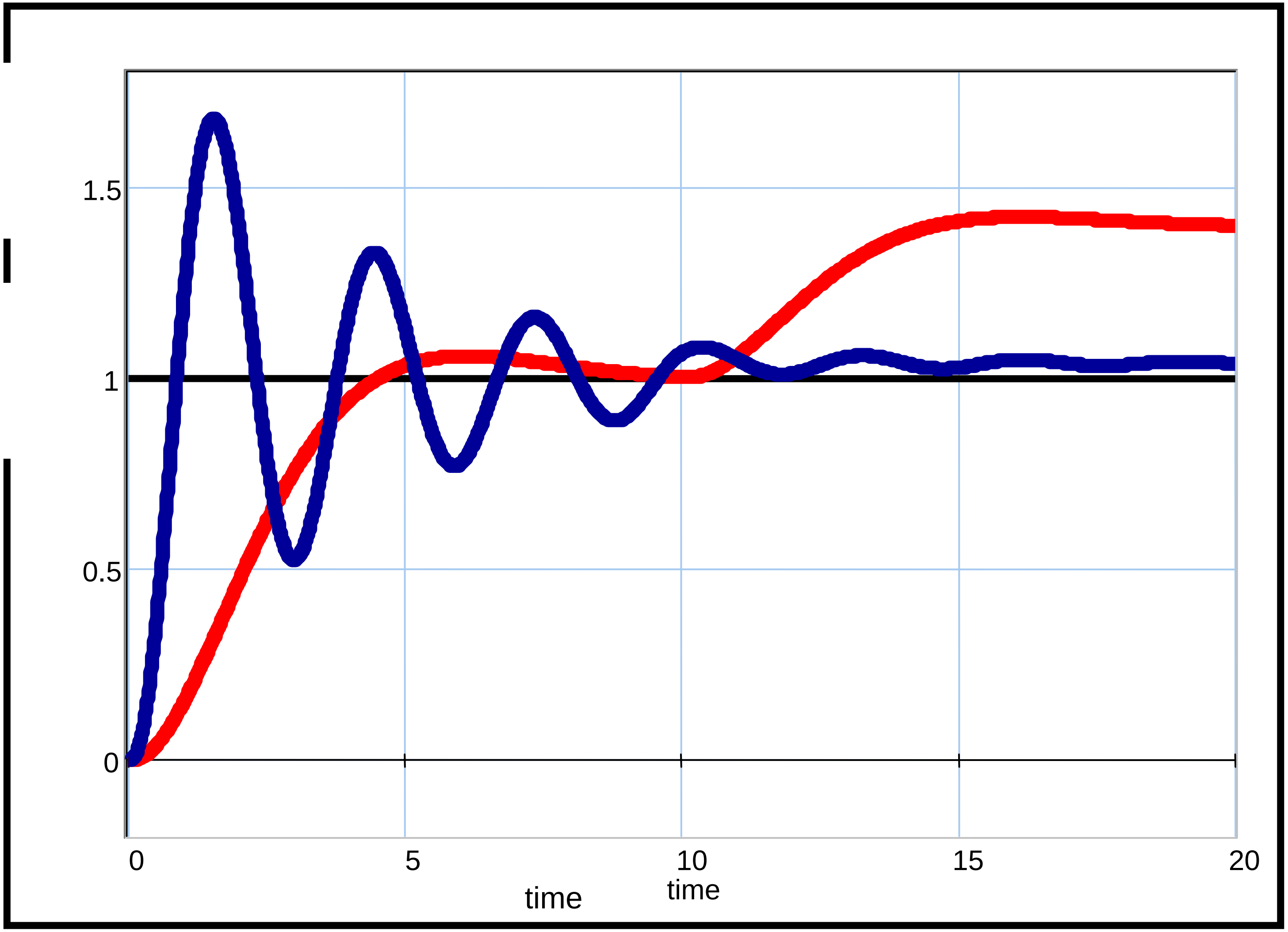
K = 0.5
(PM = 63°)



Responses

K = 0.5
(PM = 63°)

K = 5
(PM = 13°)



Responses

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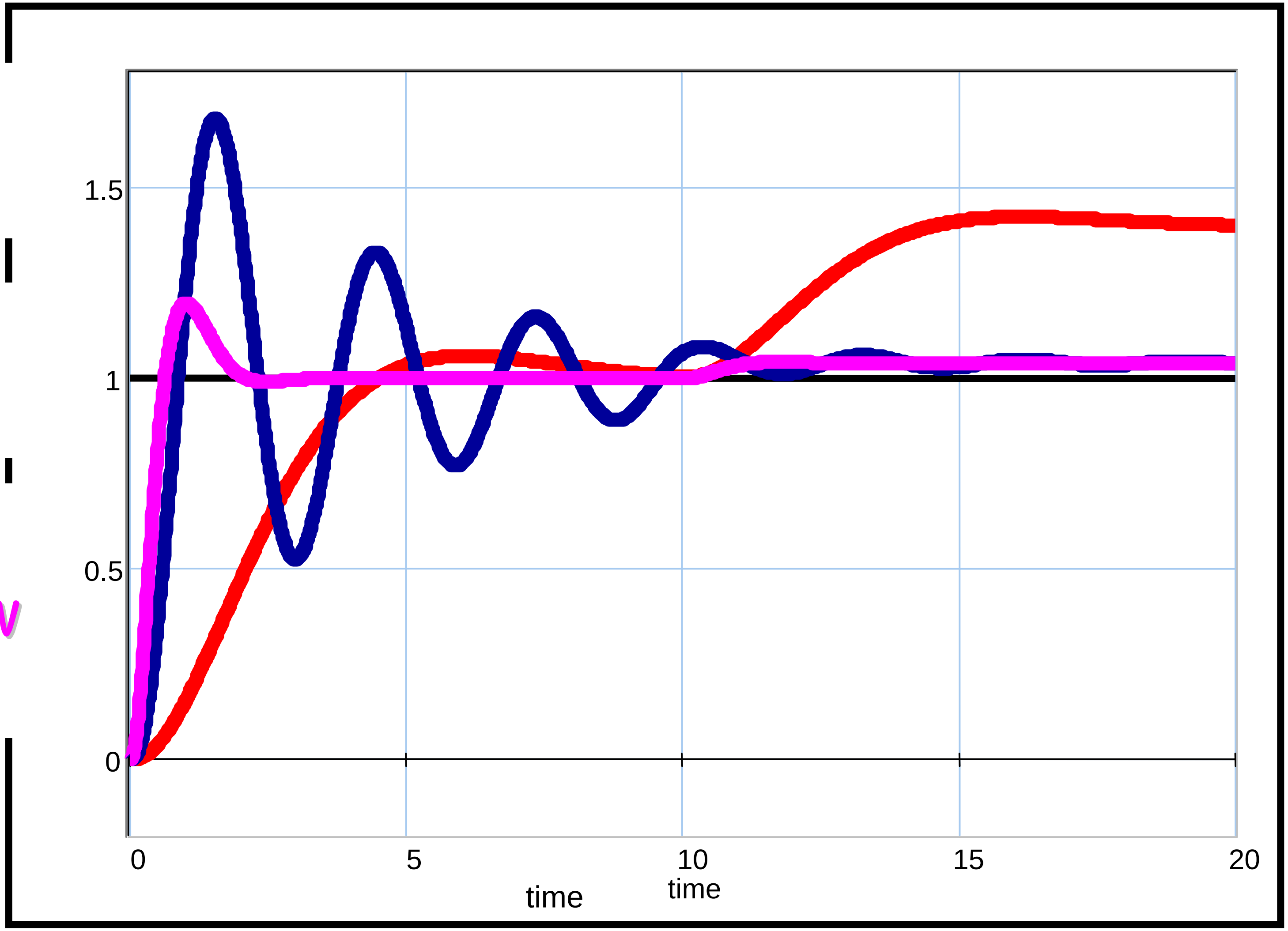
K = 0.5
(PM = 63°)

K = 5
(PM = 13°)

K = 5
+ Lead netw
(PM = 50°)



Conclusions

- Lag network:
  - dynamics approximately the same
  - (same bandwidth as low-gain system)
  - accuracy improved by increasing the low-frequency gain
- Lead network
  - Faster dynamics (increased bandwidth)
  - accuracy improved

Combination (Bode)

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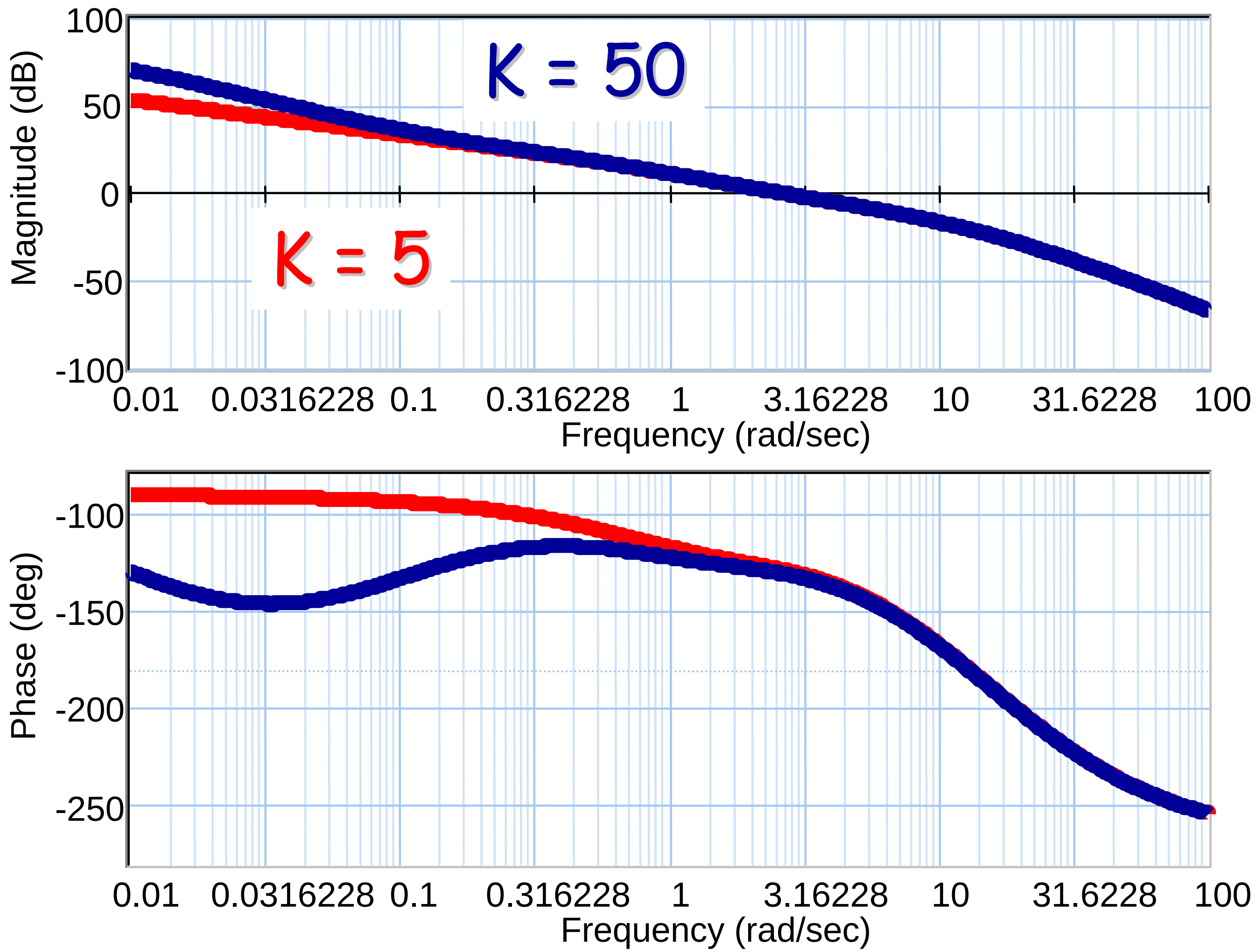
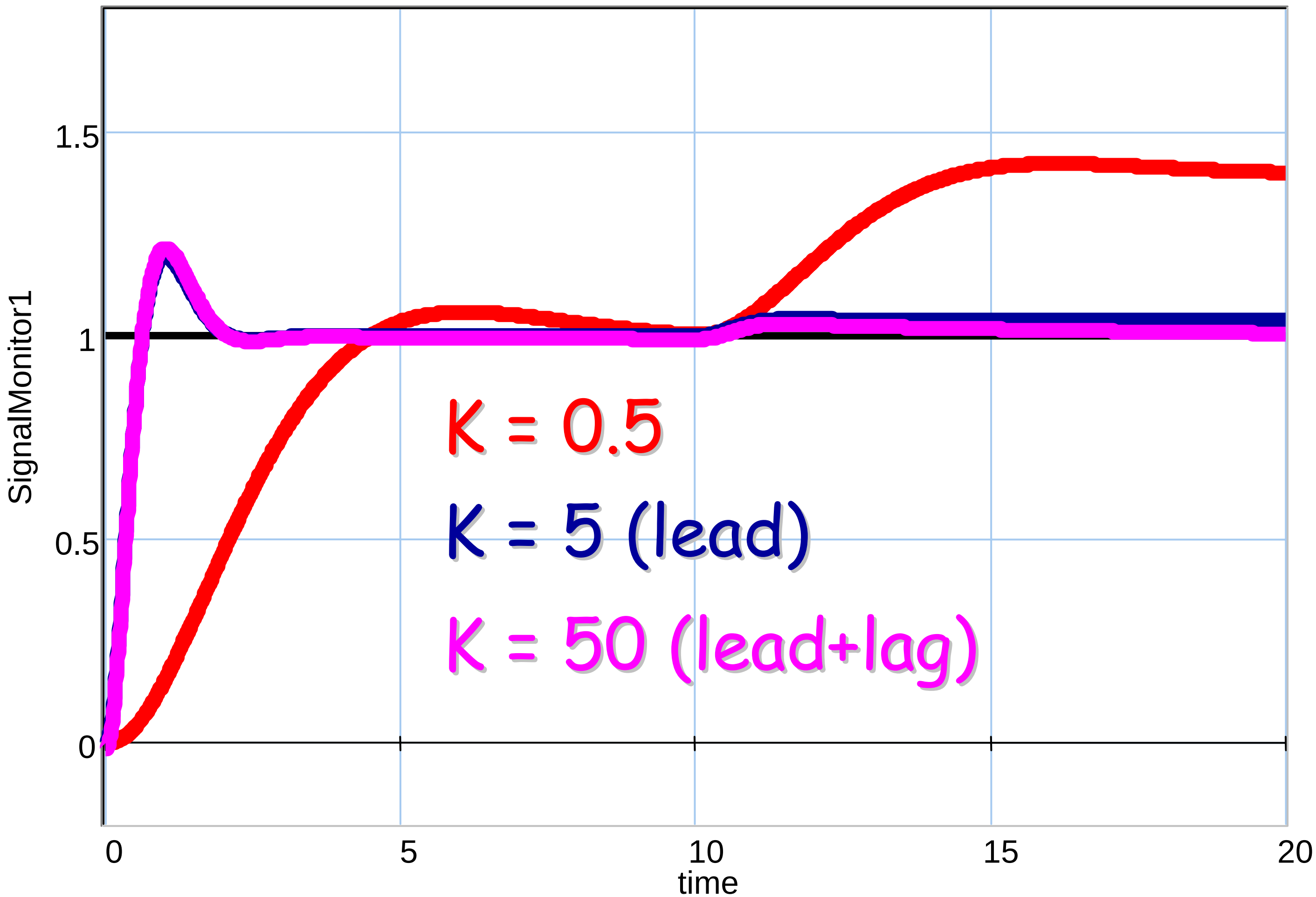
Lead

Lead + Lag



Combination (responses)



Accurate Design

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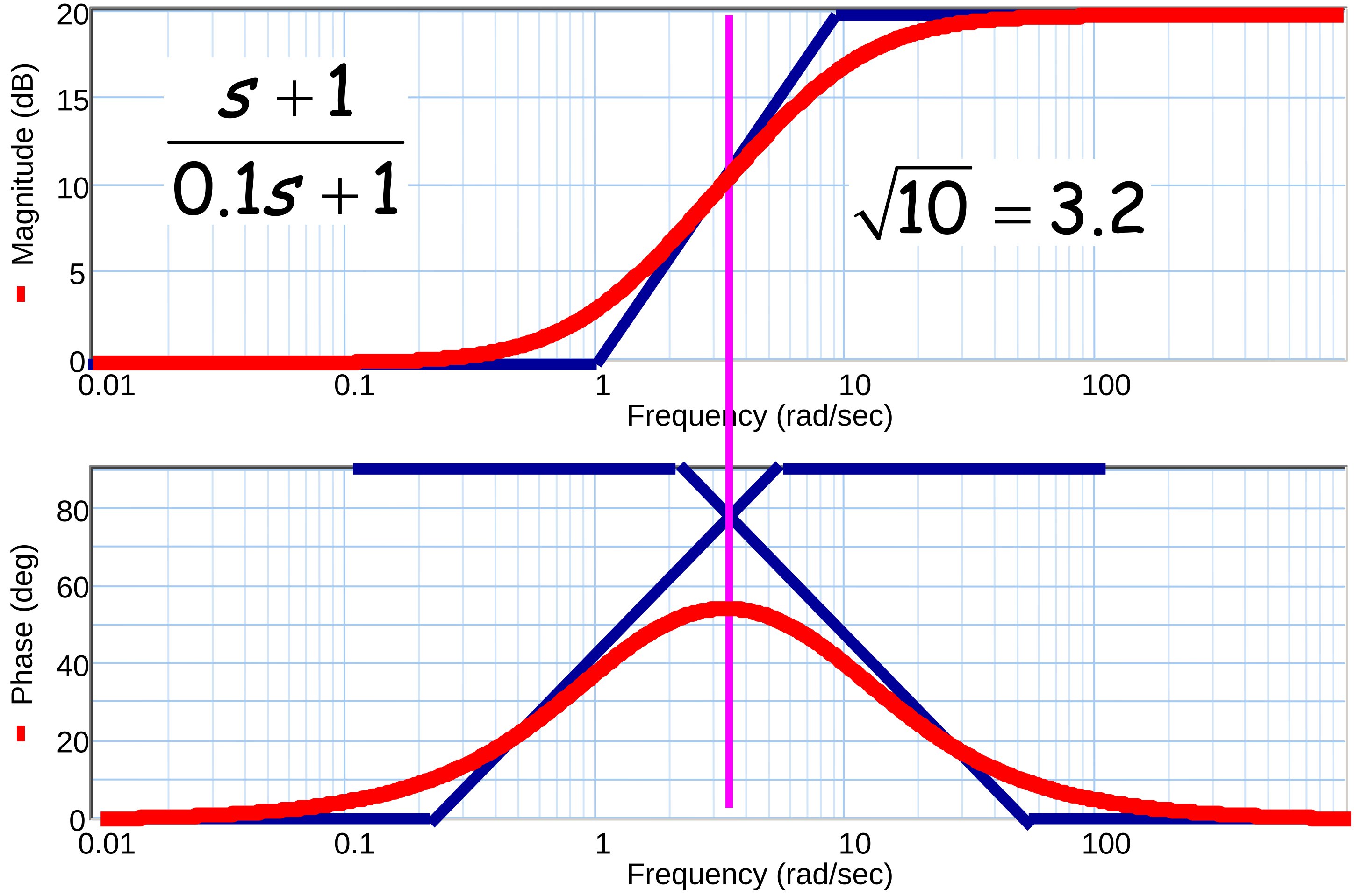
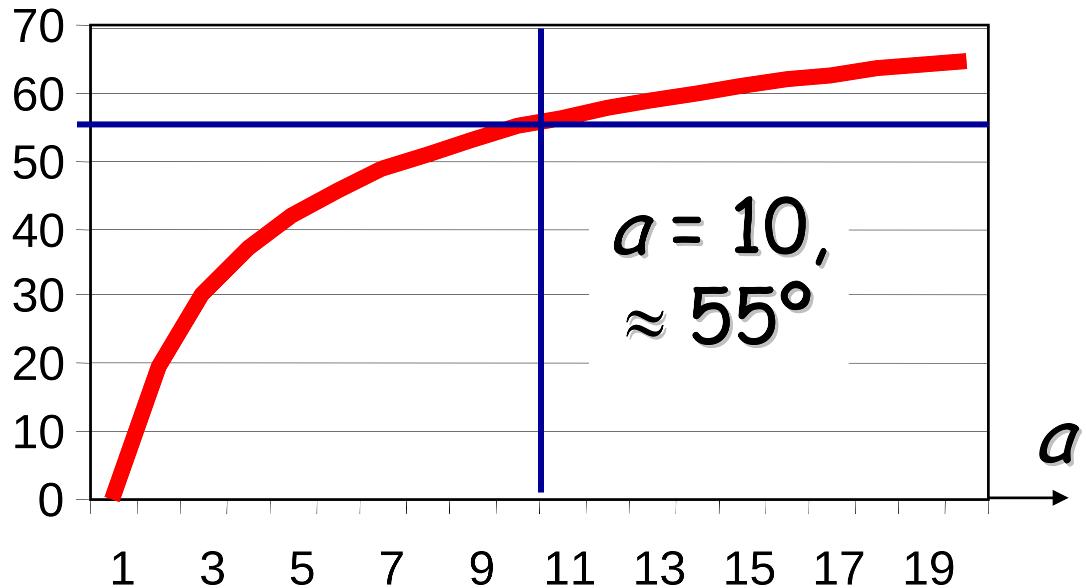


Accurate Design

φ_{max} [degree]



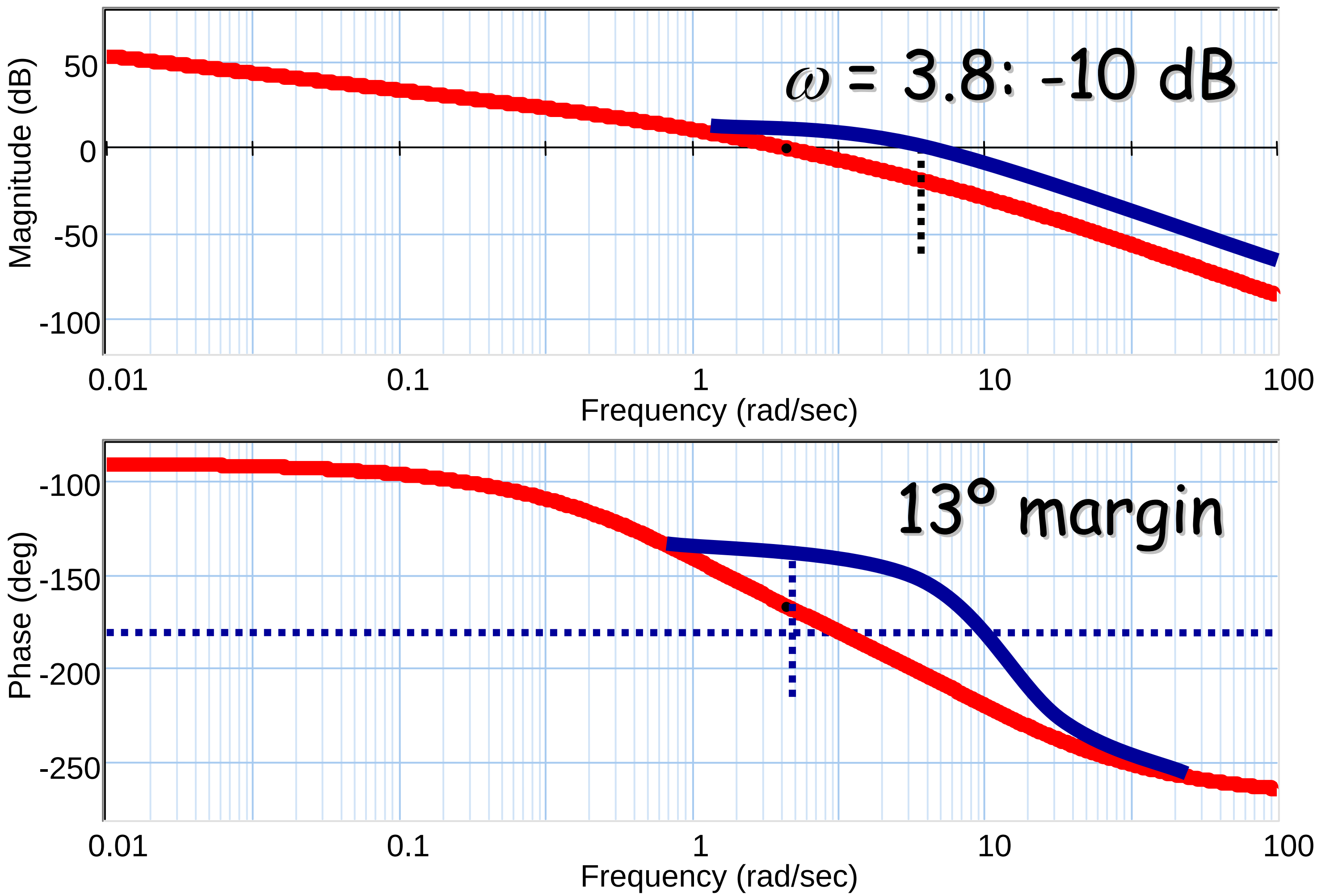
φ_{max} as a function of a

a > 10 gives only a little extra phase lead but amplifies high frequencies

Rules (1)

- Choose $a = 10$ by default
- Draw the bode plot for the desired gain
- The lead network gives 10 dB extra gain at φ_{max}
- We want φ_{max} at the new zero crossing of the modulus

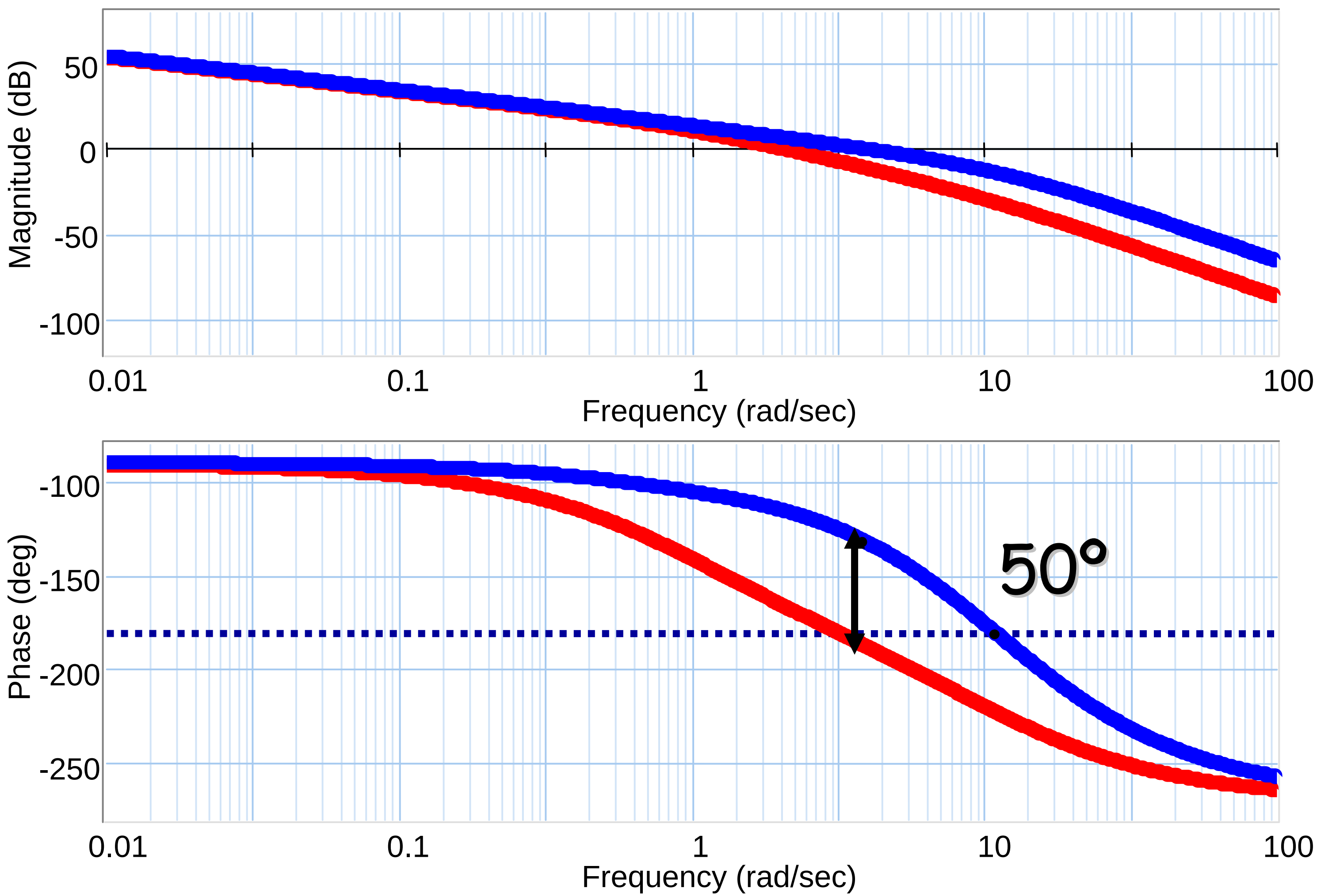
Accurate Design



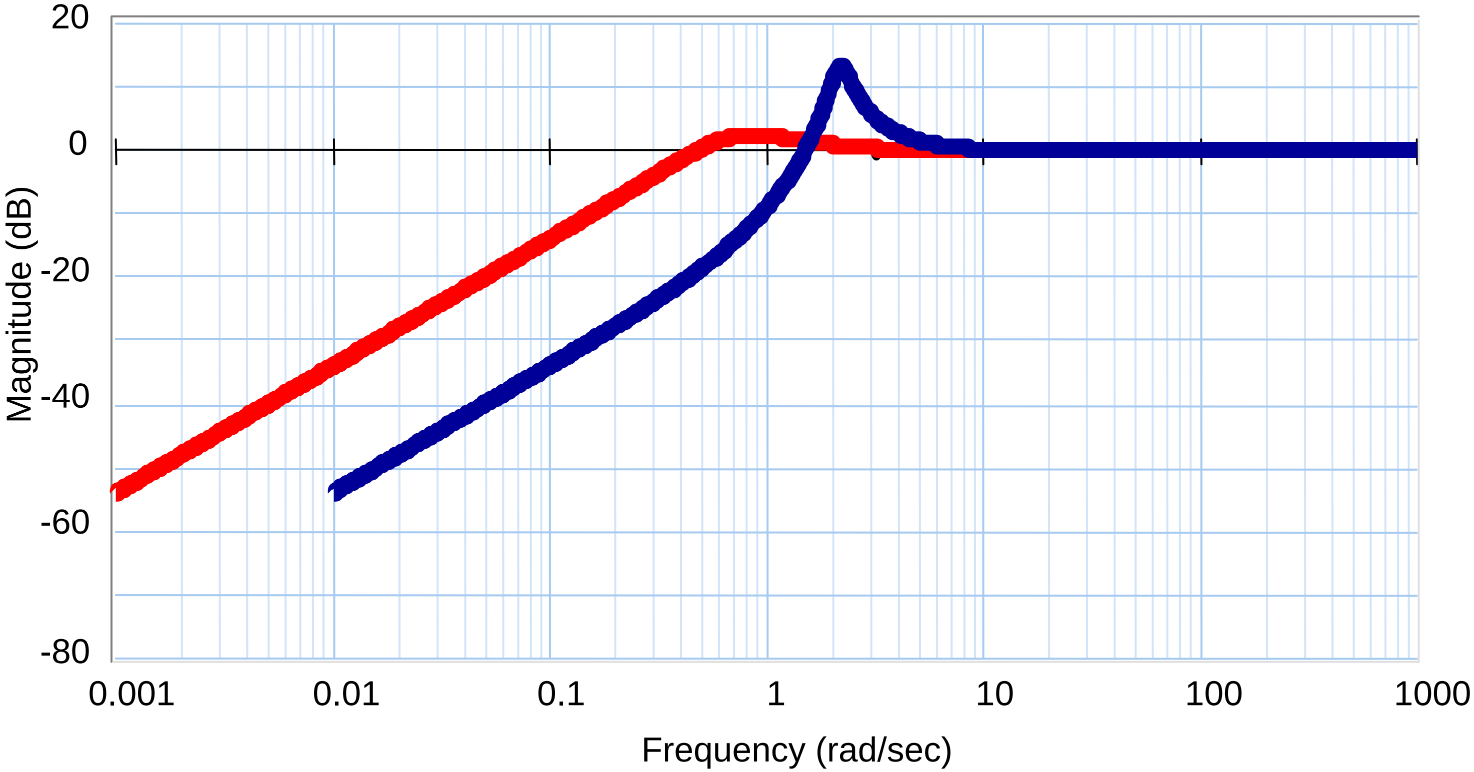
Rules (2)

- We want φ_{max} at the new zero crossing of the modulus $\omega = \omega_b$
- This implies that
  - zero should be located in $\omega = \omega_b / \sqrt{10}$
  - pole should be located in $\omega = \omega_b \cdot \sqrt{10}$
- with $\omega_b = 3.8$ it follows that
  - $\omega_b / \sqrt{10} = 1.2$
  - $\omega_b \cdot \sqrt{10} = 12$

Result

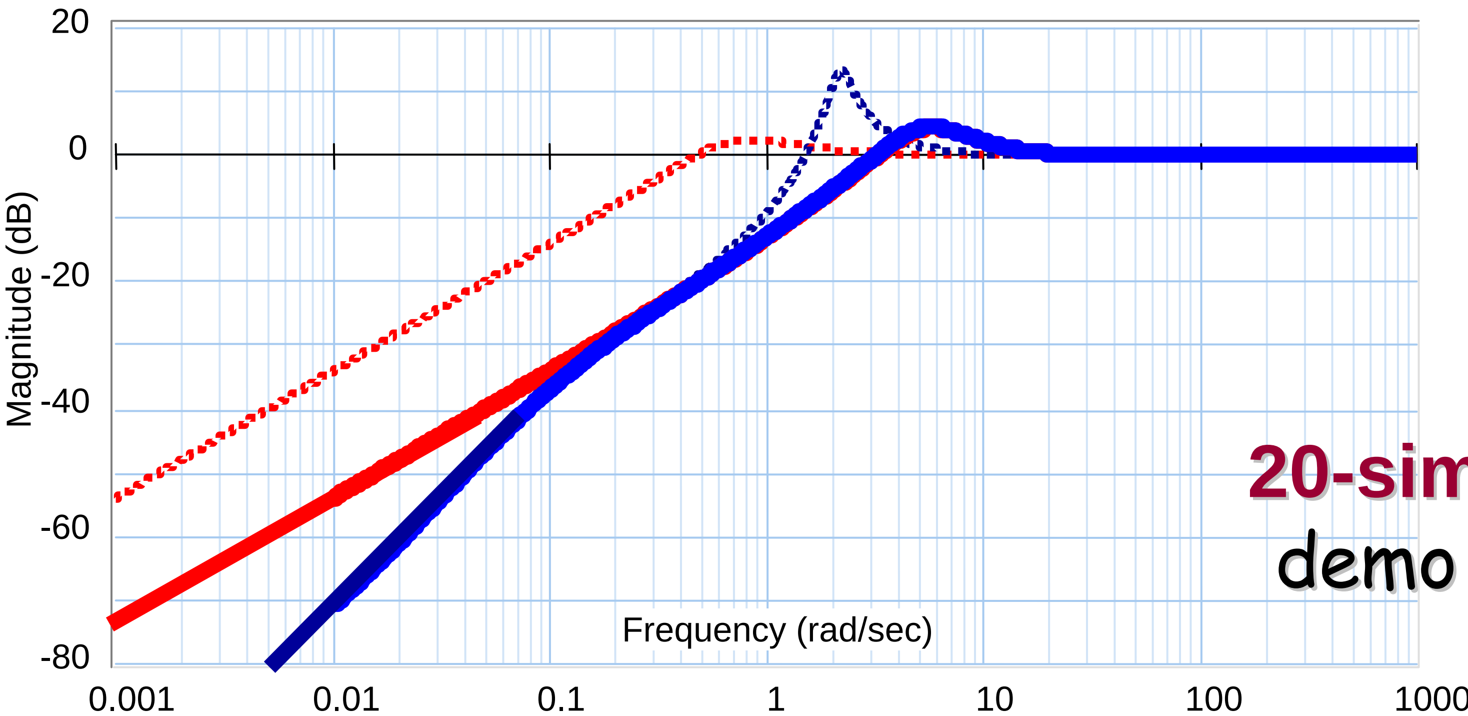

50
0
-50
-100
Magnitude (dB)
0.01
0.1
1
10
100
Frequency (rad/sec)
-100
-150
-200
-250
Phase (deg)
50°
0.01
0.1
1
10
100
Frequency (rad/sec)

Sensitivity



K = 0.5   K = 5

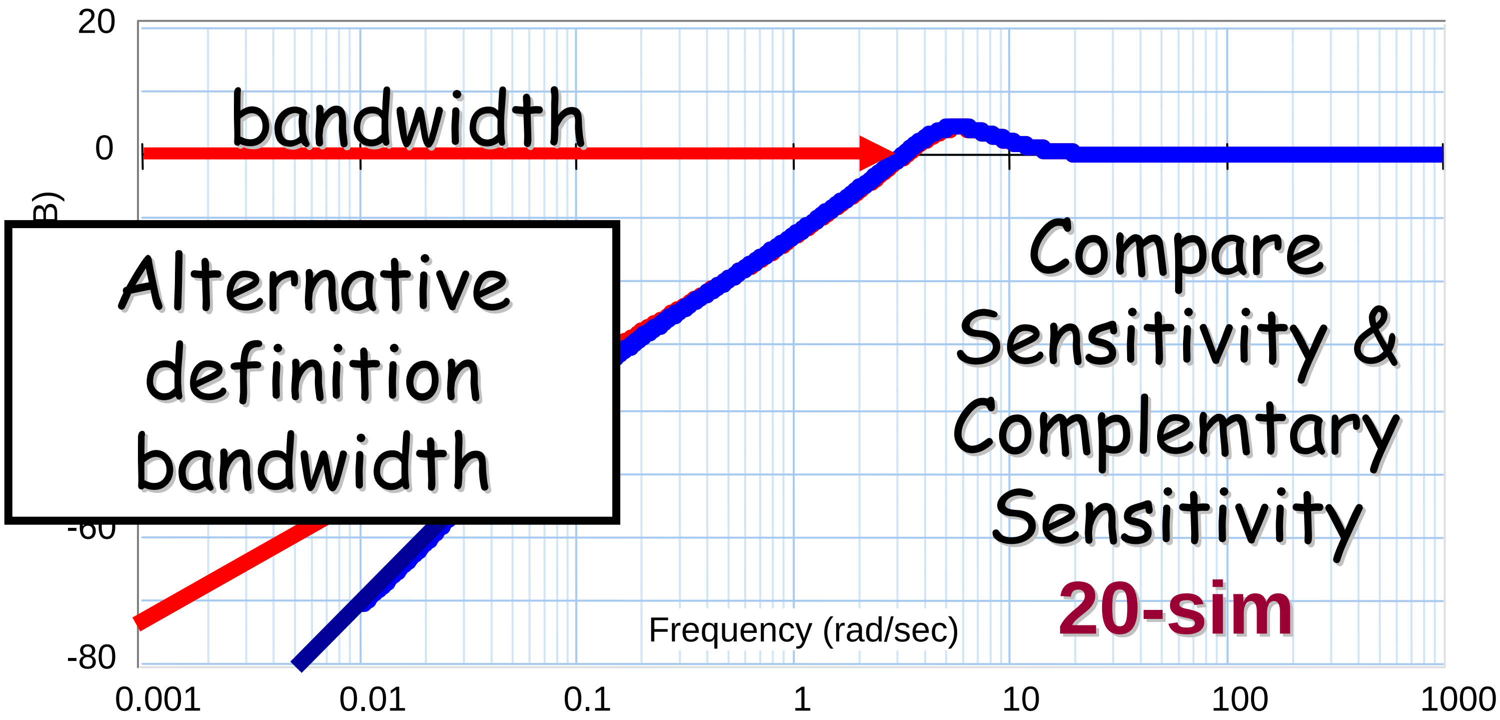
Sensitivity



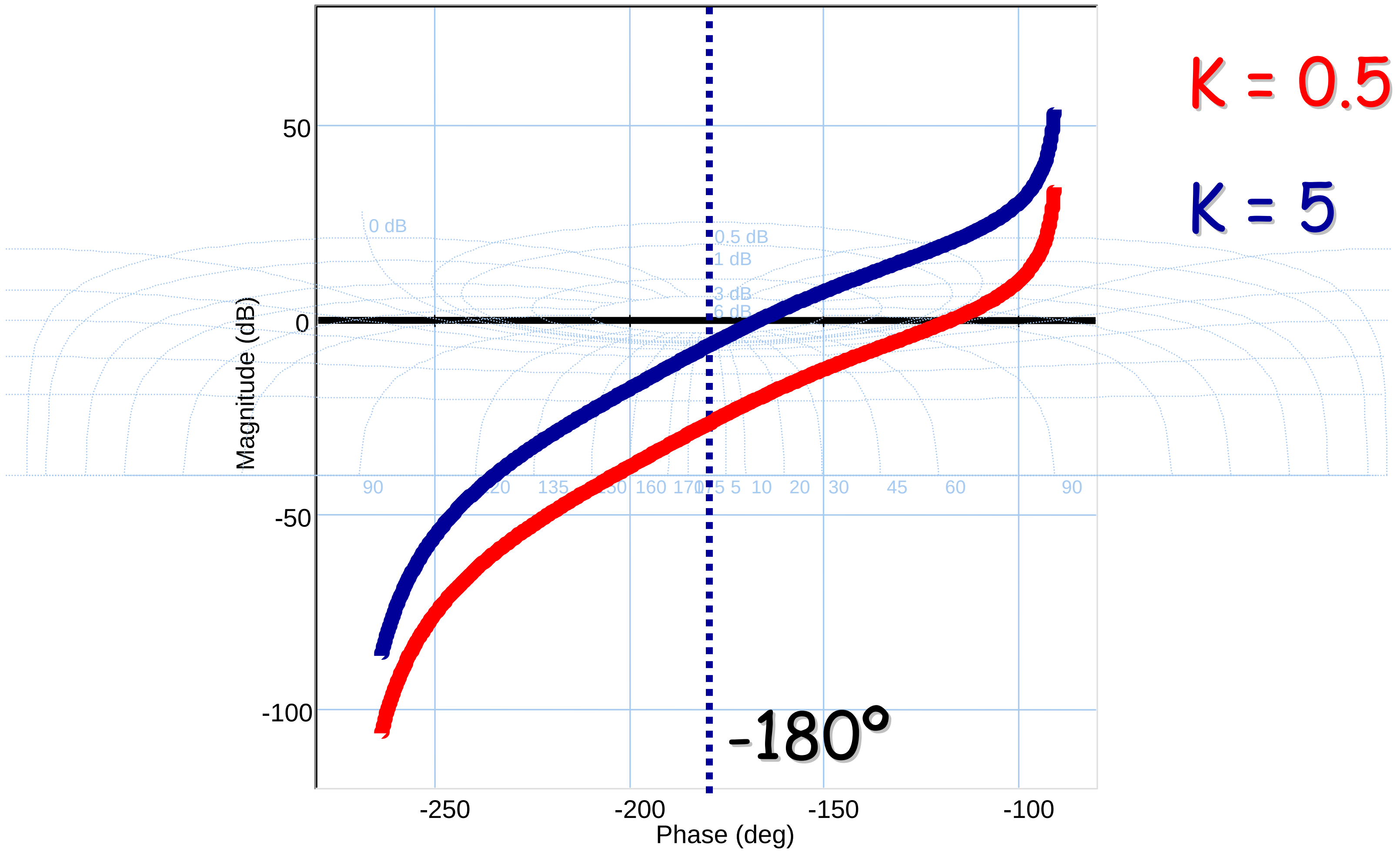
20-sim
demo

K = 0.5 K = 5 $5\,\frac{12}{1.2}\,\frac{s+1.2}{s+12}$ $10\,\frac{0.1}{0.01}\,\frac{s+0.1}{s+0.01}$

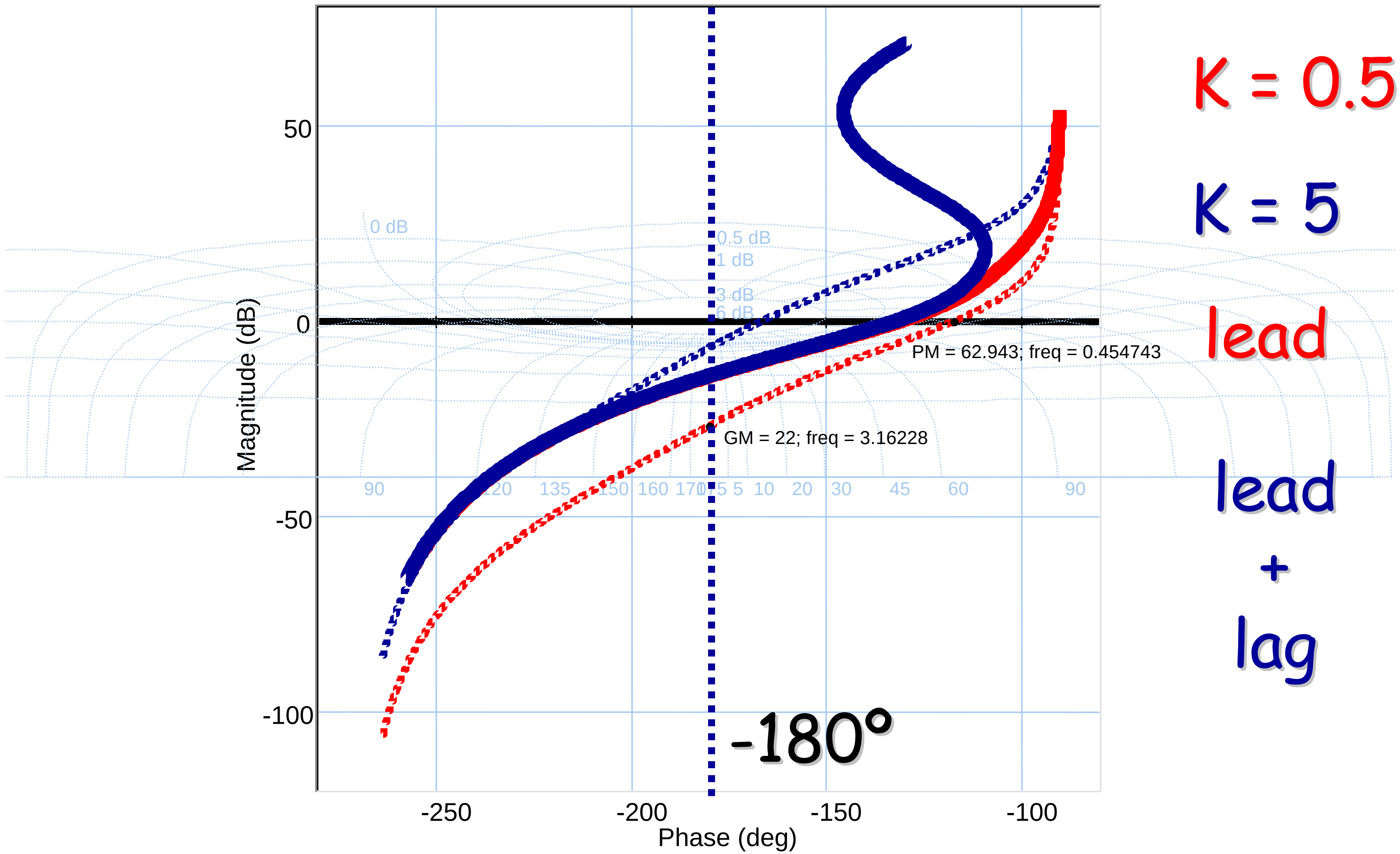
Sensitivity



Nichols


K = 0.5
K = 5
50
0
-50
-100
Magnitude (dB)
0 dB
0.5 dB
1 dB
3 dB
6 dB
90
135
160
170
5
10
20
30
45
60
90
-180°
-250
-200
-150
-100
Phase (deg)

Nichols



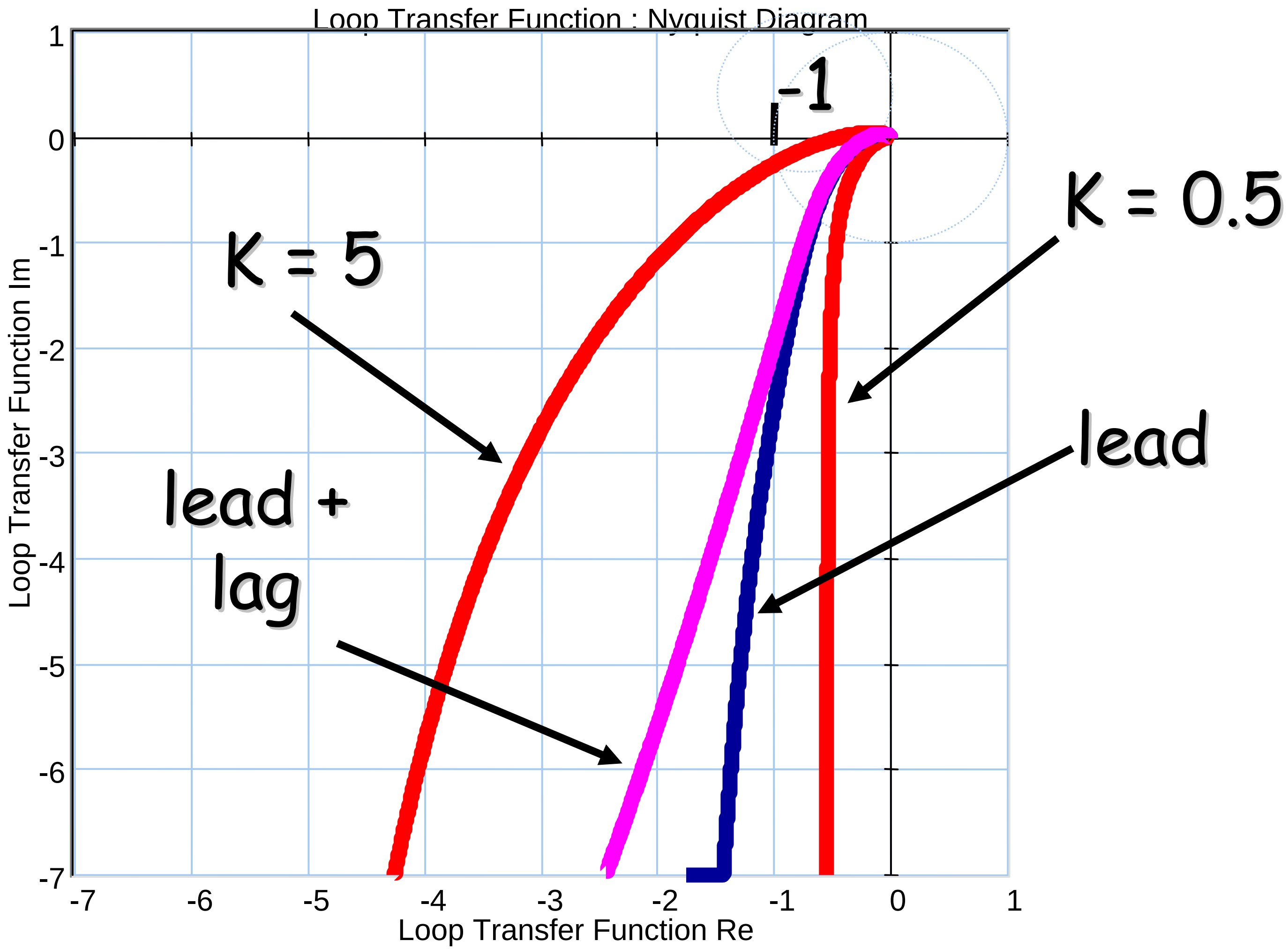
K = 0.5

K = 5

lead

lead
+
lag

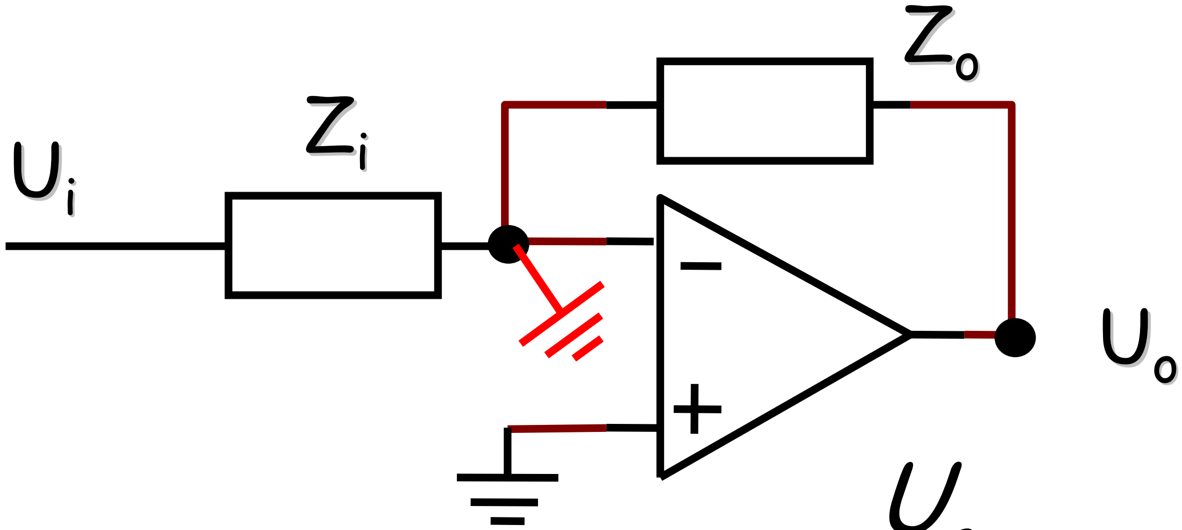
Nyquist


Loop Transfer Function : Nyquist Diagram
1
0
-1
-2
-3
-4
-5
-6
-7
-7
-6
-5
-4
-3
-2
-1
0
1
Loop Transfer Function Im
Loop Transfer Function Re
-1
K = 0.5
K = 5
lead
lead +
lag

Conclusions

- Compensation networks can improve the dynamic performance (transients) and/or the accuracy
- Lead networks: located in high-frequency region
- lag networks: located in low-frequency region

Op amp realisation



$$K \to \infty$$

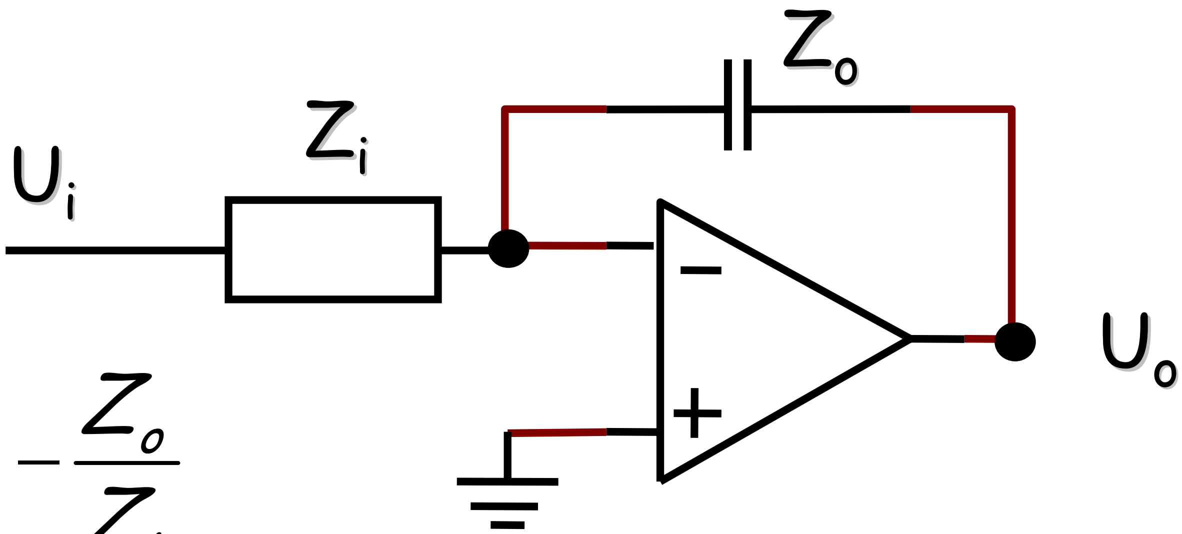
$$R_{in} \to \infty$$

$$R_{out} \to 0$$

$$\frac{U_o}{Z_o} = -\frac{U_i}{Z_i}$$

$$\frac{U_o}{U_i} = -\frac{Z_o}{Z_i}$$

Op amp realisation



$$\frac{U_o}{U_i} = -\frac{Z_o}{Z_i}$$

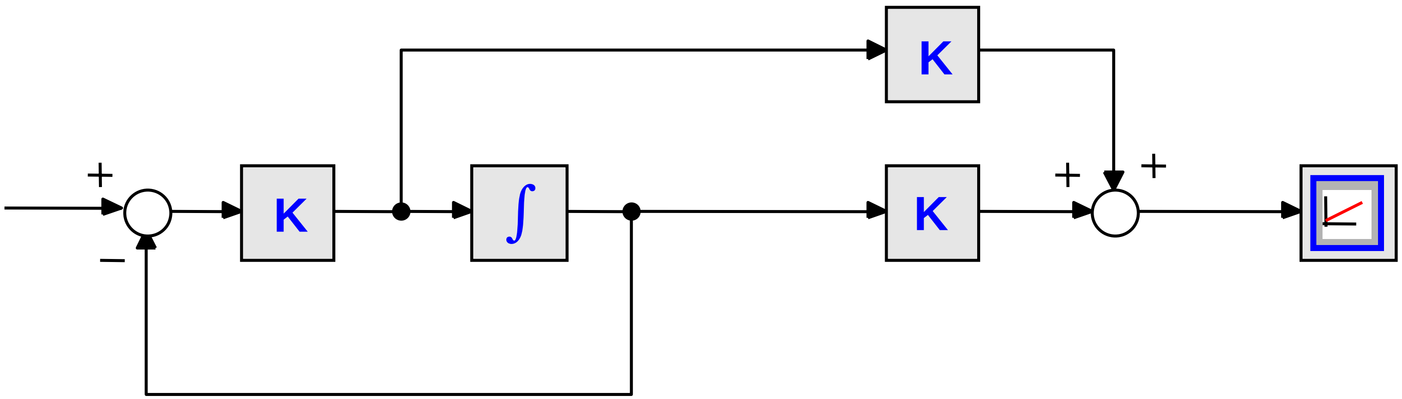
$$Z_o = \frac{1}{j\omega C}$$

$$R_i = R$$

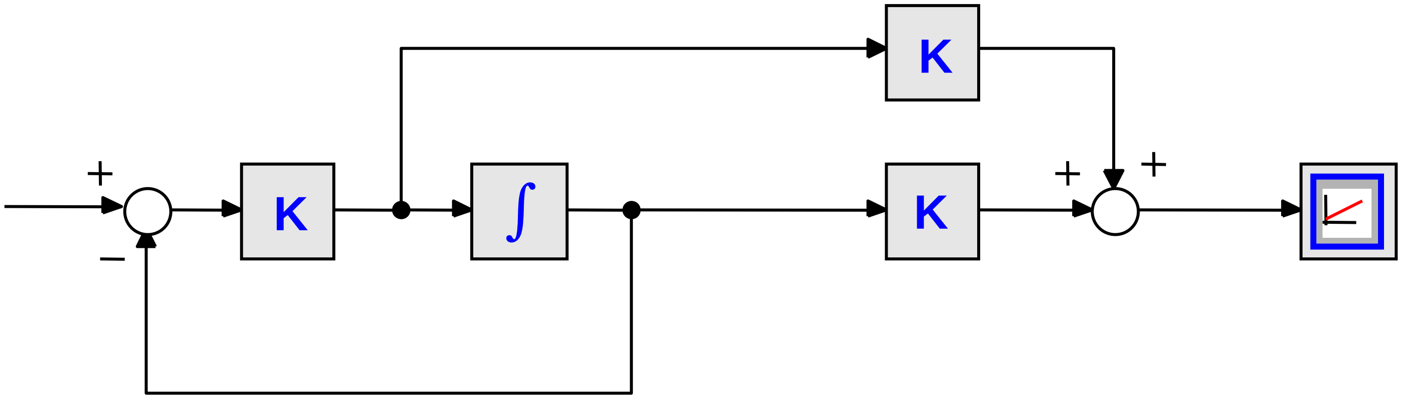
$$\frac{U_o}{U_i} = -\frac{1}{j\omega RC} = -\frac{1}{sRC} = -K\frac{1}{s}$$

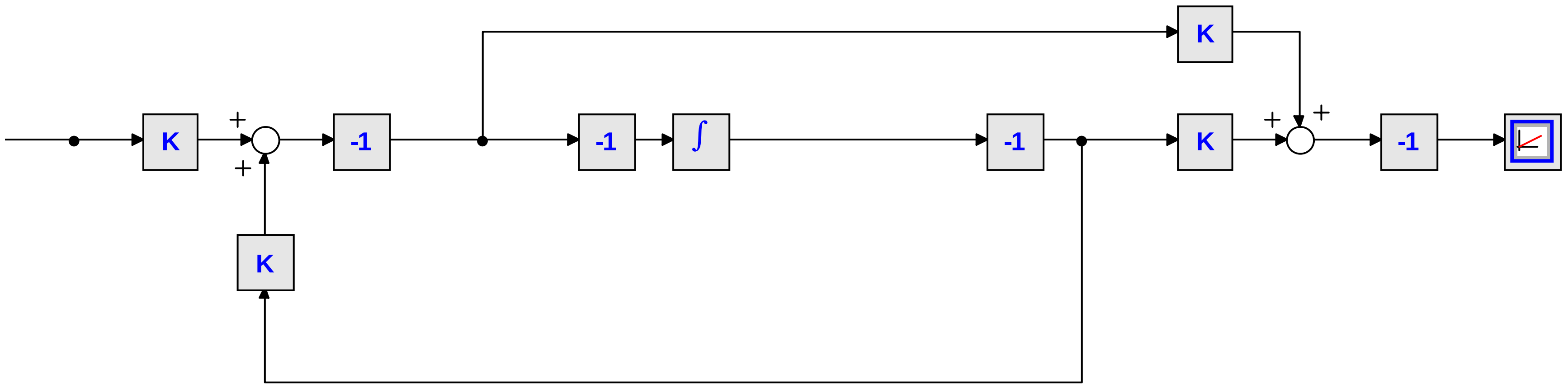
Lead/Lag network

$$K\frac{as\tau+1}{s\tau+1}=K\left(\frac{1}{s\tau+1}+s\frac{a\tau}{s\tau+1}\right)$$

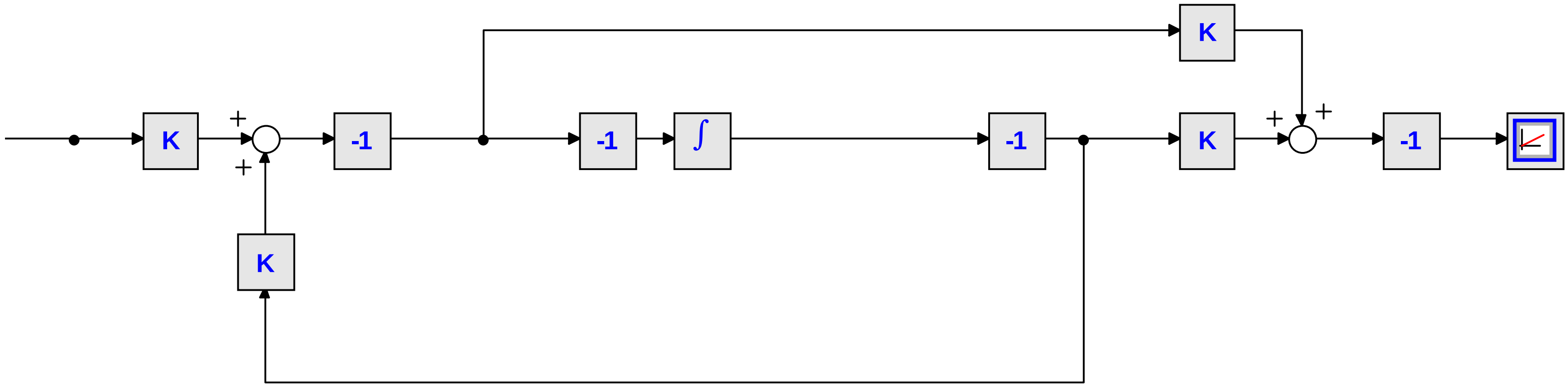


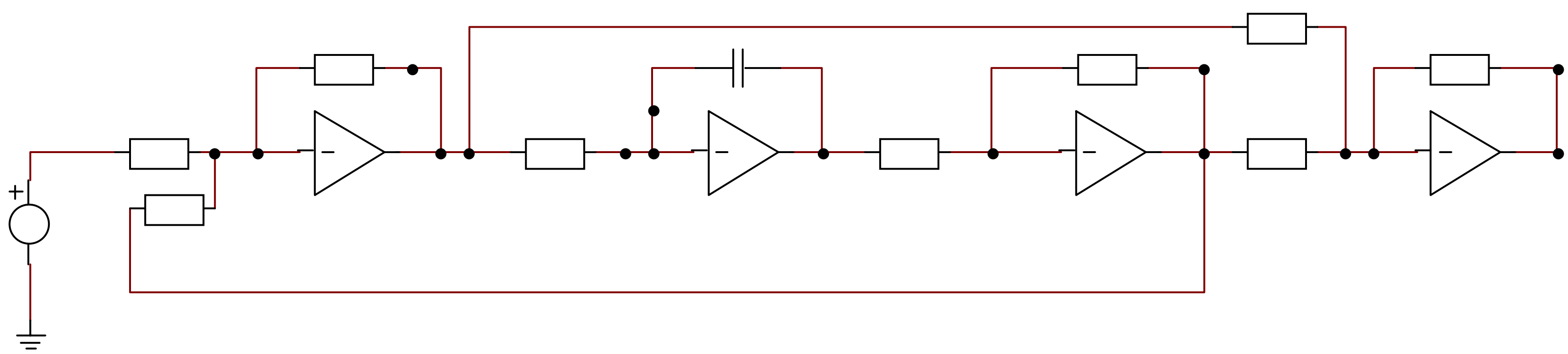
Lead/Lag network



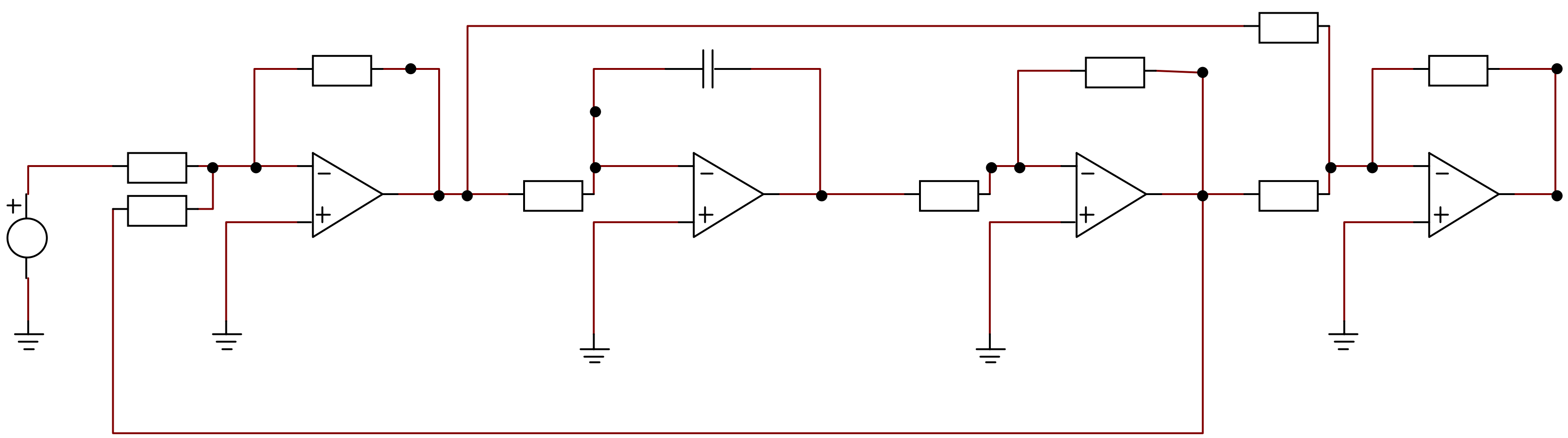


Lead/Lag network





Lead/Lag network



20 sim

opamp demo